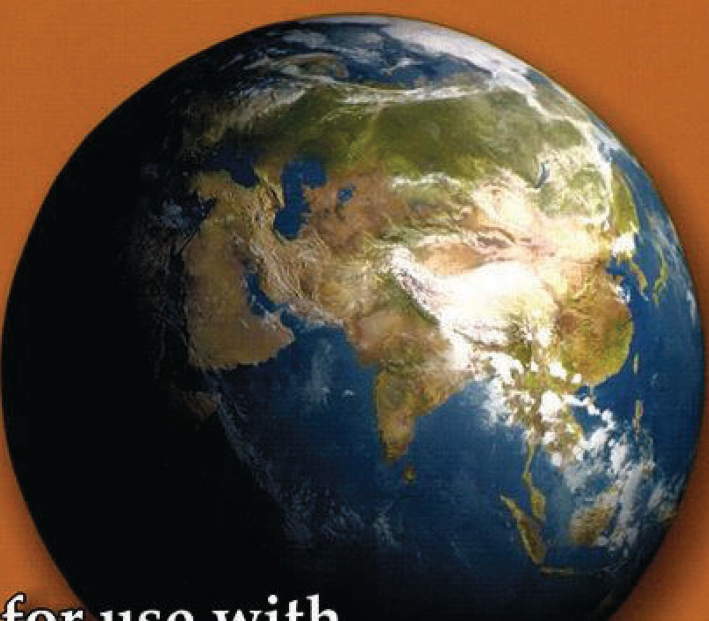
HAESE MATHEMATICS

Mathematics
for the international student

7
MYP 2

second edition



Michael Haese
Sandra Haese
Mark Humphries
Edward Kemp
Pamela Vollmar

for use with
IB Middle Years Programme

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Specialists in mathematics publishing

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MATHEMATICS FOR THE INTERNATIONAL STUDENT 7
MYP 2 second edition

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FOREWORD

MYP 2 second edition has been designed and written for the IB Middle Years Program (MYP) Mathematics framework.

This book may also be used as a general textbook at about 7th Grade level in classes where students complete a rigorous course in mathematics. We have developed this book independently of the International Baccalaureate Organization (IBO) in consultation with experienced teachers of IB Mathematics. The text is not endorsed by the IBO.

It is not our intention that each chapter be worked through in full. Teachers must select carefully, according to the abilities and prior knowledge of their students, to make the most efficient use of time and give as thorough coverage of content as possible.

Each chapter begins with an Opening Problem, offering an insight into the application of the mathematics that will be studied in the chapter. Important information and key notes are highlighted, while worked examples provide step-by-step instructions with concise and relevant explanations. Discussions, Activities, Investigations, Puzzles, and Research exercises are used throughout the chapters to develop understanding, problem solving, and reasoning, within an interactive environment.

We understand the emphasis that the IB MYP places on the six Global Contexts, and in response there are online links to ideas for projects and investigations to help busy teachers (see p. 6).

Frequent use of the interactive online features should nurture a much deeper understanding and appreciation of mathematical concepts. The inclusion of our Self Tutor software (see p. 4) is intended to help students who have been absent from classes or who experience difficulty understanding the material.

The book contains many problems to cater for a range of student abilities and interests, and efforts have been made to contextualise problems so that students can see the practical applications of the mathematics they are studying.

We welcome your feedback. Email: info@haesemathematics.com.au
Web: www.haesemathematics.com.au

PMH, SHH, MH, EK, PV

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ONLINE FEATURES

There are a range of interactive features which are available online.
With the purchase of a new hard copy textbook, you will gain 15 months subscription to our online product.
This subscription can be renewed annually for a small fee.

COMPATIBILITY

For iPads, tablets, and other mobile devices, the interactive features may not work. However, the electronic version of the textbook and additional chapters can be viewed online using any of these devices.

REGISTERING

You will need to register to access the online features of this textbook.
Visit www.haesemathematics.com.au/register and follow the instructions. Once you have registered, you can:

- activate your electronic textbook
- use your account to make additional purchases.

To activate your electronic textbook, contact Haese Mathematics. On providing proof of purchase, your electronic textbook will be activated. **It is important that you keep your receipt as proof of purchase.**

For general queries about registering and licence keys:

- Visit our Frequently Asked Questions page: www.haesemathematics.com.au/faq.asp
- Contact Haese Mathematics: info@haesemathematics.com.au

ONLINE VERSION OF THE TEXTBOOK

The entire text of the book can be viewed online, allowing you to leave your textbook at school.

SELF TUTOR

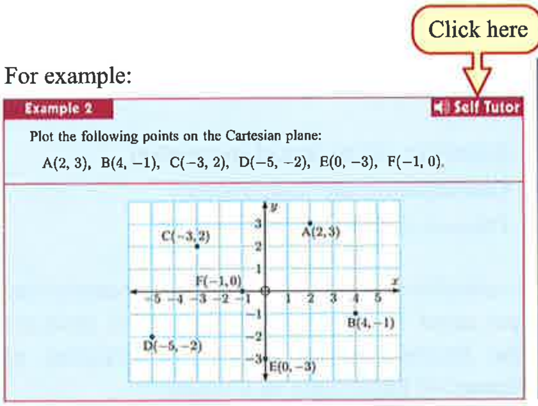
Self Tutor is an exciting feature of this book.

The Self Tutor icon on each worked example denotes an active online link.

Simply 'click' on the Self Tutor (or anywhere in the example box) to access the worked example, with a teacher's voice explaining each step necessary to reach the answer.

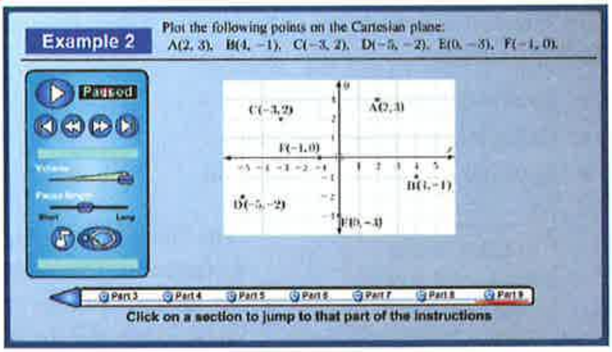
Play any line as often as you like. See how the basic processes come alive using movement and colour on the screen.

For example:



Plot the following points on the Cartesian plane:

A(2, 3), B(4, −1), C(−3, 2), D(−5, −2), E(0, −3), F(−1, 0).



Plot the following points on the Cartesian plane:
A(2, 3), B(4, −1), C(−3, 2), D(−5, −2), E(0, −3), F(−1, 0).

Click on a section to jump to that part of the instructions

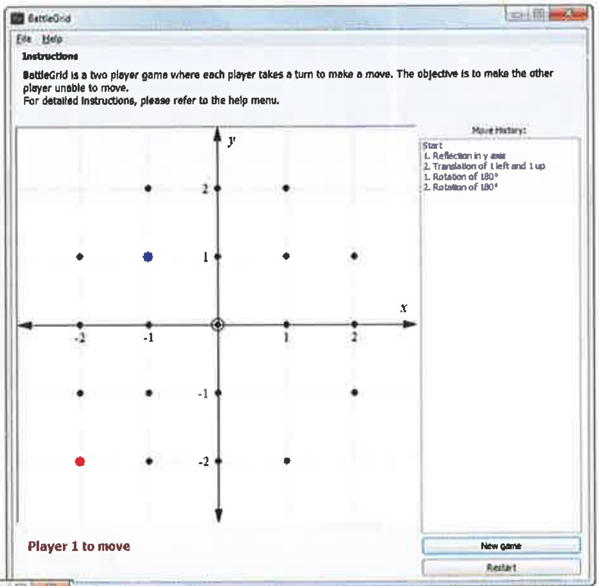
See **Chapter 12**, **Coordinate Geometry**, p. 256

INTERACTIVE LINKS

Throughout your electronic textbook, you will find interactive links to:

- Statistics packages
- Geometry packages
- Games
- Demonstrations
- Printable pages

CLICK ON THESE ICONS ONLINE

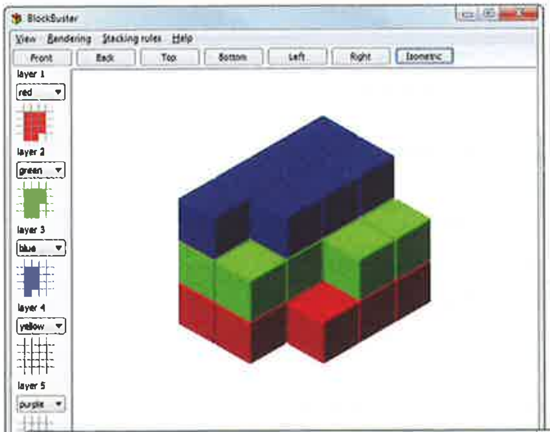


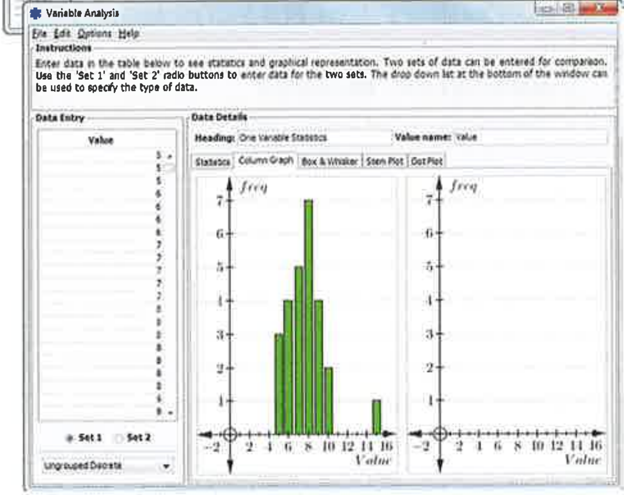
Instructions

BattleGrid is a two player game where each player takes a turn to make a move. The objective is to make the other player unable to move.
For detailed instructions, please refer to the help menu.

Start
1. Reflection in y axis
2. Translation of 1 left and 1 up
1. Rotation of 180°
2. Rotation of 180°

Player 1 to move

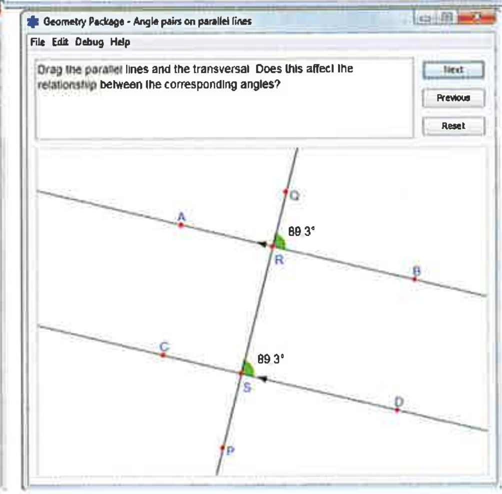




Instructions

Enter data in the table below to see statistics and graphical representation. Two sets of data can be entered for comparison. Use the 'Set 1' and 'Set 2' radio buttons to enter data for the two sets. The drop down list at the bottom of the window can be used to specify the type of data.

Heading: One Variable Statistics
Value name: Value



Drag the parallel lines and the transversal. Does this affect the relationship between the corresponding angles?

GLOBAL CONTEXTS

The International Baccalaureate Middle Years Programme focuses teaching and learning through six Global Contexts:

- Identities and relationships
- Orientation in space and time
- Personal and cultural expression
- Scientific and technical innovation
- Globalisation and sustainability
- Fairness and development

Click on the heading to access the online link.

The Global Contexts are intended as a focus for developing connections between different subject areas in the curriculum, and to promote an understanding of the interrelatedness of different branches of knowledge and the coherence of knowledge as a whole.

Global context

click here

Shikaku puzzles

Statement of inquiry:	Solving mathematical puzzles can help us to better understand mathematical concepts.
Global context:	Scientific and technical innovation
Key concept:	Logic
Related concepts:	Pattern, Measurement
Objective:	Investigating patterns
Approaches to learning:	Thinking, Social

There are six projects in this book, one for each of the Global Contexts:

Chapter 6:	Decimal numbers	p. 137	**LEAP YEARS** Orientation in space and time
Chapter 8:	Percentage	p. 178	**ELECTIONS** Fairness and development
Chapter 11:	Measurement: Length and area	p. 247	**SHIKAKU PUZZLES** Scientific and technical innovation
Chapter 14:	Ratio	p. 305	**NUTRITION** Identities and relationships
Chapter 16:	Solids	p. 337	**PAPERCRAFT AND POLYGON MODELS** Personal and cultural expression
Chapter 20:	Rates	p. 412	**POPULATION DENSITY** Globalisation and sustainability

Each project contains a series of questions, divided into:

- Factual questions (in green)
- Conceptual questions (in blue)
- Debatable questions (in red).

These questions should help guide the unit of work.

The projects are also accompanied by the general descriptor and a task-specific descriptor for each of the relevant assessment criteria, to help teachers assess the unit of work.

TABLE OF CONTENTS

EXTENSION QUESTIONS

Extension questions are marked in red.

Chapter 1

Whole numbers

Contents:

OPENING PROBLEM

A library has 17 bookcases labelled 1, 2, 3, ..., 17.

The even numbered bookcases have 4 shelves, while the odd numbered bookcases have 6 shelves.

Each shelf can hold 23 books.

Things to think about:

- **a** How many even numbered bookcases are there?
- **b** How many shelves are there, in total, in all the odd numbered bookcases?
- **c** How many books can an even numbered bookcase hold?
- **d** How many books can the library hold?
- **e** The book shelves are completely full. The average value of each book is $15. What is the total value of the books in the library?

All over the world, people use numbers. They are a vital part of our lives, and have been important to humans for thousands of years. To use numbers effectively, we need to understand the properties of numbers and the operations between them.

Over the ages, different people have created their own **number systems** to help them count. The Ancient Egyptians, Romans, and Greeks all used different symbols for their numbers, and helped to develop the more efficient systems we use today.

ACTIVITY 1 ROMAN NUMERALS

1. What do Roman numerals look like?
2. Why did the Romans use the symbols they used?
3. Where do we see Roman numerals today?
4. Write your age and the year of your birth in Roman numerals.
5. Use matchsticks to solve these puzzles:
   - **a** Move just one matchstick to make this correct: IV − II = V
   - **b** Move one matchstick to make this correct: III − II = IV
   - **c** Arrange 4 matchsticks to make a total of 15.
   - **d** Make this correct without removing any matchsticks: XI + I = X
   - **e** Remove 3 matchsticks to make this correct: VII + I = I

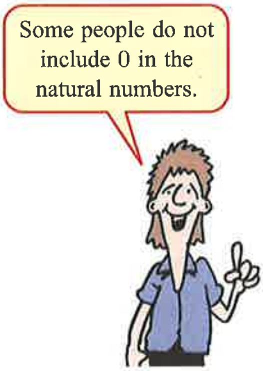
HISTORICAL NOTE

There are still many number systems in use around the world, but the most common is the **Hindu-Arabic** system. An early form of this system was established in ancient India around 3000 BC, and the first of the modern characters was developed about 2000 years ago. Use of the system slowly spread westwards, and in the 7th century AD it was adopted by the Arabs.

A THE NUMBER SYSTEM

When we write any number, we write some combination of the ten symbols: 1, 2, 3, 4, 5, 6, 7, 8, 9, and 0. These symbols are called **digits**. When we write these digits together, we form **numerals** that represent numbers.

The numbers 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, ... are called the **natural numbers**. They are also **whole numbers**, because they have no fractional or decimal part.



The set of natural numbers is endless. There is no largest natural number, so we say the set of all natural numbers is **infinite**.

There are three features of the Hindu-Arabic system that make it efficient:

- it uses only **10 digits** to construct all of the natural numbers
- it has a **place value system** where digits represent different numbers when placed in different place value columns
- it uses the digit **0** to show an empty place value.

PLACE VALUES

The **place** or position of a digit in a number determines its value.

For example, 5378 is really

5 thousand,		3 hundred	and	seventy		eight
5000	+	300	+	70	+	8

You should already be familiar with the place values:

units	1
tens	10
hundreds	100
thousands	1000

ten thousands	10 000
hundred thousands	100 000
millions	1 000 000
ten millions	10 000 000

Example 1 **Self Tutor**

a Write in numeral form the number "three thousand, two hundred and seven".

b What number is represented by the digit 6 in the numeral 1695?

a 3207 **b** six hundred or 600

EXERCISE 1A

1 Write in numeral form:

- **a** seventeen
- **b** sixty four
- **c** three hundred and twenty eight
- **d** eight hundred and ten
- **e** two thousand, nine hundred and one
- **f** five million, four hundred and two thousand, three hundred and ninety.

2 When writing out a *cheque* to pay a debt, the amount must be written in both numbers and words. Write the following amounts in words:

- **a** \$36
- **b** \$405
- **c** £6501
- **d** \$11 085
- **e** \$54 760
- **f** €285 400

3 What number is represented by the digit 7 in these numerals?

- **a** 47
- **b** 76
- **c** 217
- **d** 475
- **e** 1731
- **f** 7200
- **g** 3867
- **h** 271 000
- **i** 271 043
- **j** 7 105 839
- **k** 67 000 000
- **l** 146 070

4 Write the following numbers using numerals:

- **a** three more than nine
- **b** two less than twelve
- **c** one more than 100
- **d** 2 less than 3000
- **e** the largest two digit number.

5 Write the following quantities in order, beginning with the smallest:

- **a** one hundred and three dollars, \$130, \$113
- **b** Wendy 118 cm, Xiao 109 cm, Sarah 126 cm, Kylie 116 cm
- **c** giraffe 674 kg, hippopotamus 1872 kg, elephant 3058 kg, rhinoceros 2156 kg
- **d** Cologne 157 m, Rome 138 m, Milan 107 m, Salisbury 123 m
- **e** £4100, fourteen pounds, four thousand pounds, forty thousand pounds, fourteen thousand pounds.

Example 2 **Self Tutor**

a Express $50\,000 + 6000 + 70 + 4$ in simplest form.

b Write 6807 in expanded form.

a $50\,000 + 6000 + 70 + 4 = 56\,074$

b $6807 = 6000 + 800 + 7$

6 Express in simplest form:

- **a** $60 + 5$
- **b** $700 + 20 + 1$
- **c** $400 + 30$
- **d** $9000 + 80 + 4$
- **e** $50\,000 + 600 + 90$
- **f** $7\,000\,000 + 2000 + 60 + 3$

7 Write in expanded form:

- **a** 734
- **b** 3928
- **c** 21 080
- **d** 630 400

8

- **a** Use all of the digits 4, 2, and 5 once only to write the smallest number you can.
- **b** Write the largest number you can using the digits 6, 3, 0, 8, 2, and 5 once only.

B NUMBER STRATEGIES

There are strategies we can use to add, subtract, multiply, and divide numbers without the use of a calculator. With practice, you should be able to select the appropriate strategy then carry out the operations mentally.

STRATEGIES FOR ADDITION

Strategy	*Example*
1. Look for numbers which add to a multiple of 10. Change the order of addition if necessary.	$8 + 31 + 12$ $= \underbrace{8 + 12} + 31$ {rearrange} $= 20 + 31$ $= 51$
2. Make up one number to a multiple of 10. Balance using the other number.	$98 + 37$ $= (98 + 2) + (37 - 2)$ {$+2$ and -2 balance} $= 100 + 35$ $= 135$
3. Split one of the numbers into a multiple of 10 plus another number.	$23 + 39$ $= 23 + 30 + 9$ $= 53 + 9$ $= 62$

STRATEGIES FOR SUBTRACTION

Strategy	*Example*
1. Make both numbers larger or smaller by the same amount.	$77 - 29$ $= 78 - 30$ {add 1 to each number} $= 48$
2. Split the second number into a multiple of 10 and another number.	$87 - 45$ $= 87 - 40 - 5$ $= 47 - 5$ $= 42$

EXERCISE 1B.1

1 Find:

a $15 + 25$ **b** $25 + 17 + 15$ **c** $18 + 24 + 12$ **d** $8 + 259 + 92$

e $83 + 17 + 15$ **f** $39 + 16 + 14$ **g** $61 + 24 + 39$ **h** $137 + 342 + 63$

2 Find:

a $68 + 39$ **b** $103 + 46$ **c** $57 + 47$ **d** $1007 + 54$

e $207 + 88$ **f** $996 + 707$ **g** $2996 + 315$ **h** $148 + 86$

3 Find:

a $24 + 15$ **b** $33 + 18$ **c** $14 + 37$ **d** $35 + 26$
e $19 + 47$ **f** $67 + 24$ **g** $82 + 35$ **h** $78 + 44$

4 Find:

a $63 - 39$ **b** $80 - 21$ **c** $66 - 38$ **d** $94 - 47$
e $97 - 18$ **f** $114 - 26$ **g** $153 - 81$ **h** $196 - 88$

STRATEGIES FOR MULTIPLICATION

Strategy	*Example*
1. To multiply by powers of 10, multiply by the leading digit, then write the number of zeros which follow.	$8 \times 70\,000$ $= 8 \times 7 \times 10\,000$ $= 56 \times 10\,000$ {four zeros} $= 560\,000$ {write four zeros at the end}
2. To multiply three or more numbers we can change the order of multiplication.	$4 \times 31 \times 25$ $= 31 \times 4 \times 25$ $= 31 \times 100$ $= 3100$
3. Double one number and halve another.	14×3 $= (14 \div 2) \times (3 \times 2)$ $= 7 \times 6$ $= 42$
4. Split one number then add or subtract the two resulting products.	43×12 {12 lots of 43} $= (43 \times 10) + (43 \times 2)$ {10 lots of 43 plus 2 lots of 43} $= 430 + 86$ $= 430 + 80 + 6$ $= 516$ 65×98 {98 lots of 65} $= (65 \times 100) - (65 \times 2)$ {100 lots of 65 minus 2 lots of 65} $= 6500 - 130$ $= 6370$

EXERCISE 1B.2

1 Find:

a 7×40 **b** 60×8 **c** 4×110 **d** 9×700
e 600×12 **f** 5×2000 **g** 4000×5 **h** $9 \times 70\,000$

2 Find:

a $2 \times 13 \times 5$ **b** $4 \times 9 \times 25$ **c** $20 \times 5 \times 31$ **d** $3 \times 40 \times 5$
e $6 \times 7 \times 20$ **f** $8 \times 3 \times 25$ **g** $4 \times 7 \times 50$ **h** $8 \times 9 \times 125$

3 Find:

a 16×4
b 36×4
c 3×18
d 5×22
e 18×4
f 12×16
g 6×14
h 15×16

4 Find:

a 16×9
b 31×11
c 28×8
d 9×106
e 7×99
f 8×103
g 13×14
h 37×200
i 12×998
j 12×1005
k 16×2003
l 25×2997

STRATEGIES FOR DIVISION

Strategy	*Example*	
1. Look for a number which divides exactly into both numbers. Divide both numbers by it.	$98 \div 14$ $= 49 \div 7$ {dividing each number by 2} $= 7$	
2. Look for a multiple of the number we are dividing by which is close to the first number.	$27 \div 5$ $= (25 \div 5) + (2 \div 5)$ {25 is a multiple of 5 and is close to 27} $= 5 + \frac{2}{5}$ $= 5\frac{2}{5}$	
3. Split the first number then add or subtract the results of the two divisions.	$153 \div 3$ $= (150 \div 3) + (3 \div 3)$ $= 50 + 1$ $= 51$	$147 \div 3$ $= (150 \div 3) - (3 \div 3)$ $= 50 - 1$ $= 49$

EXERCISE 1B.3

1 Find:

a $40 \div 8$
b $72 \div 12$
c $70 \div 14$
d $56 \div 28$
e $117 \div 39$
f $208 \div 8$
g $216 \div 12$
h $408 \div 24$

2 Find:

a $16 \div 5$
b $29 \div 4$
c $46 \div 3$
d $28 \div 6$
e $205 \div 5$
f $186 \div 3$
g $294 \div 6$
h $217 \div 7$

ACTIVITY 2 STRATEGY TESTER

Click on the icon to run the strategy tester. Each set of questions should be done mentally using the strategies given in this Section.

Your score will be shown, and you will be invited to redo the questions you answered incorrectly.

STRATEGY TESTER

C ROUNDING NUMBERS

Often we do not need to know the exact value of a number, but rather we want a reasonable **estimate** or **approximation** of it.

For example, the Salt Lake Stadium in India holds about 120 000 people. This estimate gives a good idea of the stadium's size, when the exact capacity is not known or not required.

ROUNDING TO A POWER OF 10

We can round off numbers to the nearest power of ten.

For example, we can round off to the nearest 10, 100, or 1000.

157 is closer to 160 than to 150, so to round to the nearest 10, we **round up** to 160.

153 is closer to 150 than to 160, so to round to the nearest 10, we **round down** to 150.

We use the symbol $\approx$ to mean "is approximately equal to".

$$\text{So,}\quad 157 \approx 160 \quad\text{and}\quad 153 \approx 150.$$

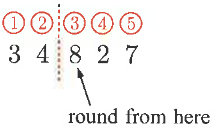
When a number is **halfway** between tens, we always **round up**. For example, $155 \approx 160$.

ROUNDING TO A NUMBER OF FIGURES

We round to a number of **significant figures** if we believe this number of digits is important.

For example, the number 34 827 has 5 significant figures.

If we round 34 827 to 2 significant figures, then we make each digit from the 3rd significant figure be zero.



Since the digit in the second place represents thousands, in this case rounding to 2 significant figures is equivalent to rounding to the nearest thousand.

34 827 is closer to 35 000 than it is to 34 000, so $34\,827 \approx 35\,000$ (to 2 significant figures).

> The rules for rounding off are:
>
> - If the digit **after** the one being rounded off is **less than 5** (0, 1, 2, 3, or 4), then we **round down**.
> - If the digit **after** the one being rounded off is **5 or more** (5, 6, 7, 8, or 9), then we **round up**.

Example 3 **Self Tutor**

Round off:

a 769 to the nearest 10 **b** 6705 to the nearest 100.

a 769 is closer to 770 than it is to 760, so $769 \approx 770$.

b 6705 is closer to 6700 than it is to 6800, so $6705 \approx 6700$.

Example 4 Self Tutor

Round off:

a 3143 to 1 significant figure **b** 15 579 to 2 significant figures.

a 3143 ≈ 3000 {the 2nd significant figure is 1, so round down}

b 15 579 ≈ 16 000 {the 3rd significant figure is 5, so round up}

EXERCISE 1C

1 Round off to the nearest 10:

a 62 **b** 43 **c** 68 **d** 127

e 99 **f** 232 **g** 305 **h** 9995

2 Round off to the nearest 100:

a 412 **b** 264 **c** 91 **d** 850

e 905 **f** 1952 **g** 18 726 **h** 25 870

3 Round off to the nearest 1000:

a 6218 **b** 2324 **c** 6587 **d** 607

e 13 500 **f** 9866 **g** 26 315 **h** 254 430

4 Round off to 1 significant figure:

a 46 **b** 205 **c** 394 **d** 467

e 863 **f** 1256 **g** 8888 **h** 49 580

5 Round off to 2 significant figures:

a 682 **b** 206 **c** 590 **d** 173

e 2019 **f** 3862 **g** 8973 **h** 16 638

6 Round off the value to the accuracy given:

a $3165 (to the nearest $100)

b a mass of 349 g (to 1 significant figure)

c an altitude of 4621 m (to the nearest 100 m)

d a crowd of 67 891 (to the nearest 1000)

e a car costs $26 990 (to 2 significant figures)

f an airport receives 695 flights each month (to the nearest 10).

7 From 2008 to 2012, 178 383 people completed the Berlin Marathon.
Round off this number to:

a the nearest 10 **b** 2 significant figures

c the nearest 1000 **d** 4 significant figures.

D ESTIMATION

To help find errors in a calculation, it is useful to accurately **estimate** the answer. The estimate will tell us if the computed answer is **reasonable**.

When estimating, we usually **round** each number to **one significant figure** and evaluate the result. We call this a **one figure approximation**.

Example 5 **Self Tutor**

Estimate the value of 7235×591.

We round each number to one significant figure.

$$\begin{aligned} 7235 \times 591 &\approx 7000 \times 600 \\ &\approx 4\,200\,000 \end{aligned}$$

The estimate tells us the correct answer should have 7 places in it.

We expect the answer to be about 4 million.

Example 6 **Self Tutor**

Estimate $3946 \div 79$.

$$\begin{aligned} &3946 \div 79 \\ \approx\ &4000 \div 80 && \{\text{rounding to one significant figure}\} \\ \approx\ &400 \div 8 && \{\text{dividing each number by } 10\} \\ \approx\ &50 \end{aligned}$$

EXERCISE 1D

1 Estimate using a one figure approximation:

a 22×394 **b** 218×74 **c** 58×3102

d 2360×21 **e** 3904×541 **f** 376×2308

g $77\,203 \times 409$ **h** $193 \times 21\,067$ **i** $38 \times 451\,807$

2 Estimate using a one figure approximation:

a $612 \div 29$ **b** $230 \div 54$ **c** $3864 \div 792$

d $8586 \div 299$ **e** $5890 \div 28$ **f** $7136 \div 19$

g $64\,183 \div 595$ **h** $38\,076 \div 822$ **i** $27\,830 \div 426$

3 A mathematics teacher gives one test a week to his class of 31 students. Estimate the number of tests the teacher will mark in 39 weeks.

4 There are 11 biscuits in a pack. Estimate how many biscuits are in a crate containing 138 packs.

5 A city car park has 9 floors, each with 79 spaces. Estimate how many parking spaces there are in total.

6 An avocado tree yields an average of 291 avocados in a season. Estimate the number of avocados that will be harvested from an orchard of 21 trees.

7 Hong Kong has an area of 1104 km^2. An average of 6349 people live in each square kilometre. Estimate the total population of Hong Kong.

8 559 eggs are divided equally between 43 baskets. Estimate the number of eggs in each basket.

9 For each of the following calculations:

- **i** estimate the answer using a one figure approximation
- **ii** use your estimate to determine whether the computed answer is reasonable.

	Calculation	*Computed answer*
a	833×6842	5 699 386
b	775×902	69 950
c	$12\,390 \div 21$	590
d	$9\,252\,250 \div 425$	2177

E OPERATING WITH NUMBERS

There are four **basic operations** that are carried out with numbers:

Addition + to find a **sum**
Subtraction − to find a **difference**
Multiplication × to find a **product**
Division ÷ to find a **quotient**

In this Section we learn some important words associated with these operations. We also practise the strategies for these operations learnt earlier in the chapter.

SUMS AND DIFFERENCES

To find the **sum** of two or more numbers, we *add* them.

For example, the sum of 3 and 16 is $3 + 16 = 19$.

To find the **difference** between two numbers, we *subtract* the smaller from the larger.

For example, the difference between 3 and 16 is $16 - 3 = 13$.

When we add or subtract **zero** (0), the number remains unchanged.

For example, $23 + 0 = 23$, $23 - 0 = 23$.

Example 7 **Self Tutor**

Find: **a** the sum of 187, 369, and 13
b the difference between 37 and 82.

a $187 + 369 + 13$
$= 187 + 13 + 369$
$= 200 + 369$
$= 569$

b The difference between 37 and 82
$= 82 - 37$
$= 45$

EXERCISE 1E.1

1 Find:

a $5 + 0$
b $4 - 0$
c $18 + 0 + 27$
d $37 + 63 + 0$
e $112 + 38 - 0$
f $61 + 0 - 47$

2 Find:

a $82 + 63 + 18$
b $63 + 241 + 37$
c $122 + 341 + 659$
d $79 + 321 + 418$
e $298 + 402 + 398$
f $604 + 95 + 296$

3 Find:

a $23 + 61 + 27 + 39$
b $624 + 218 - 324$

4 Find:

a the sum of 6, 7, and 9
b the difference between 28 and 59
c by how much 241 is greater than 158.

5 Find the sum of the first six natural numbers.

6 The Burj Khalifa building in the United Arab Emirates is 828 m tall, whereas Taipei 101 in Taiwan is 509 m tall. How much taller is Burj Khalifa than Taipei 101?

7 What number must be increased by 137 to get 912?

8 What number must be decreased by 415 to get 288?

9

In an equestrian event Marcus came first, winning €2300. Megan came second, winning €1250.

a What was the difference between the prizes?
b If their scores had been equal, they would have shared the total prize money for first and second. How much would they have each received?

10 Alysia stands on some scales with a 15 kg dumbbell in each hand. If the scales read 92 kg, what does she weigh?

11 My bank account balance was €1080. I withdrew amounts of €427 and €173. I then banked my pay cheque of €769. What is my bank balance now?

PRODUCTS

The word **product** is used to represent the result of a multiplication.

For example, the product of 3 and 5 is $3 \times 5 = 15$.

Multiplying by **one** (1) does not change the value of a number.

For example, $17 \times 1 = 17$, $1 \times 17 = 17$.

Multiplying by **zero** (0) produces zero.

For example, $17 \times 0 = 0$.

Example 8 Self Tutor

Find the products:

a 7×8
b 7×80
c 70×800

a 7×8
$= 56$

b 7×80
$= 7 \times 8 \times 10$
$= 56 \times 10$
$= 560$

c 70×800
$= 7 \times 10 \times 8 \times 100$
$= 56 \times 1000$
$= 56\,000$

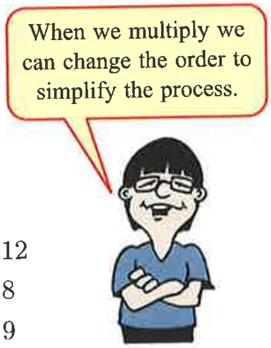
Example 9 Self Tutor

Simplify these products:

a 15×11
b $17 \times 8 \times 125$

a 15×11
$= 15 \times 10 + 15 \times 1$
$= 150 + 15$
$= 165$

b $17 \times 8 \times 125$
$= 17 \times 1000$
$= 17\,000$



EXERCISE 1E.2

1 Find the product:

a 5×9
b 8×3
c 6×1
d 4×12
e 11×0
f 3×14
g 0×157
h 8×8
i 4×11
j 11×4
k 9×6
l 6×9
m $2 \times 5 \times 7$
n $5 \times 7 \times 2$
o $7 \times 2 \times 5$
p $3 \times 3 \times 4$

2 Find the product:

a 4×7 **b** 40×7 **c** 40×70
d 8×5 **e** 80×5 **f** 80×500
g 6×14 **h** 6×1400 **i** $60 \times 14\,000$

3 Simplify:

a 11×35 **b** 17×9 **c** 13×101
d 99×21 **e** 12×32 **f** 5×97

4 Evaluate:

a $25 \times 3 \times 4$ **b** $50 \times 7 \times 2$
c $11 \times 5 \times 20$ **d** $8 \times 14 \times 125$
e $40 \times 25 \times 8$ **f** $200 \times 37 \times 50$
g $40 \times 8 \times 125 \times 25$ **h** $5 \times 80 \times 20 \times 125$

In each question select the most appropriate number strategy.

Example 10 **Self Tutor**

Simplify the product: 87×15

$$\begin{array}{rl} 87 & \\ \times\ \ 15 & \\ \hline 435 & \{\text{multiplying 87 by 5}\} \\ 870 & \{\text{multiplying 87 by 10}\} \\ \hline 1305 & \{\text{adding}\} \end{array}$$

$\therefore\ 87 \times 15 = 1305$

Estimate the answer first, so you can check whether the result you obtain is reasonable.

5 Simplify:

a 37×15 **b** 120×7 **c** 24×45 **d** 209×13
e 67×84 **f** 405×32 **g** 612×18 **h** 193×47

6 Find the product of:

a 23 and 19 **b** 6, 8, and 9 **c** 5, 4, 3, 2, and 1.

7 My mum gave me twelve $5 notes for my birthday. How much did she give me in total?

8 Start with the number 27. Add on 4, 16 times. What is the result?

9 A concert hall has 39 rows of seats. There are 35 seats in each row. How many people can be seated in the concert hall?

10 Adrian sold 14 crates of oranges. The oranges in each crate weighed 22 kg. If Adrian received $3 per kg of oranges, how much did he receive in total?

11 A squad of 18 rugby players attend a training camp. Each player takes 4 pairs of socks with them. How many socks in total are taken to the camp?

QUOTIENTS

The word **quotient** is used to represent the result of a division.

The number being divided is called the **dividend** and the number we are dividing by is called the **divisor**.

For example,

$$\underset{\substack{\uparrow \\ \text{dividend}}}{15} \div \underset{\substack{\uparrow \\ \text{divisor}}}{3} = \underset{\substack{\uparrow \\ \text{quotient}}}{5}$$

We say "The quotient of 15 and 3 is 5."

Dividing by **one** (1) does not change the value of a number.

For example, $38 \div 1 = 38$.

Division by **zero** (0) is meaningless. We say it is **undefined**.

For example, $0 \div 4 = 0$ but $4 \div 0$ is undefined.

Neither the Greeks nor the Romans had a symbol to represent nothing, but other ancient peoples such as the Babylonians did. The symbol 0 was called *zephirum* in Arabic. Our word zero comes from this.

EXERCISE 1E.3

1 Find the quotient:

a $21 \div 7$
b $45 \div 5$
c $36 \div 4$
d $29 \div 1$
e $0 \div 5$
f $88 \div 8$
g $0 \div 1$
h $56 \div 7$
i $0 \div 93$
j $132 \div 12$
k $7 \div 0$
l $96 \div 8$

2 Find the quotient:

a $9 \div 3$
b $90 \div 3$
c $9000 \div 30$
d $48 \div 6$
e $480 \div 60$
f $4800 \div 6$
g $72 \div 8$
h $7200 \div 800$
i $720\,000 \div 80$

Example 11

Self Tutor

Find the quotients:

a $180 \div 15$
b $416 \div 8$

a

$$\begin{aligned} &180 \div 15 \\ &= 60 \div 5 \quad \{\text{dividing each number by } 3\} \\ &= 12 \end{aligned}$$

b

$$\begin{aligned} &416 \div 8 \\ &= (400 \div 8) + (16 \div 8) \\ &= 50 + 2 \\ &= 52 \end{aligned}$$

3 Perform the following divisions:

a $300 \div 12$
b $140 \div 4$
c $210 \div 14$
d $360 \div 8$
e $84 \div 6$
f $450 \div 18$
g $126 \div 6$
h $515 \div 5$
i $189 \div 3$
j $154 \div 7$
k $324 \div 6$
l $196 \div 4$

Example 12 **Self Tutor**

Simplify: $456 \div 19$

$$\begin{array}{r} 24\\ 19\overline{)456}\\ \underline{38}\\ 76\\ \underline{76}\\ 0\end{array}$$

{19 goes into 45 twice}

$\{19 \times 2 = 38\}$

$\{45 - 38 = 7,$ bring 6 down$\}$

{19 goes into 76 four times}

$\{76 - 76 = 0\}$

$\therefore\ 456 \div 19 = 24$

4 Find:

- **a** $91 \div 7$
- **b** $644 \div 28$
- **c** $364 \div 13$
- **d** $306 \div 17$
- **e** $665 \div 35$
- **f** $1292 \div 34$

5 How many 38 seat buses are needed to transport 646 students to the athletics stadium?

6 A truck is transporting 35 chairs to a school. The total mass of the chairs is 770 kg. Find the mass of each chair.

7 Dana sells coal in 15 kg bags. If she has 480 kg of coal, how many bags will she need to sell?

8 Lincoln wants to buy a new mobile phone which costs $189. It will take him exactly 21 weeks to save this amount. How much money is Lincoln able to save each week?

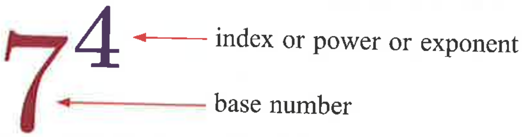
F INDEX NOTATION

A convenient way to write a product of *identical numbers* is to use **index notation**.

For example, instead of writing $7 \times 7 \times 7 \times 7$, we can write 7^4.

The 7 is called the **base number**.

The 4 is called the **index** or **power** or **exponent**. It is the number of times the base number appears in the product.



The following table demonstrates correct language when talking about index notation:

Natural number	*Product form*	*Index form*	*Spoken form*
3	3	3^1	three
9	3×3	3^2	three squared
27	$3 \times 3 \times 3$	3^3	three cubed
81	$3 \times 3 \times 3 \times 3$	3^4	three to the fourth
243	$3 \times 3 \times 3 \times 3 \times 3$	3^5	three to the fifth

Example 13 **Self Tutor**

Write in index form:

$2\times 2\times 2\times 2\times 3\times 3\times 3$

$$\underbrace{2\times 2\times 2\times 2}\times\underbrace{3\times 3\times 3}$$
$$= \quad 2^4 \quad \times \quad 3^3$$
$$= 2^4\times 3^3$$

Example 14 **Self Tutor**

Write as a natural number:

$2^3\times 3^2\times 5$

$$2^3\times 3^2\times 5$$
$$= 2\times 2\times 2\times 3\times 3\times 5$$
$$= 8\times 9\times 5$$
$$= 40\times 9$$
$$= 360$$

The **power key** of your calculator may look like $\boxed{\wedge}$, $\boxed{x^y}$, or $\boxed{y^x}$. It can be used to enter numbers in index form into the calculator.

Example 15 **Self Tutor**

Use your calculator to convert $2^3\times 3^4\times 11^2$ into natural number form.

Key in 2 $\boxed{\wedge}$ 3 $\boxed{\times}$ 3 $\boxed{\wedge}$ 4 $\boxed{\times}$ 11 $\boxed{\wedge}$ 2 $\boxed{=}$. *Answer*: 78 408

EXERCISE 1F

1 Match the following numbers in index form with the correct product:

a	5^4	**A**	$5\times 5\times 5\times 5\times 5\times 5\times 5$
b	7^5	**B**	$7\times 7\times 7$
c	5^7	**C**	$7\times 7\times 7\times 7\times 7$
d	7^1	**D**	$5\times 5\times 5\times 5$
e	7^3	**E**	7

2 Write in index form:

a $2\times 3\times 3$
b $2\times 2\times 3\times 5$
c $2\times 5\times 5\times 5$
d $3\times 3\times 5\times 5\times 5$
e $2\times 2\times 2\times 5\times 7$
f $3\times 3\times 3\times 7\times 7$
g $3\times 3\times 3\times 3\times 5\times 5$
h $7\times 7\times 7\times 7\times 7\times 11\times 11\times 11$

3 Write as a natural number:

a 2^3
b 2×3^2
c $2^2\times 3^2$
d $2^3\times 5\times 7$
e $3^2\times 7^2\times 11$
f $2^2\times 3\times 7^3$

4 Use your calculator to convert each product into natural number form:

a $2^4\times 3^6$
b $2^2\times 5^4\times 7^5$
c $2^5\times 3^3\times 11^2$
d $2^3\times 3^4\times 5^2\times 11$
e $3^4\times 7^2\times 11^3$
f $2^3\times 5^5\times 13^3$

5 Write in index form with 2 as a base:

a 2 **b** 4 **c** 16 **d** 64

6 Write in index form with 3 as a base:

a 3 **b** 27 **c** 81 **d** 729

7 Write in index form with 10 as a base:

a 100 **b** 1000 **c** 100 000 **d** 1 000 000

8 Write in index form with a single digit base:

a 25 **b** 36 **c** 125 **d** 343

ACTIVITY 3 INDEX CROSSWORD

Click on the icon to obtain a printable version of this crossword.

1		2	■	3	
	■	4			■
■	5		■	6	
7		■	8		■
■	9			■	10
11		■	12		

Across

1	19^2	**8**	9^2
3	2^4	**9**	22^2
4	4^4	**11**	4^3
5	5^2	**12**	13^2
6	3^4		
7	3^3		

Down

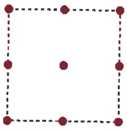
1	6^2
2	5^3
3	41^2
5	14^3
8	29^2
10	7^2

CROSSWORD

G SQUARES AND CUBES

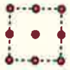
If a number can be represented by a square arrangement of dots, it is called a **square number** or **perfect square**.

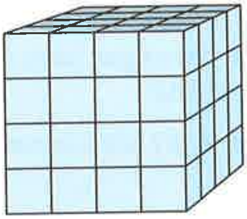
For example, 9 is a square number as it can be represented by the 3×3 square shown.



We say 'three squared is equal to nine' and we write $3^2 = 9$.

The following table shows the first four square numbers:

Square number	*Geometric form*	*Symbolic form*	*Product form*	*Value*
1st		1^2	1×1	1
2nd		2^2	2×2	4
3rd		3^2	3×3	9
4th		4^2	4×4	16



The diagram alongside shows a cube made up of smaller blocks.

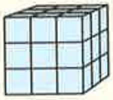
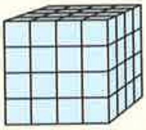
There are 4 layers of blocks.

Each layer is 4 blocks wide and 4 blocks deep.

So, in total there are $4 \times 4 \times 4 = 4^3$ blocks.

This is why a number to the power 3 is called a **cubic number**.

The following table shows the first four cubic numbers:

Cubic number	*Geometric form*	*Symbolic form*	*Product form*	*Value*
1st		1^3	$1 \times 1 \times 1$	1
2nd		2^3	$2 \times 2 \times 2$	8
3rd		3^3	$3 \times 3 \times 3$	27
4th		4^3	$4 \times 4 \times 4$	64

EXERCISE 1G

Try to memorise the first 20 square numbers.

1 For each of the 5th and 6th square numbers:

a draw a diagram **b** state its value.

2 **a** Without using a calculator, find the 7th, 8th, 9th, and 10th square numbers.

b Use your calculator to find the 15th, 25th, and 40th square numbers.

3 **a** Write down two numbers between 20 and 50 that are both odd and square.

b Write down two numbers between 50 and 120 that are both even and square.

4 **a** Use a calculator to complete the following:

$1^2 =$

$11^2 =$

$111^2 =$

$1111^2 =$

b Have you noticed a pattern? Complete the following *without* using your calculator:

i $11\,111^2 =$ **ii** $111\,111^2 =$

c Investigate other such patterns with square numbers. If you find any, share them with your class!

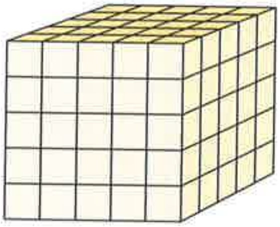
5 **a** Copy and complete the following pattern:

$$\begin{aligned} 1 &= 1 = 1^2 \\ 1+3 &= 4 = 2^2 \\ 1+3+5 &= 9 = 3^2 \\ 1+3+5+7 &= \\ 1+3+5+7+9 &= \end{aligned}$$

b Use the pattern to find the sum of the first:

i 6 odd numbers **ii** 10 odd numbers.

6



By considering the number of blocks in this cube, find the value of the 5th cubic number.

7 Use your calculator to find the 6th, 7th, 10th, and 13th cubic numbers.

8 How many cubic numbers are less than 10 000?

9 Find two consecutive numbers such that one number is a perfect square and the other is a cubic number.

10 **a** Copy and complete the following pattern:

$$\begin{aligned} 1^3 &= 1 && = 1 && = 1^2 \\ 1^3+2^3 &= 1+8 && = 9 && = (1+2)^2 \\ 1^3+2^3+3^3 &= && = && = \\ 1^3+2^3+3^3+4^3 &= && = && = \end{aligned}$$

b Predict the value of:

i $1^3+2^3+3^3+4^3+5^3$ **ii** $1^3+2^3+3^3+\ldots.+10^3$

Check your answers using a calculator.

H ORDER OF OPERATIONS

When two or more different operations are carried out, the answer could vary depending on the **order** in which the operations are performed.

For example, consider the expression $16-10\div 2$.

Ilsa subtracted first then divided:

$$\begin{aligned} &16-10\div 2 \\ =\ &6\div 2 \\ =\ &3 \end{aligned}$$

Lily divided first then subtracted:

$$\begin{aligned} &16-10\div 2 \\ =\ &16-5 \\ =\ &11 \end{aligned}$$

Which answer is correct, 3 or 11?

To avoid this problem, a set of rules for the **order of operations** has been agreed upon by all mathematicians.

THE RULE OF BEDMAS

- Perform operations within **B**rackets first.
- Then, calculate any part involving **E**xponents.
- Then, starting from the left, perform all **D**ivisions and **M**ultiplications as you come to them.
- Finally, working from the left, perform all **A**dditions and **S**ubtractions.

We note that:

- The rule of BEDMAS does *not* mean that division should be performed before multiplication, or that addition should be performed before subtraction.
  - If an expression contains only $\times$ and $\div$ operations, we work from left to right.
  - If an expression contains only $+$ and $-$ operations, we work from left to right.
- If an expression contains more than one set of brackets, we evaluate the innermost brackets first.

Using these rules, Lily's method is correct, and $16 - 10 \div 2 = 11$.

Example 16 Self Tutor

Evaluate: $35 - 10 \div 2 \times 5 + 3$

$$\begin{aligned} & 35 - 10 \div 2 \times 5 + 3 && \text{\{division and multiplication working from left\}} \\ &= 35 - 5 \times 5 + 3 \\ &= 35 - 25 + 3 && \text{\{subtraction and addition working from left\}} \\ &= 10 + 3 \\ &= 13 \end{aligned}$$

Example 17 Self Tutor

Evaluate: $2 \times (3 \times 6 - 4) + 7$

$$\begin{aligned} & 2 \times (3 \times 6 - 4) + 7 && \text{\{inside brackets, multiply\}} \\ &= 2 \times (18 - 4) + 7 && \text{\{complete brackets\}} \\ &= 2 \times 14 + 7 && \text{\{multiplication next\}} \\ &= 28 + 7 && \text{\{addition last\}} \\ &= 35 \end{aligned}$$

If you do not follow the order rules, you are likely to get the wrong answer.

EXERCISE 1H

1 Evaluate:

a $3 + 7 - 5$
b $6 + 9 \div 3$
c $8 - 3 + 2$
d $4 \times 3 - 11$
e $6 \div 2 \times 3$
f $5 \times 8 \div 2$
g $12 \div 4 + 2 \times 5$
h $13 - 2 \times 6 + 4$
i $3 \times 5 + 4 \times 6$
j $5 + 6 \times 3 \div 9$
k $18 - 5 \times 2 + 7$
l $5 \times 4 - 24 \div 6$

2 Evaluate, remembering to complete the brackets first:

a $(5+4)\div 3$
b $3\times(4-2)$
c $(4+7)\times 8$
d $12+(3+7)\div 5$
e $9+6\times(8-5)$
f $18-(7+4)$
g $(6-3)\times 11-12$
h $16+(17-11)$
i $(3+8)\times(6-2)$

3 Evaluate:

a $(12+3)\div 5+2\times 4$
b $(13-5)\div(1+3)+2$
c $23-(6\div 2+7)+4$
d $(5\times 2-6)\times(3-6\div 2)$
e $7-(4\times 3-8)+18\div 3$
f $(3+4)\times 5+6\times 7-8$

Example 18 **Self Tutor**

Evaluate: $5+[13-(8\div 4)]$

$5+[13-(8\div 4)]$ {innermost brackets first}
$=5+[13-2]$ {remaining bracket next}
$=5+11$ {addition last}
$=16$

4 Evaluate:

a $[(3+5)\times 8]-4$
b $3+[(5\times 8)-4]$
c $3+[5\times(8-4)]$
d $[(15-12)\div 3]+3$
e $15-[(12\div 3)+3]$
f $15-[12\div(3+3)]$

Example 19 **Self Tutor**

Evaluate: $3\times(6-2)^2$

$3\times(6-2)^2$ {brackets first}
$=3\times 4^2$ {exponent next}
$=3\times 16$ {multiplication last}
$=48$

5 Evaluate:

a 2×5^2
b 4^3+2^2
c $18-(9\div 3)^2$
d $18-9\div 3^2$
e $(18-9)\div 3^2$
f $(18-9\div 3)^2$
g $(7-2)^2-4^2$
h $3\times(4^2-9)-2$
i $16-2^3+3^2$

6 Replace each $*$ with either $+$, $-$, $\times$, or $\div$ to make each statement true:

a $5*9\div 3=8$
b $7*11-21=56$
c $18-16*2=10$
d $17*3^2=8$
e $13*4\times 2=5$
f $4*13-6*7=10$

7 Insert brackets to make each statement true:

a $3\times 4+2\times 5=90$
b $3\times 4-5\times 4=28$
c $4\times 16-1-6=54$
d $6+7\times 2\div 5=4$
e $4+4\div 2+2=5$
f $3+11-5\div 3=3$

ACTIVITY 4

BEDMAS CHALLENGE

Click on the icon to run the BEDMAS Challenge.

How fast can you go?

BEDMAS CHALLENGE

KEY WORDS USED IN THIS CHAPTER

- approximation
- difference
- estimate
- Hindu-Arabic system
- numeral
- power
- round down
- square number
- BEDMAS
- dividend
- expanded form
- index
- perfect square
- product
- round up
- sum
- cubic number
- divisor
- exponent
- natural number
- place value
- quotient
- significant figures
- whole number

REVIEW SET 1A

1 Write 9602 in word form.

2 What number is represented by the digit 7 in 17 260?

3 Simplify:

a 15×0 **b** $11 - 0$ **c** $12 \div 0$

4 What is the difference between 792 and 2895?

5 Round off 49 552 to the nearest:

a 10 **b** 100 **c** 1000.

6 By how much is 838 greater than 562?

7 Use the digits 1, 5, 9, 6, 0, and 2 exactly once each to make the largest number you can.

8 Use a one figure approximation to estimate 4067×905.

9 How many buses would be required to transport 517 students if each bus holds a maximum of 47 students?

10 Find the value of:

a $3 + 16 \div 2^2$ **b** $5 \times 4 + 18 \div 3$ **c** $3 \times 25 + 3 \times (7 - 2)$

11 A woman has $350 in her purse. She gives $22 to each of her five children. How much money does she have left?

12 Use your calculator to find the 17th cubic number.

13 How many square numbers are there between 40 and 110?

14 Insert brackets to make each statement true:

a $2 + 12 \div 4 - 2 = 8$ **b** $30 \div 5 + 1 + 4 = 9$

15 Answer the questions in the **Opening Problem** on page **12**.

REVIEW SET 1B

1 Write the numeral $40\,701$ in words.

2 Write $30\,502$ in expanded form.

3 Use an appropriate strategy to find:

a $97 + 56$ **b** $19 + 26 + 41$ **c** $96 \div 16$

d $77 - 39$ **e** $6 \times 90\,000$ **f** $36 \div 7$

4 Simplify:

a $5 \times 14 \times 2$ **b** $20 \times 17 \times 5$ **c** $8 \times 9 \times 125$

5 If a pie cart vendor sells 11 dozen pies at \$3 each, how much money does he make?

6 Round off to 2 significant figures:

a 573 **b** $37\,193$ **c** 4239 **d** $119\,603$

7 Find:

a 2^7 **b** 5^5 **c** $3^4 \times 7^2$

8 There are 29 students in a class. Each student has an average of 11 books. *Estimate* the total number of books.

9 My bank account contains €227, and my fortnightly pay of €540 is added directly into my bank account. If there are no withdrawals, what is my bank balance after 12 weeks?

10 Insert brackets to make a true statement: $44 - 8 \div 4 + 2 = 6$

11 Simplify:

a 32×13 **b** 527×24 **c** $459 \div 17$ **d** $1081 \div 23$

12 Find the sum of the 8th square number and the 7th cubic number.

13 Simplify:

a $2^2 \times (7 - 3) + 5 \times 8$ **b** $(13 - 2 \times 5)^2$

14 **a** Use the digits 2, 9, 8, and 5 exactly once each to make the largest and smallest numbers you can.

b Find the difference between your answers in **a**.

15 A student receives 12 assignments during a term. On average he spends 98 minutes on each one.

a *Estimate*, using a one figure approximation, the total amount of time (in minutes) that the student spends on assignments over the term.

b Use a suitable multiplication strategy to find the *exact* amount of time spent on assignments.

c Find the difference between your estimate and the actual value.

d The teacher spends a total of 292 minutes marking these 12 assignments. *Estimate*, using a one figure approximation, the average time spent marking each assignment.

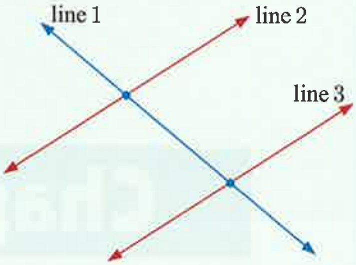
Chapter 2

Angles and lines

Contents:

OPENING PROBLEM

This diagram shows three lines. We can see points of **intersection** where line 1 meets the other lines.



Things to think about:

- **a** How can we describe the point where:
  - **i** line 1 meets line 2
  - **ii** line 1 meets line 3?
- **b** Lines 2 and 3 do not meet in this diagram.
  - **i** If we were to extend the lines, do you think they would eventually meet?
  - **ii** By measuring angles in the diagram, can we test whether the lines will eventually meet?

If we look carefully, we can see **angles** in many objects and situations. We see them in the framework of buildings, the pitches of roof structures, the steepness of ramps, and the positions of boats from a harbour and aeroplanes from an airport.

The measurement of angles dates back more than 2500 years and is still very important today in architecture, building, surveying, engineering, navigation, space research, and many other industries.

RESEARCH DEGREE MEASURE

The Babylonian Empire was founded in the 18th century BC by Hammurabi in lower Mesopotamia, which is today in southern Iraq. It lasted over a thousand years, finally being absorbed into the Persian Empire of Darius in the 6th century BC.

1. The Babylonians invented the **astrolabe**.
   Find out what an astrolabe measures.
2. How many degrees did the Babylonians decide should be in one full turn? Why did they choose this number?

A POINTS AND LINES

POINTS

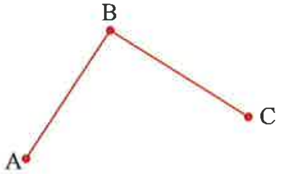
We use a **point** to mark a location or position.

Examples of points are:

- the corner of your desk
- the tip of your compass needle.

Points do not have size. We say they are **infinitely small**. In geometry, however, a point is represented by a small dot so we can see it. To help identify the point, we label it with a capital letter.

For example:



The letters A, B, and C identify the points.

The letters allow us to make statements like:
"the distance from A to B is ...." or
"the angle at B measures ....".

STRAIGHT LINES

A **straight line**, usually just called a **line**, is a continuous infinite collection of points which lie in a particular direction. A line has no beginning or end.

(AB) is the **line** which passes through points A and B. We can call it "**line** AB" or "**line** BA". There is only one straight line which passes through both A and B.

[AB] is the **line segment** which joins points A and B. We call it "**line segment** AB" or "**line segment** BA". It is only part of the line (AB). The *length* of line segment [AB] is written AB.

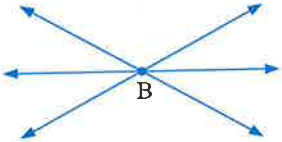
[AB) is the **ray** which starts at A, passes through B, then continues on forever in that direction.

If three or more *points* lie on a single straight line, we say that the points are **collinear**.

For example, in the diagram the points A, B, C, and D are collinear.

If three or more *lines* meet or intersect at the same point, we say that the lines are **concurrent**.

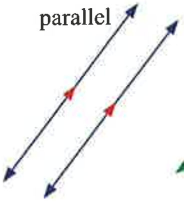
For example, the lines shown are concurrent at point B.

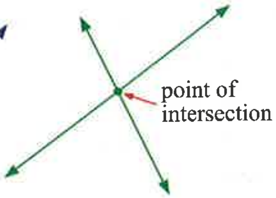


PARALLEL AND INTERSECTING LINES

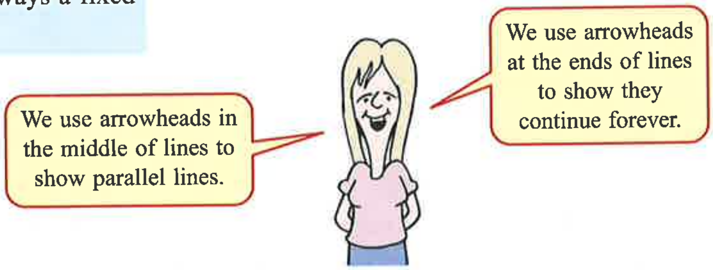
In mathematics, a **plane** is a flat surface like a table top or a sheet of paper. It goes on indefinitely in all directions, so it has no boundaries.

Two straight lines in the same plane may either be **parallel** or **intersecting**.





Parallel lines are lines which are always a fixed distance apart and never meet.



DISCUSSION

1 Give *two* examples in the classroom of:

a a point **b** a line segment **c** a flat surface
d an angle **e** parallel lines.

2 How many different lines can you draw through:

a two distinct points A and B **b** all three distinct collinear points A, B, and C
c one point A **d** all three distinct non-collinear points A, B, and C?

EXERCISE 2A

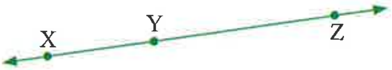
1 Describe, with a sketch, the meaning of:

a a line segment **b** a ray **c** a point of intersection
d parallel lines **e** collinear points **f** concurrent lines.

2 Give *all* ways of naming the following lines:

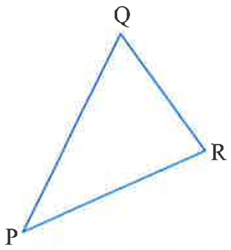
a

b



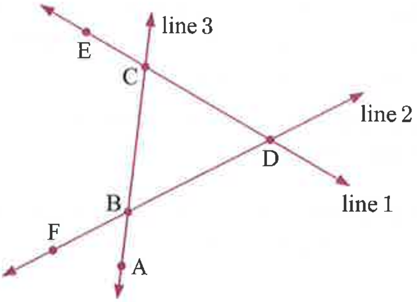
3 PQR is a triangle.

a Name the three sides of the triangle.
b Which sides intersect at point P?



4 Name the point of intersection between:

a line 2 and line 3
b line 1 and line 3
c (AB) and [DE]
d [AC] and [DF].



5 Draw a diagram for each statement:

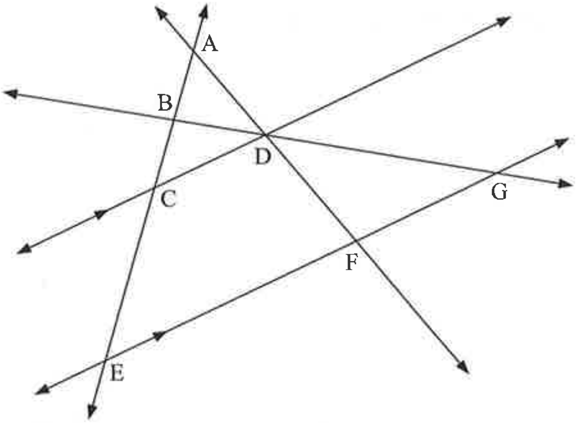
a X is a point on [PQ]. **b** [EF] and (GH) meet at point M.
c S, T, U, and V are collinear. **d** (JK) and (MN) are parallel.
e [AB], (CD), and (EF) are concurrent at G.

6 **a** Name the line (AB) in three other ways.

b How many lines go through point D?

c What can be said about:

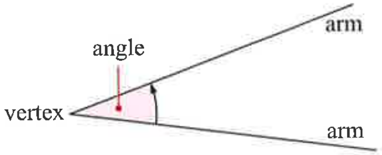
- **i** lines (EF) and (AD)
- **ii** points A, D, and F
- **iii** lines (CD) and (EG)?



B MEASURING AND CLASSIFYING ANGLES

Whenever two lines or edges meet, an **angle** is formed between them. In mathematics, an angle is made up of two **arms** which meet at a point called the **vertex**.

The **size** or **measure** of the angle is the amount of turning or rotation from one arm to the other.

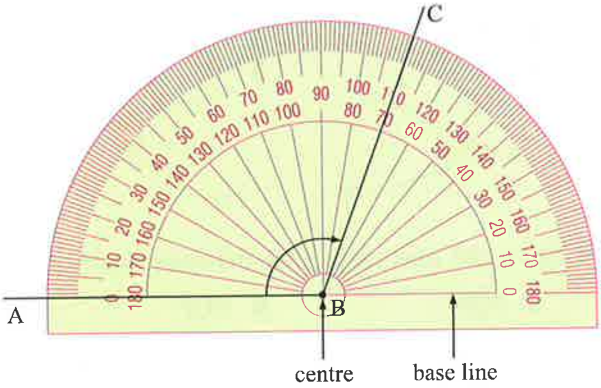


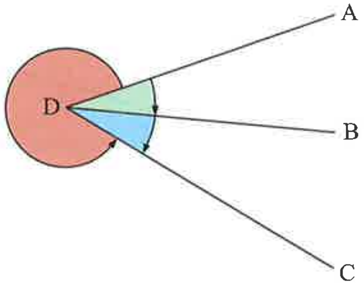
THE PROTRACTOR

Alongside is a **protractor** placed with its centre at B and its base line on [AB]. The amount of turning from [AB] to [BC] is 110 degrees.

We write $A\hat{B}C = 110°$ which reads "the angle ABC measures 110 degrees".

$A\hat{B}C$ is called **three point notation**. We use it to make it clear which angle we are referring to.





For example, if we want to talk about the angle shaded green in this figure, we cannot just say the angle at D. This could refer to many angles, including the blue one and the red one.

The green angle is $A\hat{D}B$ or $B\hat{D}A$.

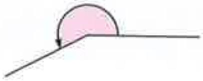
The blue angle is $B\hat{D}C$ or $C\hat{D}B$.

$A\hat{D}C$ is made up of the green angle *and* the blue angle.

The red angle is called the *reflex* $A\hat{D}C$, since its size is more than 180°.

CLASSIFYING ANGLES

Angles are classified according to their **size**.

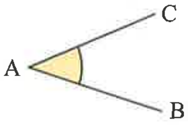
Revolution	**Straight Angle**	**Right Angle**
 One complete turn. One revolution = 360°.	 $\frac{1}{2}$ turn 1 straight angle = 180°.	 $\frac{1}{4}$ turn 1 right angle = 90°.
Acute Angle	**Obtuse Angle**	**Reflex Angle**
 Less than a $\frac{1}{4}$ turn. An acute angle has size between 0° and 90°.	 Between $\frac{1}{4}$ turn and $\frac{1}{2}$ turn. An obtuse angle has size between 90° and 180°.	 Between $\frac{1}{2}$ turn and 1 turn. A reflex angle has size between 180° and 360°.

EXERCISE 2B

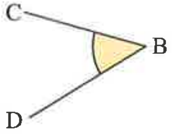
1 Match the names to the correct angles:

a $A\hat{B}C$ **b** $C\hat{A}B$ **c** $B\hat{C}A$ **d** $C\hat{B}D$

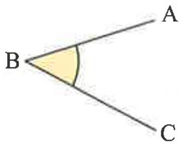
A



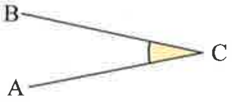
B



C



D



2 Draw and label each of the following angles:

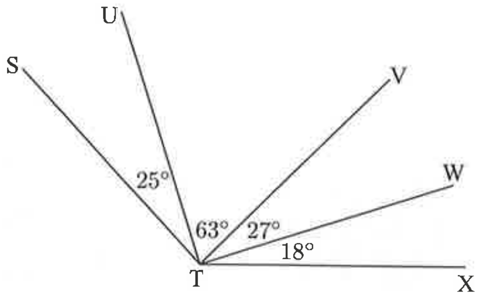
a $P\hat{Q}R$ **b** $R\hat{Q}P$ **c** reflex $E\hat{F}G$ **d** $C\hat{A}B$

3 **a** Find the sizes of these angles without using your protractor:

i $S\hat{T}U$ **ii** $W\hat{T}U$

iii $X\hat{T}V$ **iv** $S\hat{T}W$

b Classify each angle in **a** as acute, right, or obtuse.



4 Use your ruler and protractor to draw angles with the following sizes:

a 38° **b** 89° **c** 120°

Ask a friend to check the accuracy of your angles.

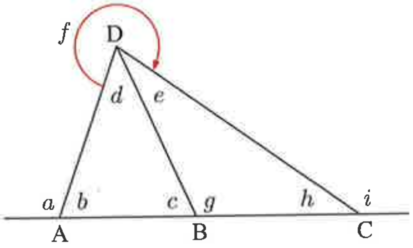
5 Consider the figure alongside.

a Find the angles corresponding to:

i $B\widehat{A}D$ **ii** $D\widehat{B}C$ **iii** $A\widehat{D}B$

b Classify the following angles as acute, obtuse, or reflex:

i f **ii** a **iii** h



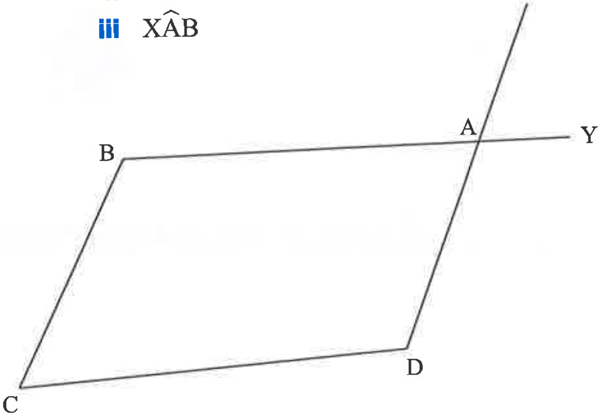
6 Draw a free-hand sketch of:

a obtuse $X\widehat{Y}Z$ **b** revolution B **c** reflex $Y\widehat{S}P$

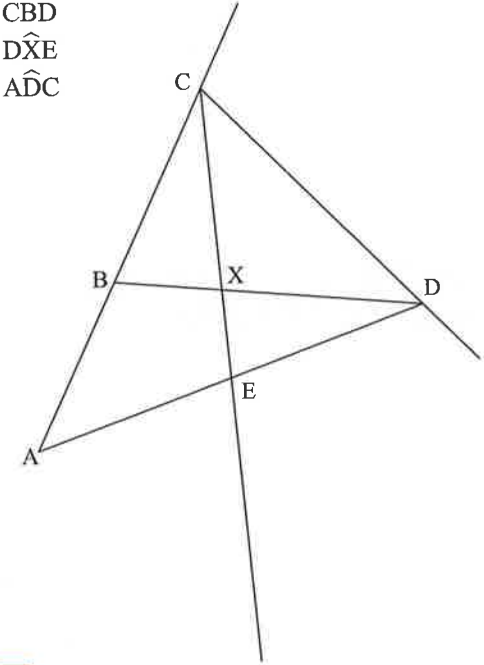
d right $C\widehat{Z}R$ **e** acute $J\widehat{K}L$ **f** straight $E\widehat{F}G$.

7 Use a protractor to measure the named angles:

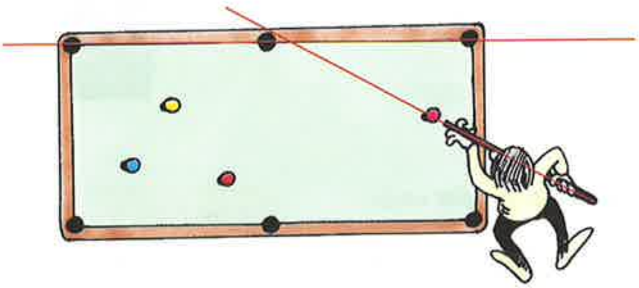
a **i** $B\widehat{A}D$
ii $C\widehat{B}A$
iii $X\widehat{A}B$



b **i** $C\widehat{B}D$
ii $D\widehat{X}E$
iii $A\widehat{D}C$



8 Kit hits the billiard ball so that it follows the path shown. What *acute* angle will it make with the edge of the table?

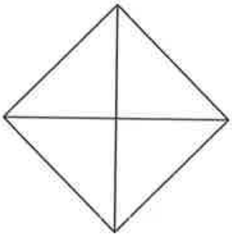


9 An awning is extended out the front of a café as shown. Measure the angle between the top of the awning and the support post.

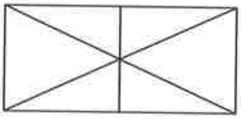
10 For each figure, find the total number of:

i right angles **ii** acute angles.

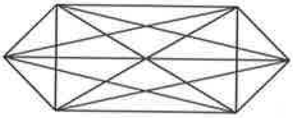
a



b



c



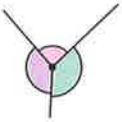
ACTIVITY 1 MAKING A PROTRACTOR

Click on the icon to obtain instructions for this Activity.

It explains how to make a protractor of your own, and provides activities for you to do in class.

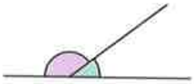
INSTRUCTIONS

C ANGLE PROPERTIES



These angles are **angles at a point**.

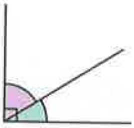
Angles at a point add to 360°.



These angles are **angles on a line**.

Angles on a line add to 180°.

Angles which add to 180° are called **supplementary angles**.

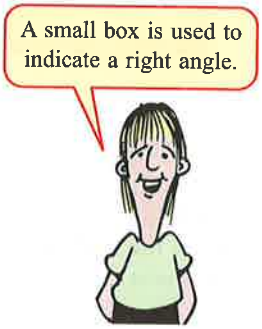


These angles are **angles in a right angle**.

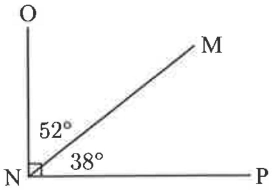
Angles in a right angle add to 90°.

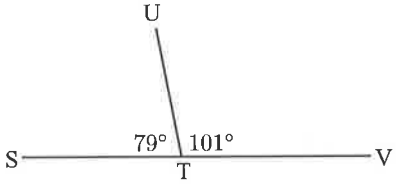
Angles which add to 90° are called **complementary angles**.

Lines or line segments which meet at 90° are said to be **perpendicular**.



For example:





$M\widehat{N}O$ and $M\widehat{N}P$ are complementary because $52° + 38° = 90°$.

[ON] and [NP] meet in a right angle, so they are perpendicular.

$S\widehat{T}U$ and $U\widehat{T}V$ are supplementary because $79° + 101° = 180°$.

Two angles are **equal** if they have the same size or degree measure.

DISCUSSION

For any angle ABC, what is the relationship between the sizes of $A\widehat{B}C$ and reflex $A\widehat{B}C$?

Example 1 **Self Tutor**

a Are angles with sizes 37° and 53° complementary?

b What angle size is supplementary to 48°?

a $37° + 53° = 90°$. So, the angles are complementary.

b The angle size supplementary to 48° is $180° - 48° = 132°$.

EXERCISE 2C

1 Add the following pairs of angles and state whether they are complementary, supplementary, or neither:

a 109°, 71°
b 67°, 117°
c 62°, 28°
d 155°, 31°
e 25°, 55°
f 64°, 116°

2 Find the size of the angle complementary to:

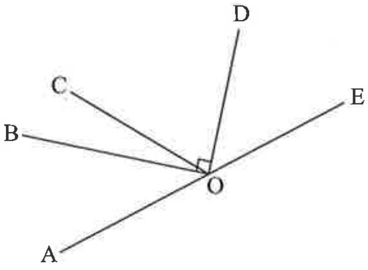
a 15°
b 87°
c 43°

3 Find the size of the angle supplementary to:

a 129°
b 57°
c 90°

4 Classify the following angle pairs as complementary, supplementary, or neither:

a $C\widehat{O}A$ and $C\widehat{O}E$
b $A\widehat{O}D$ and $E\widehat{O}C$
c $B\widehat{O}C$ and $C\widehat{O}D$
d $C\widehat{O}E$ and $D\widehat{O}B$



5 Copy and complete:

a the size of the angle complementary to $x°$ is ......

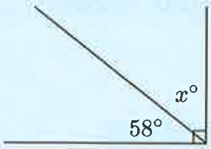
b the size of the angle supplementary to $y°$ is ......

Example 2

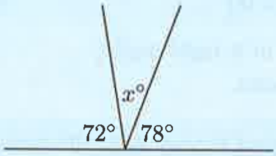
Self Tutor

Find the value of the unknown:

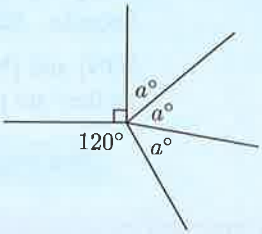
a



b



c



a The angles 58° and $x°$ are complementary.

$\therefore\ x = 90 - 58$

$\therefore\ x = 32$

b The three angles are supplementary, so they add to 180°.

$\therefore\ x = 180 - 72 - 78$

$\therefore\ x = 30$

c We have five angles at a point, so the sum of the five angles is 360°.

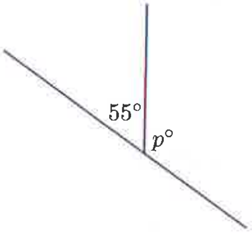
So, the three equal angles add to

$360° - 90° - 120° = 150°$

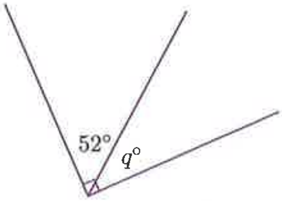
$\therefore\ \ a = 150 \div 3 = 50$

6 Find the value of the unknown:

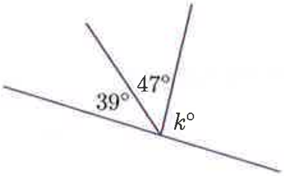
a



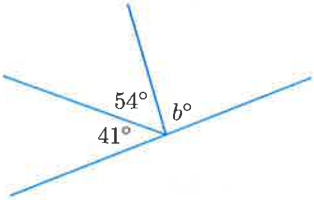
b



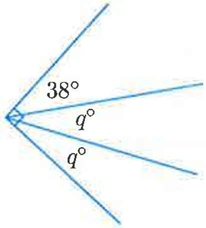
c



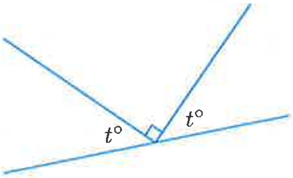
d



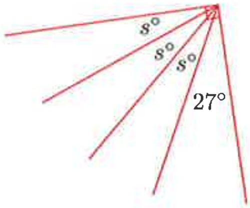
e



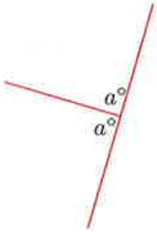
f



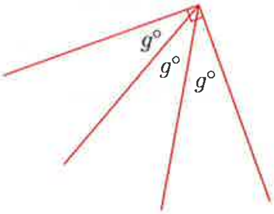
g



h

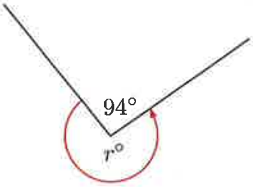


i

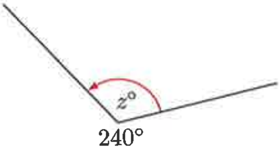


7 Find the sizes of the unknown angles:

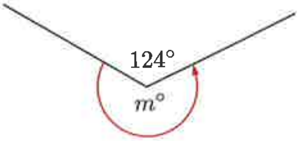
a



b

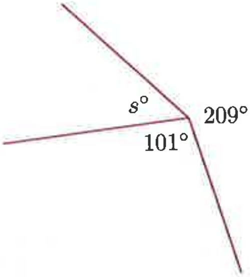


c

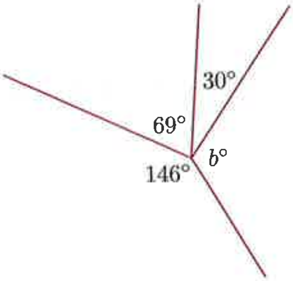


8 Find the values of the unknowns:

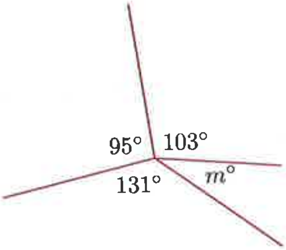
a



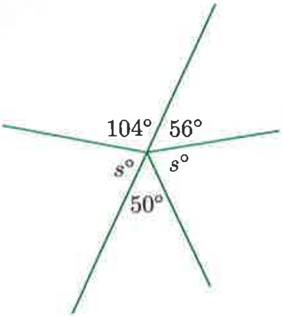
b



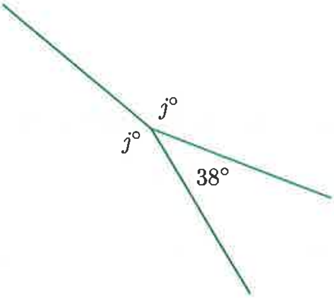
c



d



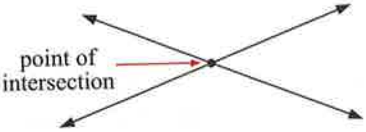
e



D ANGLE PAIRS

We have already seen how lines drawn in a plane are either **parallel** or **intersecting**.

parallel



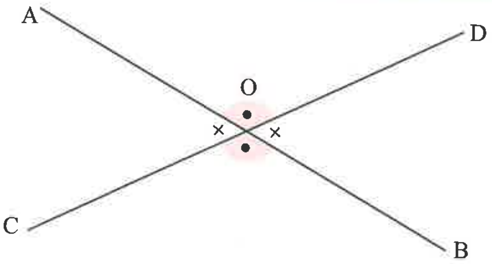
intersecting

When we are dealing with several lines in a plane, we can identify a number of **angle pairs**.

VERTICALLY OPPOSITE ANGLES

Vertically opposite angles are formed when two straight lines intersect. The two angles are directly opposite each other through the vertex.

For example:

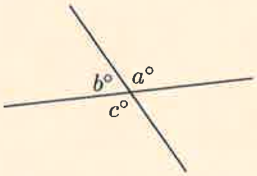


$A\widehat{O}C$ and $D\widehat{O}B$ are vertically opposite.
$A\widehat{O}D$ and $C\widehat{O}B$ are vertically opposite.

Measure the pairs of vertically opposite angles carefully. You should find that:

When two straight lines intersect, **vertically opposite** angles are *equal* in size.

Proof:



$a + b = 180$ {angles on a line}

and $c + b = 180$ {angles on a line}

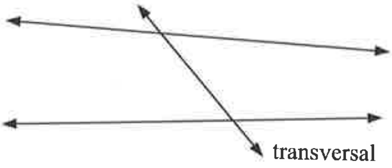
$\therefore \quad a = c$

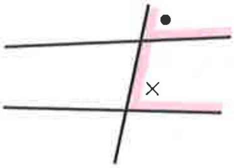
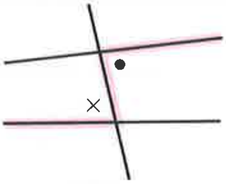
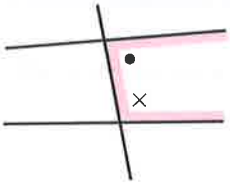
GEOMETRY PACKAGE

CORRESPONDING, ALTERNATE, AND CO-INTERIOR ANGLES

A third line that crosses two other straight lines is called a **transversal**.

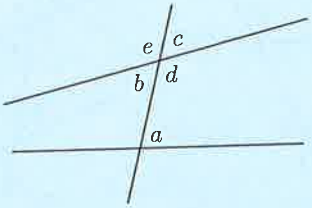
When two or more straight lines are cut by a transversal, three different angle pairs are formed:



Corresponding angle pairs	Alternate angle pairs	Co-interior angle pairs
		
The angles marked ● and × are **corresponding angles** because they are both in the *same position*. They are on the *same side* of the transversal and the *same side* of the two straight lines.	The angles marked ● and × are **alternate angles**. They are on *opposite sides* of the transversal and *between* the two straight lines.	The angles marked ● and × are **co-interior angles**. They are on the *same side* of the transversal and *between* the two straight lines. Co-interior angles can also be called **allied angles**.

Example 3

Self Tutor



Describe the following angle pairs:

a a and d **b** a and b

c d and e **d** a and c

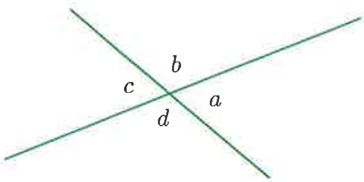
a a and d are co-interior angles. **b** a and b are alternate angles.

c d and e are vertically opposite angles. **d** a and c are corresponding angles.

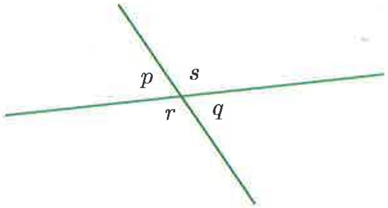
EXERCISE 2D

1 In each diagram, list the pairs of vertically opposite angles:

a

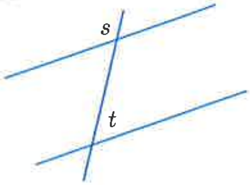


b

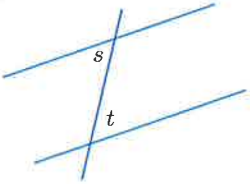


2 In which diagrams are s and t alternate angles?

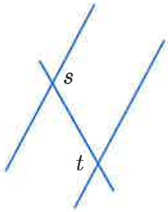
A



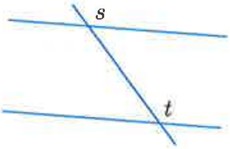
B



C

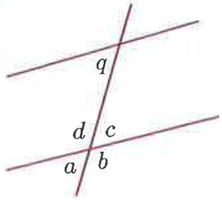


D

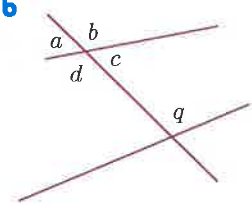


3 Which angle is alternate to angle q?

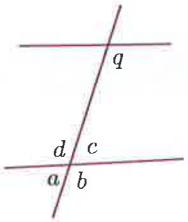
a



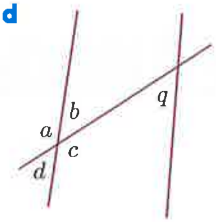
b



c

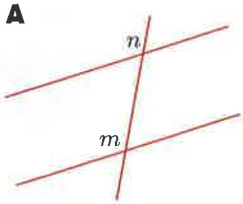


d

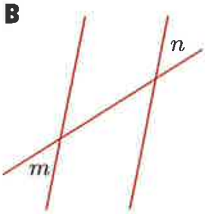


4 In which diagrams are m and n corresponding angles?

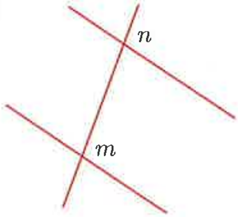
A



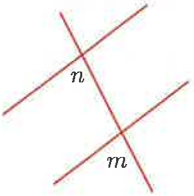
B



C

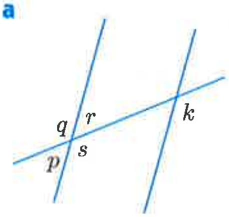


D

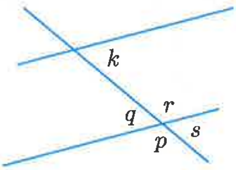


5 Which angle is corresponding to angle k?

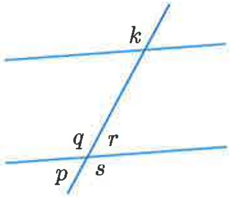
a



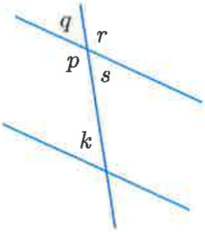
b



c

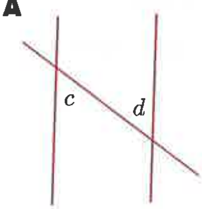


d

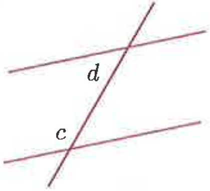


6 In which diagrams are c and d co-interior angles?

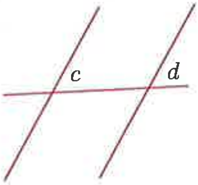
A



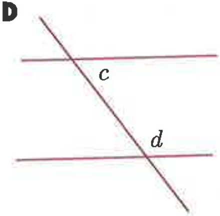
B



C

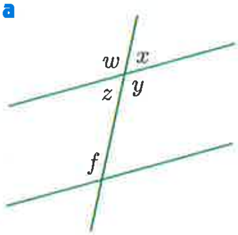


D

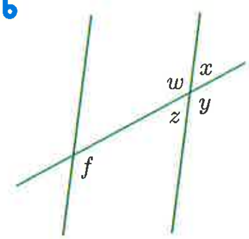


7 Which angle is co-interior with angle f?

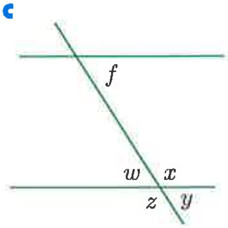
a



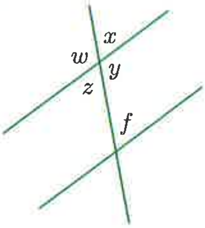
b



c

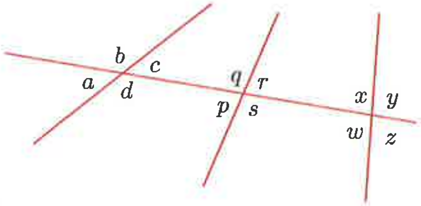


d



8 Classify the following angle pairs as either corresponding, alternate, co-interior, or vertically opposite:

a a and p
b r and w
c r and x
d z and s
e b and q
f a and c
g x and z
h w and s
i c and p



E PARALLEL LINES

If the two lines cut by a transversal are parallel, then corresponding, alternate, and co-interior angle pairs have special properties. We will discover these properties in the following **Investigation**.

INVESTIGATION ANGLE PAIRS ON PARALLEL LINES

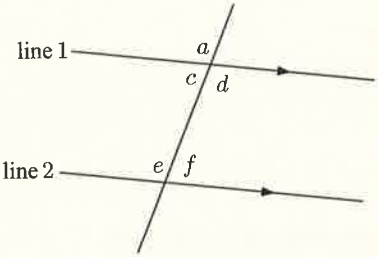
What to do:

WORKSHEET

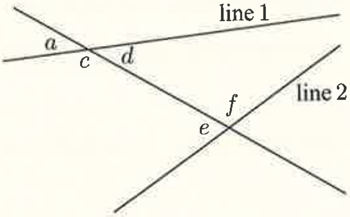
1 Print this worksheet so that you can write directly onto it.

2 In each diagram, measure the angles marked and answer the related questions in the table below:

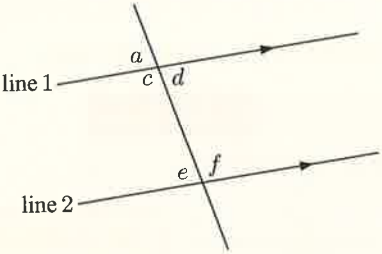
a



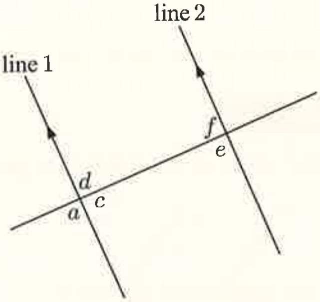
b



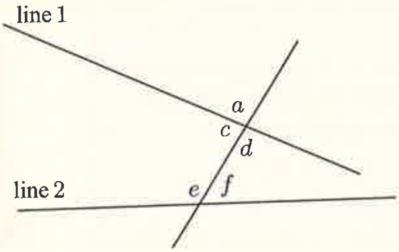
c



d



e



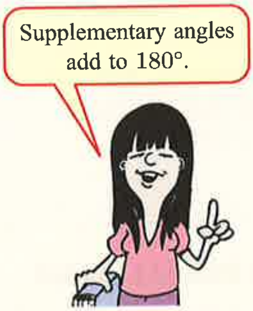


Diagram	Are lines 1 and 2 parallel?	Are the corresponding angles a and e equal?	Are the alternate angles c and f equal?	Are the co-interior angles d and f equal? Are they supplementary?	
				equal	supplementary
a					
b					
c					
d					
e					

3 What can you conclude from your results?

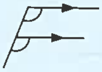
GEOMETRY PACKAGE

4 Click on the icon to further investigate angle pairs on parallel lines, and hence confirm your conclusions.

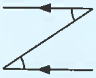
From the **Investigation** you should have discovered the following important facts:

When **parallel lines** are cut by a **transversal**:

- corresponding angles are equal in size



- alternate angles are equal in size



- co-interior angles are supplementary, which means they add up to 180°.

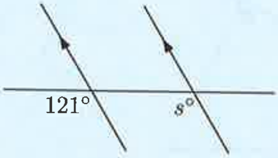
Using these geometrical facts, we can find unknown values for angles on parallel lines.

Example 4

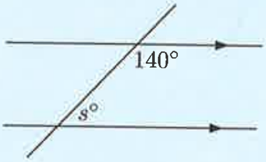
Self Tutor

Find the value of the unknown, giving a brief reason for your answer:

a



b



a Corresponding angles on parallel lines are equal.

∴ $s = 121$

b Co-interior angles on parallel lines are supplementary.

∴ $s = 180 - 140$

∴ $s = 40$

The special properties only apply if the lines cut by the transversal are *parallel.*

TESTS FOR PARALLELISM

Suppose two lines are cut by a transversal.

- If pairs of corresponding angles are equal in size then the lines are parallel.
- If pairs of alternate angles are equal in size then the lines are parallel.
- If pairs of co-interior angles are supplementary then the lines are parallel.

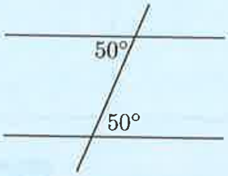
GEOMETRY PACKAGE

Example 5

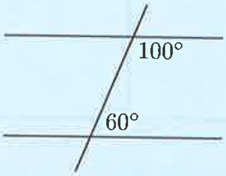
Self Tutor

Decide if the figure contains parallel lines, giving a brief reason for your answer:

a



b



a These alternate angles are equal, so the lines are parallel.

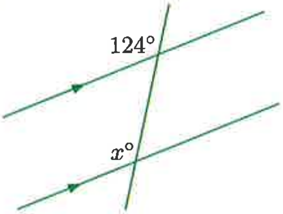
b These co-interior angles add to 160°, so they are not supplementary.

∴ the lines are *not* parallel.

EXERCISE 2E

1 Find, giving brief reasons, the values of the unknowns:

a



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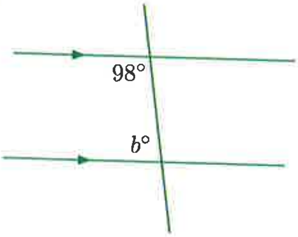

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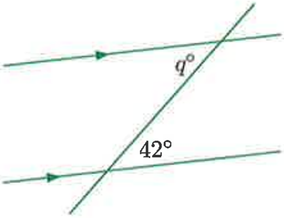
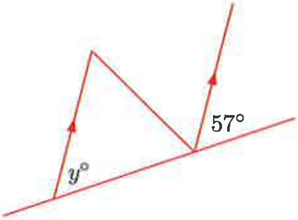
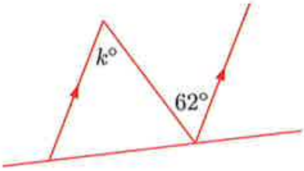


d



e



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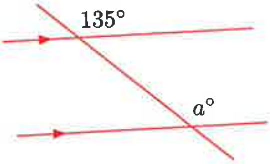

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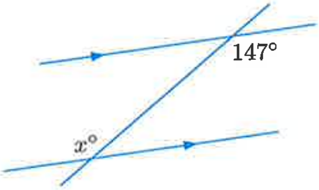
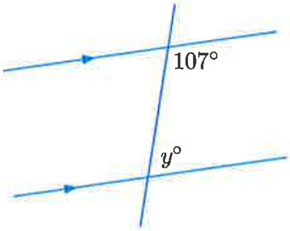
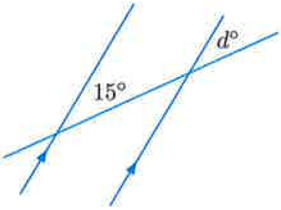


h

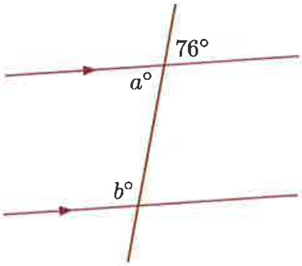


i

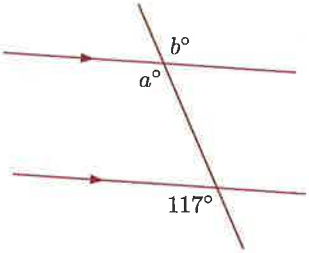


2 Find the values of the unknowns in alphabetical order, giving brief reasons:

a



b



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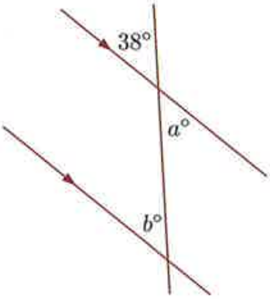

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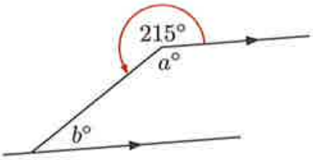
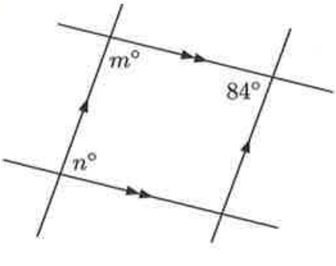


e



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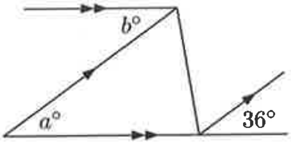
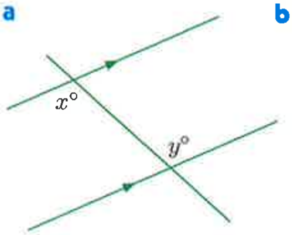
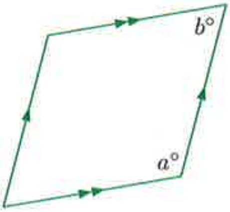


3 Write a statement connecting the unknowns, giving a brief reason:

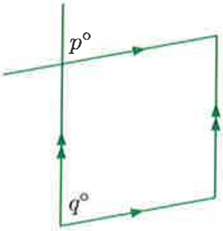
a



b



c



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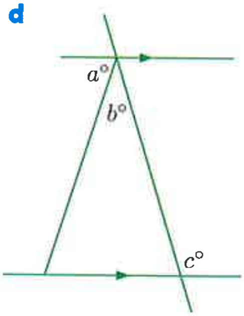
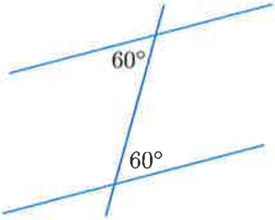


4 The following figures are not drawn to scale.
Decide if each figure contains a pair of parallel lines, giving brief reasons for your answers.

a



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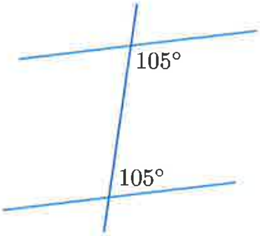

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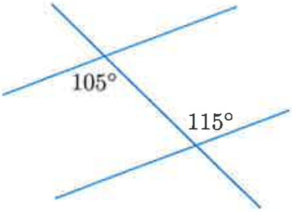

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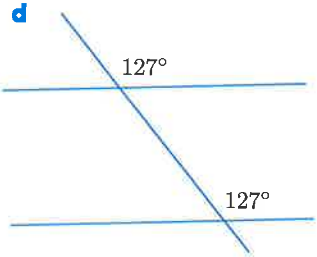
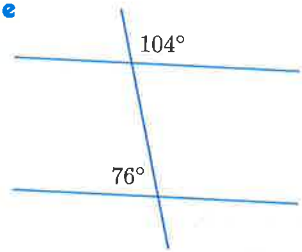
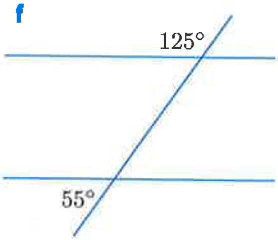


e

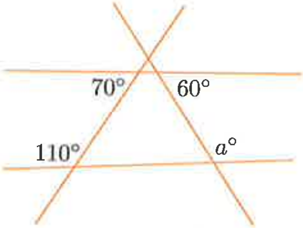


f

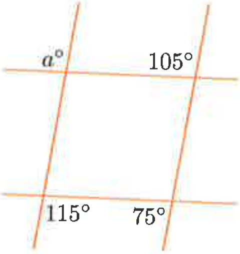


5 The following figures are not drawn to scale. Find the value of a, giving reasons for your answer.

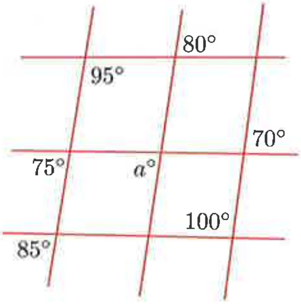
a



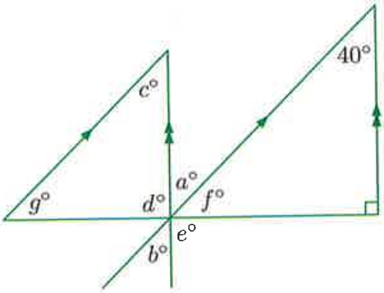
b



6 This figure is not drawn to scale.
Find the value of a.



7 Working in alphabetical order, find the values of the unknowns.
Give a reason for each answer.



PUZZLE

PRINTABLE

1 When a transversal intersects two parallel lines, ... angles are on the same side of the transversal and on the same side of the parallel lines.

2 Points that lie in a straight line are ......

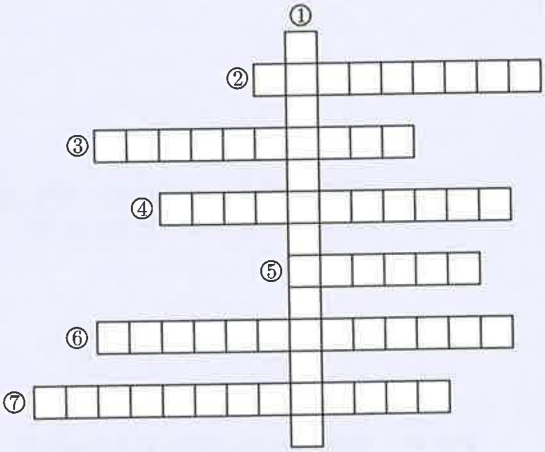
3 Angles between parallel lines on the same side of a transversal are ...... angles.

4 A line which intersects two parallel lines is a ......

5 An angle which measures between 90° and 180° is ......

6 Lines which intersect at right angles are ......

7 Two angles on a straight line are ...... angles.



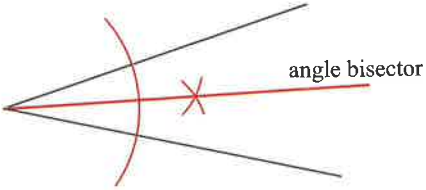
F GEOMETRIC CONSTRUCTION

In **geometric constructions** we use a ruler and compass to accurately draw diagrams. When you perform geometric constructions, **do not erase** the construction lines.

In previous courses you should have learnt how to construct **angle bisectors**.

If necessary click on the icon to review this construction.

DEMO



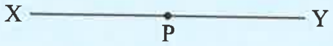
CONSTRUCTING A 90° ANGLE TO A LINE

A **right angle** or 90° **angle** can be constructed without a protractor or set square. This allows us to construct a line which is **perpendicular** to another line.

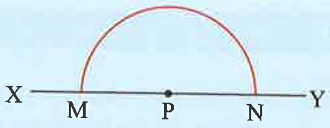
Example 6

Self Tutor

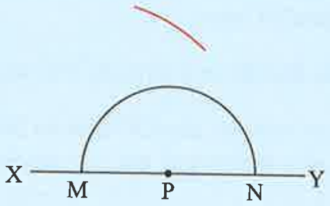
Construct an angle of 90° at P on the line segment [XY].



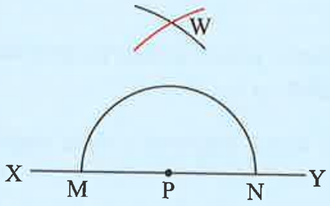
Step 1: On the line segment [XY], draw a semi-circle with centre P and convenient radius which cuts [XY] at M and N.



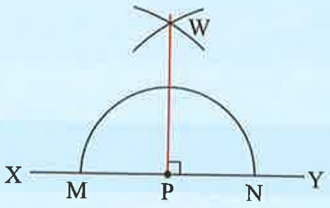
Step 2: With centre M and radius MN, draw an arc above P.

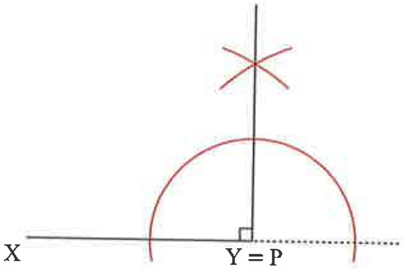


Step 3: With centre N and radius MN, draw an arc to cut the first one at W.



Step 4: Draw the line from P through W.
$W\widehat{P}Y$ and $W\widehat{P}X$ are both 90°.

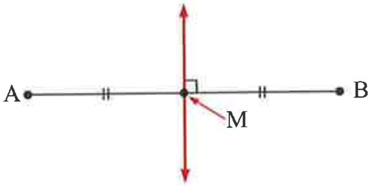




In some cases the point P may be close to one end of the line segment, or indeed be the end of the line segment. In these cases you may need to extend the line segment.

For example, in the construction alongside, Y and P are the same point. We say they **coincide**. We can still construct a right angle at P, but we need to extend the line segment first.

CONSTRUCTING A PERPENDICULAR BISECTOR



The red line in this figure is at right angles to [AB] so it is **perpendicular** to [AB].

It passes through M which is midway between A and B, so it **bisects** [AB].

We therefore say the red line is the **perpendicular bisector** of [AB].

We can use a ruler and compass to construct the perpendicular bisector of a line segment.

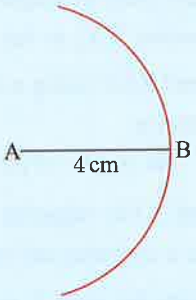
Example 7

Self Tutor

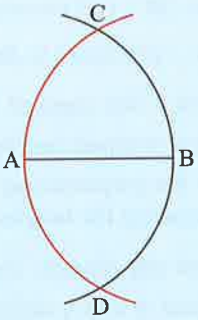
Draw a line segment [AB] with length 4 cm. Construct the perpendicular bisector of [AB].

Step 1: Draw the line segment [AB] with length 4 cm.

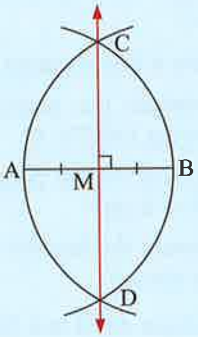
Step 2: With centre A and radius AB, draw an arc of a circle as shown.



Step 3: Repeat *Step 2* but with centre B. Make sure that the first arc is crossed twice, at C and D.



Step 4: With pencil and ruler, join C and D. (CD) and [AB] are perpendicular, and meet at M, the midpoint of [AB].



(CD) is the perpendicular bisector of [AB].

EXERCISE 2F

1 **a** Use your protractor to accurately draw $A\widehat{B}C$ of size 70°.

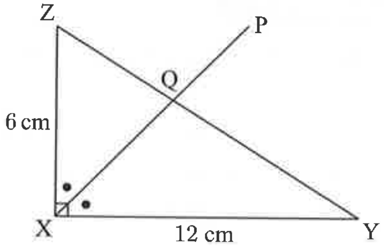
b Use a compass and ruler only to bisect $A\widehat{B}C$.

c Use a protractor to check the accuracy of your construction.

2 **a** Accurately draw the line segment [AB] with the dimensions shown. Mark on it the points A, B, and C.

b Use a compass and ruler to construct a right angle at C below the line segment [AB].

3 **a** Draw a line segment [XY] of length 12 cm.



b Use a compass and ruler to construct a 90° angle at X.

c Draw [XZ] of length 6 cm, and join [ZY] as shown.

d Measure [ZY] to the nearest mm.

e Bisect $Z\widehat{X}Y$ using a compass and ruler only.

f Measure:

i the length of [QY] **ii** the size of $X\widehat{Q}Y$.

4 **a** Draw a line segment [AB] of length 4 cm.

b Use a compass and ruler to construct a 90° angle at A.

c Locate the point C so that $C\widehat{A}B = 90°$ and the length of [AC] is 3 cm.

d Join [BC] and measure its length.

e Use a protractor to measure $C\widehat{B}A$.

5 **a** Draw a line segment [PQ] of length 5 cm.

b Use a compass and ruler to construct the perpendicular bisector of [PQ].

c Let the perpendicular bisector of [PQ] meet [PQ] at Y. Check your perpendicular bisector by measuring the lengths of [PY] and [QY].

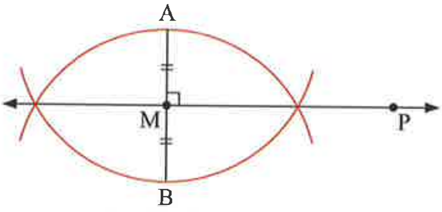
6 **a** Draw any triangle, and construct the perpendicular bisectors of its three sides.

b Repeat **a** with a different triangle.

c Hence, copy and complete: "The three perpendicular bisectors of the sides of a triangle are ......".

7 **a** Draw a line segment [AB] of length 2 cm.

b Construct the perpendicular bisector of [AB], meeting [AB] at M.



c Locate P on the perpendicular bisector such that MP = 3 cm.

d Measure the lengths of [AP] and [BP]. What do you notice?

e Measure $A\widehat{P}M$ and $B\widehat{P}M$ with a protractor. What do you notice?

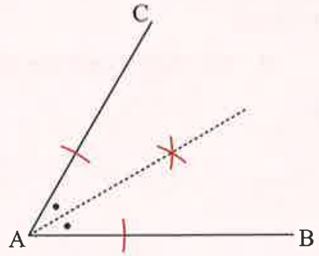
ACTIVITY 2

TRISECTING AN ANGLE

In previous years we learnt how to **bisect** an angle using a compass and pencil.

In the diagram, angle CAB is bisected into two equal angles.

It has been proven that an angle cannot be **trisected** using a compass and pencil only. In other words, we cannot perform a geometric construction to divide angle CAB into *three* equal angles.



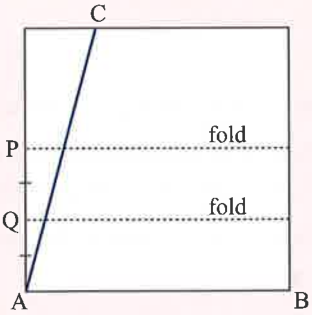
However, Japanese mathematician **Hisashi Abe** showed how to trisect an angle using **origami** or paper-folding.

What to do:

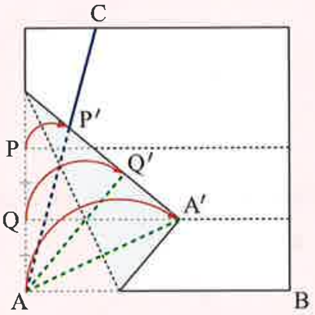
PRINTABLE PAPER

VIDEO DEMO

1. Start with a square piece of paper 20 cm by 20 cm. Label the base [AB], and choose any point C on the top. We will trisect angle CAB.
2. Draw [AC].
3. Fold the page parallel to [AB] at some point P about half-way down the page.
4. Fold the page parallel to [AB] at the point Q which is half-way between A and P.



5. Now fold the paper one more time so that P is placed onto the line [AC] **and** A is placed onto the fold through Q you made in **4**.
6. Mark on the page the positions of Q′ and A′.
7. Unfold the paper, and draw [AQ′] and [AA′]. Angle CAB is trisected by [AQ′] and [AA′].
8. Check your result using a protractor.



KEY WORDS USED IN THIS CHAPTER

- acute angle
- alternate angles
- angle
- co-interior angles
- collinear
- complementary angles
- concurrent
- corresponding angles
- intersecting lines
- line
- line segment
- obtuse angle
- parallel lines
- perpendicular
- perpendicular bisector
- point
- protractor
- ray
- reflex angle
- revolution
- right angle
- straight angle
- supplementary angles
- three point notation
- transversal
- vertex
- vertically opposite

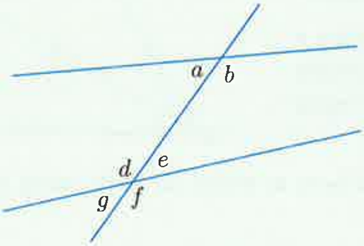
REVIEW SET 2A

1 Find:

a the angle complementary to 53°

b the angle supplementary to 130°.

2



Name the angle:

a corresponding to b

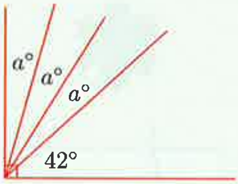
b alternate to e

c co-interior to a

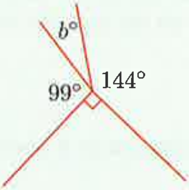
d vertically opposite f.

3 Find the value of the unknown:

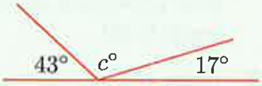
a



b



c

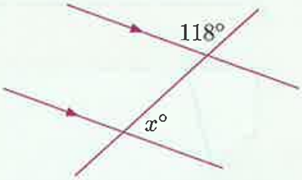


4 How many points are needed to determine the position of a line?

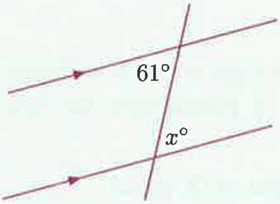
5 Draw a diagram to illustrate the following statement:
"Line segments [AB] and [CD] intersect at P."

6 Find, giving a reason, the value of x:

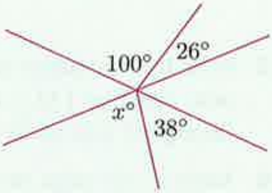
a



b

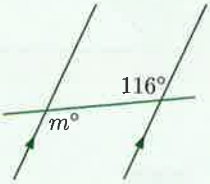


c

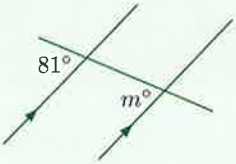


7 Find, giving a reason, the value of m:

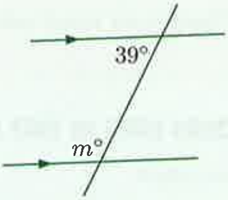
a



b

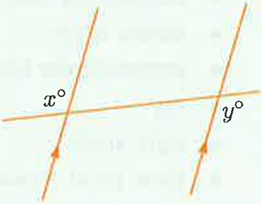


c



8 Write down an equation connecting the unknowns. Give reasons for your answers.

a



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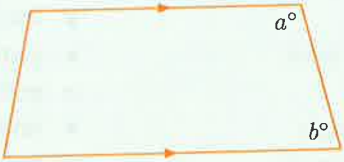
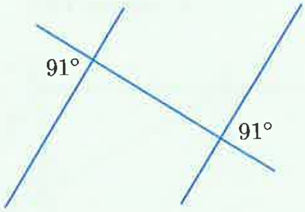
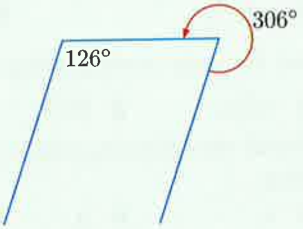


9 State whether each figure contains a pair of parallel lines. Give reasons for your answers.

a



b



10 **a** Draw a line segment [PQ] of length 6 cm.

b Construct the perpendicular bisector of [PQ], meeting [PQ] at X.

c Check your perpendicular bisector by measuring the lengths of [PX] and [QX].

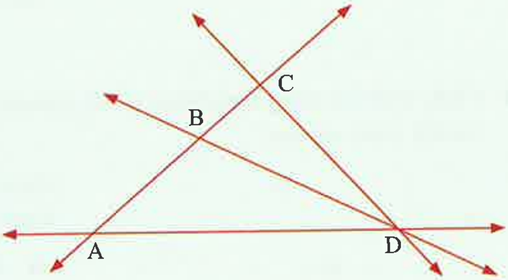
REVIEW SET 2B

1 Consider the diagram alongside.

a Name (AB) in two other ways.

b What can be said about:

- **i** points A, B, and C
- **ii** lines (AD) and (BD)?



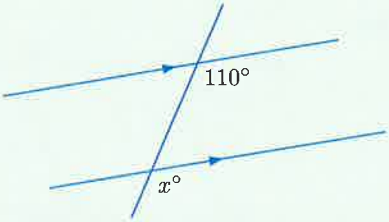
2 Find:

a the angle complementary to 65°

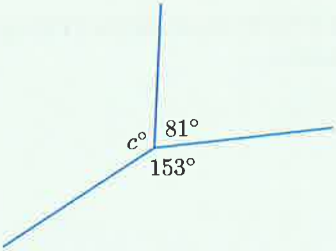
b the angle supplementary to 88°.

3 Find, giving a reason, the value of the unknown:

a

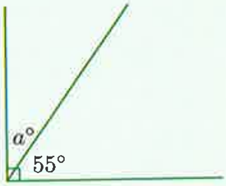


b

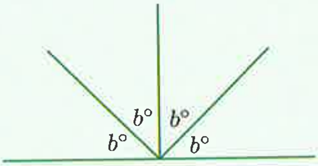


4 Find, giving a reason, the value of the unknown:

a



b



5 Draw [AB] of length 8 cm. Construct an angle of 90° at B using a compass and ruler only. Hence draw [BC] of length 6 cm which is perpendicular to [AB].

6 Draw and label the following angles:

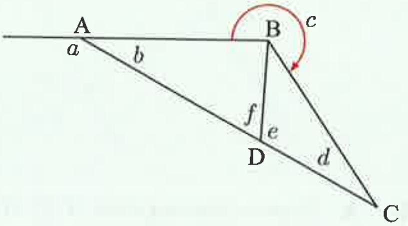
a reflex $B\widehat{A}C$ **b** acute $P\widehat{Q}R$ **c** obtuse $T\widehat{R}S$

7 **a** Find the angle corresponding to:

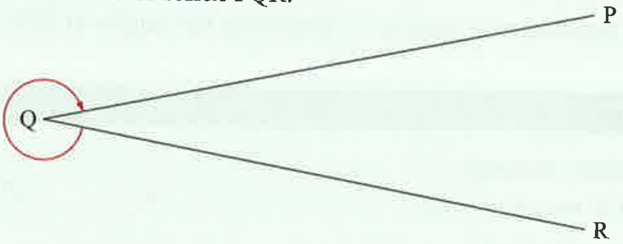
i $B\widehat{D}A$ **ii** $D\widehat{C}B$ **iii** $B\widehat{A}C$

b Classify the following angles as acute, obtuse, or reflex:

i c **ii** a **iii** d

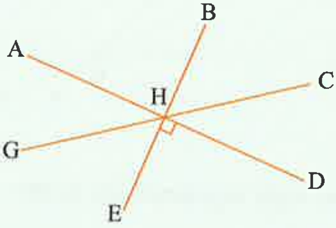


8 **a** Determine the measure of reflex $P\widehat{Q}R$.



b Find, without using a protractor, the measure of acute $P\widehat{Q}R$. Justify your answer.

9



Classify the following angles as complementary, supplementary, or neither:

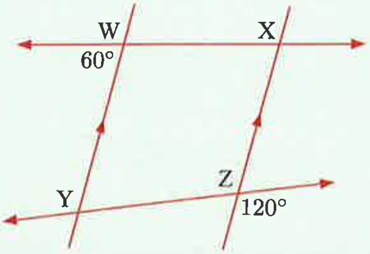
a $C\widehat{H}A$ and $C\widehat{H}D$

b $A\widehat{H}G$ and $A\widehat{H}B$

c $B\widehat{H}C$ and $C\widehat{H}D$

d $B\widehat{H}C$ and $E\widehat{H}G$

10 Decide if (WX) is parallel to (YZ), giving reasons for your answer.



Chapter 3

Positive and negative numbers

Contents:

OPENING PROBLEM

Credit cards are a common way to pay for things. When you buy something, its value is *subtracted* from the card balance. When you make payments onto the card, the amount is *added* to the card balance.

Things to think about:

a Graham's credit card has a balance of $-\$1230$. He purchases a table for \$799 using his card. What will his new balance be?

b Jill's credit card has a balance of $-\$271$. She pays some money onto the card, and her balance now reads $+\$105$. How much money did Jill put on the card?

c Kate buys \$75 worth of groceries each week using her credit card. If her starting balance is $-\$330$, what will her balance be after 5 weeks?

The **natural numbers** 0, 1, 2, 3, 4, 5, ... are useful for solving many mathematical problems. However, there are certain situations where these numbers are not sufficient.

You are probably familiar with the **countdown** for a rocket:
10, 9, 8, 7, 6, 5, 4, 3, 2, 1, BLAST OFF!

But if we keep counting backwards, what comes after zero?

It may seem that we have 'run out' of numbers when we reach zero. However, there are many situations where we need to be able to keep counting and where an answer of less than zero has a sensible meaning.

For example, which of these ideas can you explain, either in words or with a diagram?

- 10 metres below sea level
- 5 degrees below freezing
- owing \$30
- 3 floors below ground level

A OPPOSITES

Many mathematical problems involve **opposites**.

These include:

- *having* money in a bank account and *owing* money to a bank account
- temperature *above* zero and temperature *below* zero
- height *above* sea level and height *below* sea level
- putting purchases on a credit card and paying off the credit card debt.

DISCUSSION

Prepare a list of *ten* opposites which involve numbers.

Instead of using words to distinguish between opposites, we can use **positive** and **negative** numbers.

POSITIVE NUMBERS

Positive numbers are numbers which are greater than zero.

They can be written with a **positive sign** (+) before the number, but we normally see them with no sign at all and we *assume* the number is positive.

For instance:
- '10 metres above sea level' would be written as +10 or just 10
- 'having $30' would be written as +30
- '3 floors above ground level' would be written as +3.

In each case a measurement is being taken from a reference position of zero such as sea level or ground level.

NEGATIVE NUMBERS

Negative numbers are numbers which are less than zero. They are written with a **negative sign** (−) before the number.

For instance:
- '10 metres below sea level' would be written as −10
- 'owing $30' would be written as −30
- '3 floors below ground level' would be written as −3.

Again, the measurement is being taken from a reference position of zero.

Some common uses of positive and negative signs are listed in the table below:

Positive (+)	*Negative* (−)	*Positive* (+)	*Negative* (−)
above	below	fast	slow
increase	decrease	win	lose
profit	loss	north	south
right	left	east	west

EXERCISE 3A

1 Copy and complete the following table:

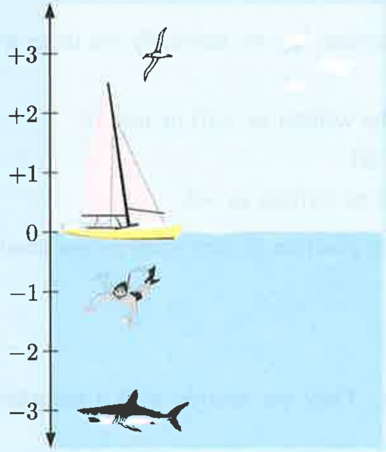
	Statement	*Number*	*Opposite of statement*	*Number*
a	winning by 5 goals	+5		
b	25 m east of a building			
c	a clock is 3 min slow			
d	a gain of 4 kg			
e	a loss of $1250			
f	20 km south of the city			
g	200 m above sea level			
h	11°C below zero			
i	a decrease of $100			
j	one floor above ground level			

Example 1

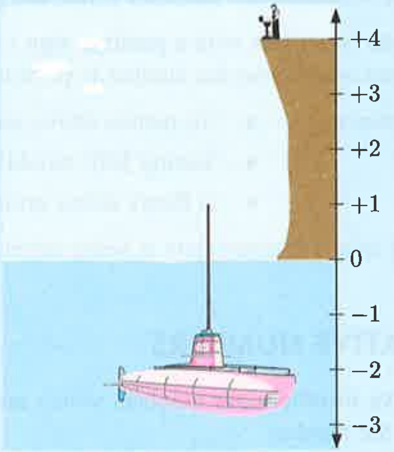
Self Tutor

Write the positive or negative number for the position of each object.
The reference position is the water level.

a



b

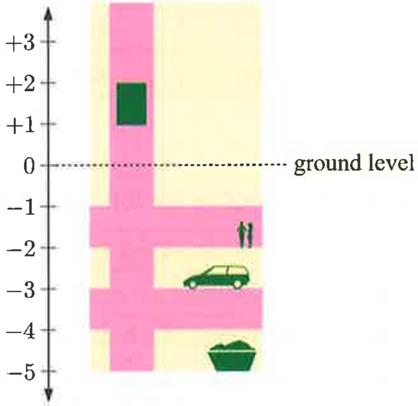


a Positions *above* the water level are marked with *positive numbers.*
∴ the bird is at +3.
The boat is level with the water, so it is at 0.
Positions *below* the water level are marked with *negative numbers.*
∴ the diver is at −1 and the shark is at −3.

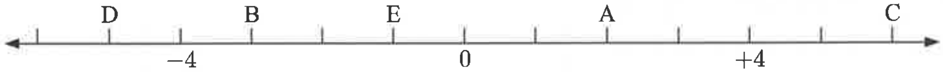
b The clifftop is at +4, the top of the periscope is at +1, the water level is at 0, and the submarine is at −2.

2 Write positive or negative numbers for the position of each object. Use the bottom of each object to make your measurement.

- **a** lift
- **b** car
- **c** people
- **d** rubbish skip



3 With zero as the reference position, right is positive and left is negative. Write numbers for the positions of A, B, C, D, and E.



4 Write these temperatures as positive or negative numbers:

a 4°C below zero	**b** 21°C above zero	**c** 13°C above zero
d 17°C below zero	**e** 32°C above zero	

5 If north is the positive direction, write these positions as positive or negative numbers:

a 3 km south **b** 15 km north **c** 250 km south

d 2000 km south **e** 57 km north

6 State the combined effect of:

- **a** a withdrawal of $70 followed by a deposit of $100
- **b** a rise in temperature of 2°C followed by a fall of 5°C
- **c** a 4 km trip east followed by a 3 km trip west
- **d** 9 steps south followed by 9 steps north
- **e** going up 8 floors in a lift then coming down 9 floors
- **f** a loss of 6 kg followed by a gain of 4 kg.

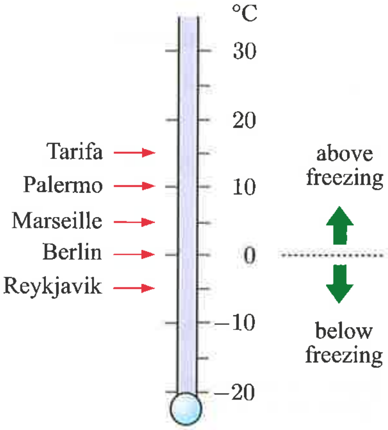
7 A baby girl weighed 3270 grams at birth. The record of her weight for the first five days is shown opposite.

- **a** Write each day's gain or loss as a positive or negative number.
- **b** What was the baby's weight at the end of the five days?

Day 1:	56 g gain
Day 2:	16 g loss
Day 3:	28 g loss
Day 4:	73 g gain
Day 5:	19 g loss

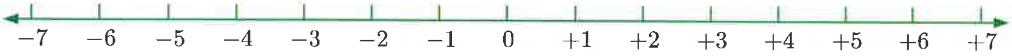
8 The minimum temperatures of some European cities were recorded on December 1st. The results are shown on the thermometer.

- **a** What was the temperature for each city?
- **b** How many °C was Tarifa warmer than:
  - **i** Palermo **ii** Berlin **iii** Reykjavik?
- **c** How many °C was Reykjavik cooler than:
  - **i** Berlin **ii** Marseille **iii** Palermo?
- **d** What was the difference in temperature between:
  - **i** Berlin and Palermo
  - **ii** Marseille and Tarifa?



B THE NUMBER LINE

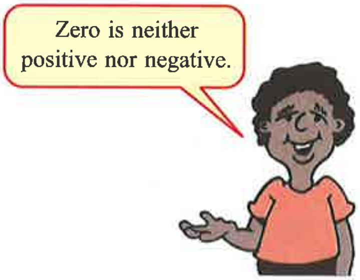
We can represent all whole numbers on a **number line**. This line extends forever in both directions.



The numbers to the **right of zero** are the **positive** numbers.
The numbers to the **left of zero** are the **negative** numbers.

The negative whole numbers, zero, and the positive whole numbers are together known as **integers**.

Pairs of numbers like −7 and 7 are exactly the same distance from 0 but on opposite sides of zero. They are therefore called **opposites**.



Example 2

Self Tutor

What is the opposite of: **a** $+4$ **b** -9?

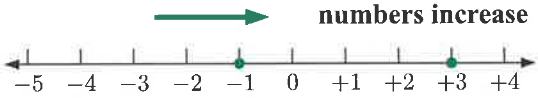
a The opposite of $+4$ is -4.

b The opposite of -9 is $+9$.

Numbers which are **opposites** are the same distance from zero on the number line, but on different sides.

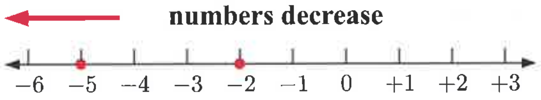
We can use a number line to compare the sizes of different numbers and arrange them in order.

As you move along the number line from *left* to *right*, the numbers increase. In a group of numbers, the number furthest to the right is the greatest number.



$+3$ is *greater* than -1 because it is further to the right on the number line.

As you move along the number line from *right* to *left*, the numbers decrease in size. In a group of numbers, the number furthest to the left is the least number.



-5 is *less* than -2 because it is further to the left on the number line.

We can use the symbols $>$ and $<$ when comparing numbers.

$>$ stands for '**is greater than**'

$<$ stands for '**is less than**'

So, we could write these two statements as $+3 > -1$ and $-5 < -2$.

Example 3

Self Tutor

a Show $+3$ and -2 on a number line and write a sentence comparing their size.

b Write the statement $-7 > -4$ in words, then state whether it is true or false.

a

Since $+3$ is further to the right, $+3$ is greater than -2.
We could also say -2 is less than $+3$.

b The statement reads 'negative 7 is greater than negative 4'.
This is false because -7 is to the **left** of -4, and so it is less than -4.

EXERCISE 3B

1 Write the opposite of each number:

a -3 **b** 15 **c** -10 **d** -9 **e** 38 **f** -6 **g** $+7$ **h** 0

2 Show -1 and -6 on a number line and write a sentence comparing their size.

3 Write the statement $2 > -5$ in words, then state whether it is true or false.

4 Write *true* or *false* for each of the following statements:

a $6 > 2$ **b** $-4 < 15$ **c** $17 > 18$

d $-2 < 19$ **e** $-13 > 5$ **f** $-20 < -12$

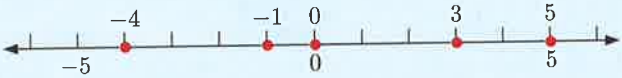
5 Insert either $<$ or $>$ in place of ☐ to make each statement true:

a 8 ☐ 6 **b** 18 ☐ 7 **c** −9 ☐ −4

d −3 ☐ 15 **e** 20 ☐ −15 **f** −6 ☐ −2

Example 4 🔊 **Self Tutor**

Locate the values of $5, 3, 0, -1,$ and -4 on a number line.



6 Draw number lines to show the following sets of numbers. Use a different number line for each set.

a $-9, 2, 5$ **b** $2, 6, 8, -3, -4$

c $9, -4, -9, 1, 6, -6$ **d** $-3, 2, 5, -5, 0, -1$

7 Display the number set $4, -2, 1, -1$ on a number line.
Hence arrange the numbers from least to greatest.

8 Display the number set $5, -3, 0, 2, -4, 6, -1$ on a number line.
Hence arrange the numbers from least to greatest.

9 **a** Arrange in descending order:
$0, -5, 8, -7, -2,$ and 6.

b Arrange in ascending order:
$0, -10, 8, 7, -7,$ and -2.

Descending means downwards. Ascending means upwards.

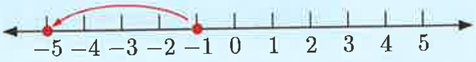
10 The temperatures of six cities were:
Ulaanbaatar 3°C, Singapore 33°C, Melbourne 19°C, Oslo −4°C, Moscow −6°C, Tokyo 1°C.
Place them in order from coldest to hottest.

Example 5 🔊 **Self Tutor**

Use a number line to: **a** increase -2 by 5 **b** decrease -1 by 4.

a To *increase* -2 by 5, we move along the number line 5 units to the *right*.
The result is $+3$.

b To *decrease* -1 by 4, we move along the number line 4 units to the *left*.
The result is -5.



11 Use a number line to:

a increase -5 by 1
b decrease 3 by 4
c decrease -5 by 6
d increase -4 by 3
e increase -5 by 5
f decrease -1 by 6

12 Use a number line to find:

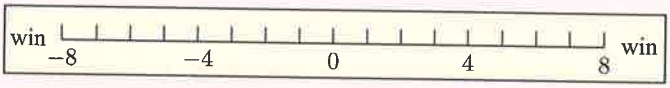
a $3+7$
b $-3+5$
c $5-9$
d $4-8$
e $0-6$
f $-11+7$
g $-2-6$
h $-3-9$

ACTIVITY 1 NUMBER GAME FOR 2 PLAYERS

You will need:

PRINTABLE NUMBER LINE

- 2 different coloured dice
- 2 counters
- a number line



How to play:

1. Choose one die to represent the positive numbers 1 to 6 and the other to represent the negative numbers -1 to -6.
2. Start the game with both counters on zero.
3. Take it in turns to throw both dice and move your own counter according to the sum of the numbers thrown.
4. Keep going until one player goes over either end. Their score must be greater than 8 or less than -8. That person wins!

C ADDING AND SUBTRACTING NEGATIVES

ADDING A NEGATIVE NUMBER

We know that $4+3=7$, but what is the value of $4+-3$?

Consider the following true statements:

$$4+3=7$$
$$4+2=6$$
$$4+1=5$$
$$4+0=4$$

As the number being added to 4 decreases by 1, the final answer also decreases by 1.

Continuing this pattern gives:

$$4+-1=3$$
$$4+-2=2$$
$$4+-3=1$$
$$4+-4=0$$

Compare with:

$$4-1=3$$
$$4-2=2$$
$$4-3=1$$
$$4-4=0$$

Adding a **negative** number is equivalent to **subtracting** its opposite.

For example, $2+-6$ is equivalent to $2-6$.

SUBTRACTING A NEGATIVE NUMBER

We know that $4-3=1$, but what is the value of $4--3$?

Consider the following true statements:

$$4-3=1$$
$$4-2=2$$
$$4-1=3$$
$$4-0=4$$

As the number being subtracted decreases by 1, the answer increases by 1.

Continuing this pattern gives:

Continuing this pattern gives:	$4--1=5$	Compare with:	$4+1=5$
	$4--2=6$		$4+2=6$
	$4--3=7$		$4+3=7$
	$4--4=8$		$4+4=8$

Subtracting a **negative** number is equivalent to **adding** its opposite.

For example, $3--5$ is equivalent to $3+5$.

Example 6 **Self Tutor**

Simplify and then evaluate:

a $2+-5$ **b** $2--5$ **c** $-2+-5$ **d** $-2--5$

a $2+-5$
$=2-5$
$=-3$

b $2--5$
$=2+5$
$=7$

c $-2+-5$
$=-2-5$
$=-7$

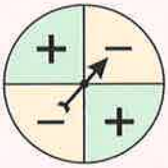
d $-2--5$
$=-2+5$
$=3$

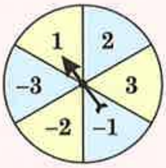
ACTIVITY 2 SHARKS

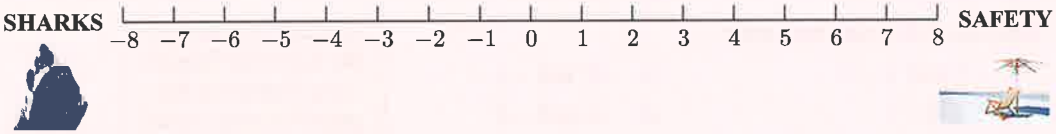
This Activity builds on from the Number game for 2 players on page **68**.

You will need:

- 2 spinners
- 2 counters
- a number line







How to play:

PRINTABLE NUMBER LINE

1 Both players start at zero.

2 One player spins the **direction** spinner.
If you spin a positive direction (+) you **face** towards the SAFETY end.
If you spin a negative direction (−) you **face** towards the SHARKS end.

3 The same player then spins the **steps** spinner.
If you spin a positive value, step **forward** the number of steps indicated.
If you spin a negative value, step **backward** the number of steps indicated.

4 Players take it in turns to move.

5 Keep playing until a player reaches either the SHARKS or SAFETY. If they reach the SHARKS, they lose. If they reach SAFETY, they win!

Discussion

What combinations of spins allow you to head to safety?

EXERCISE 3C

1 Simplify if possible, and then evaluate:

a $4-3$ **b** $4+3$ **c** $-4-3$ **d** $-4+3$
e $4+-3$ **f** $4--3$ **g** $-4+-3$ **h** $-4--3$

2 Simplify if possible, and then evaluate:

a $2-6$ **b** $2+6$ **c** $-2-6$ **d** $-2+6$
e $2+-6$ **f** $2--6$ **g** $-2+-6$ **h** $-2--6$

3 Evaluate:

a $1+-2$ **b** $-2+-6$ **c** $6+-8$ **d** $-3--7$
e $3+-13$ **f** $4--5$ **g** $-7--10$ **h** $-9+-8$

4 A maintenance man in an office block starts his working day on the ground floor. To fulfil his duties he goes up 12 floors, down 3 floors, up 5 floors, down 7 floors, up 1 floor, and then down 6 floors. What floor does he end up on?

5 Evaluate:

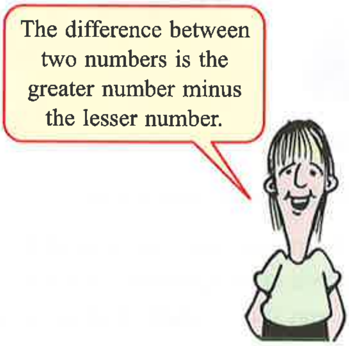
a $-2-7$ **b** $-7+5$ **c** $-6+-3$
d $-13-8$ **e** $-7+-5$ **f** $-6--9$
g $2+-12$ **h** $-4--3$ **i** $-11--17$

6 Simplify and hence evaluate:

a $4+7+-10$ **b** $8--2+-4$ **c** $-4--6--1$
d $-12--9+5$ **e** $-4--5+-8$ **f** $-3--10+10$
g $-3-7+-7$ **h** $-10+-8+-9$ **i** $-1+10--7$

7 Find the difference between:

a -5 and 4 **b** -2 and -5
c 8 and -1 **d** -4 and -2
e 8 and -8 **f** -7 and 8



The difference between two numbers is the greater number minus the lesser number.

8 Salt Lake City recorded the following maximum temperatures for a week:

Mon 5°C, Tues −3°C, Wed −7°C,
Thurs 2°C, Fri 1°C, Sat −4°C,
Sun −1°C.

What was the *average* daily maximum temperature for the week?

To find the *average*, add the 7 temperatures and then divide this sum by 7.

ACTIVITY 3 MAGIC SQUARES

4	3	8
9	5	1
2	7	6

A **magic square** is a square grid filled with **consecutive whole numbers** so that each row, column, and diagonal has the same sum.

For example, this magic square contains the numbers 1 to 9, and has the **magic sum** of 15 along every row, column, and diagonal.

What to do:

1 Copy and complete the following magic squares:

a

4		8
	7	
		10

b

		7	12
15		9	6
	5		
8	11	2	

2 Magic squares may also contain negative numbers.

2	−5	0
−3	−1	1
−2	3	−4

a Is the square alongside a magic square? If so, what is the magic sum?

b Make a new magic square by adding 2 to each number in the magic square given. State the new magic sum.

c Make a new magic square by subtracting 3 from each number in the magic square given. State the new magic sum.

3 If 3 was added to each number in a 3×3 magic square, what would happen to the magic sum? Use your answers in **2** to help you.

4 Copy and complete the following magic squares:

a

−4		0
	−1	
		2

b

3			−9
−8			
−7		−4	5
6		−1	−6

c

4	11		−5	2
		−6	1	
−9	−7	0	7	
−3			8	
−2			−11	−4

D MULTIPLYING NEGATIVE NUMBERS

We have already seen how to add and subtract negative numbers. In this Section we look for rules for their **multiplication**.

Consider the following true statements:

$$\begin{array}{ll} 3 \times 3 = 9 & \\ & -3 \\ 3 \times 2 = 6 & \\ & -3 \\ 3 \times 1 = 3 & \\ & -3 \\ 3 \times 0 = 0 & \end{array}$$

As the number being multiplied by 3 decreases by 1, the final answer decreases by 3.

Continuing this pattern gives:

$$\begin{array}{ll} 3 \times -1 = -3 & \\ & -3 \\ 3 \times -2 = -6 & \\ & -3 \\ 3 \times -3 = -9 & \end{array}$$

We can change the order in which numbers are multiplied, so we can also say that

$$\begin{array}{l} -1 \times 3 = -3 \\ -2 \times 3 = -6 \\ -3 \times 3 = -9 \end{array}$$

These suggest that (**positive**) × (**negative**) = (**negative**) and (**negative**) × (**positive**) = (**negative**).

A similar pattern shows that:

$$\begin{array}{ll} -3 \times 3 = -9 & \\ & +3 \\ -3 \times 2 = -6 & \\ & +3 \\ -3 \times 1 = -3 & \\ & +3 \\ -3 \times 0 = 0 & \end{array}$$

As the number being multiplied by -3 decreases by 1, the final answer increases by 3.

Continuing this pattern gives:

$$\begin{array}{ll} -3 \times -1 = 3 & \\ & +3 \\ -3 \times -2 = 6 & \\ & +3 \\ -3 \times -3 = 9 & \end{array}$$

This suggests that (**negative**) × (**negative**) = (**positive**).

RULES FOR MULTIPLICATION

- (**positive**) × (**positive**) = (**positive**)
- (**positive**) × (**negative**) = (**negative**)
- (**negative**) × (**positive**) = (**negative**)
- (**negative**) × (**negative**) = (**positive**)

When the signs are the **same**,
the answer is **positive**.
When the signs are **different**,
the answer is **negative**.

Example 7

Self Tutor

Evaluate:

a 2×5 **b** 2×-5 **c** -2×5 **d** -2×-5

a $2 \times 5 = 10$ **b** $2 \times -5 = -10$ **c** $-2 \times 5 = -10$ **d** $-2 \times -5 = 10$

EXERCISE 3D

1 Evaluate:

a 6×4 **b** 6×-4 **c** -6×4 **d** -6×-4
e 4×-6 **f** -4×6 **g** 4×6 **h** -4×-6

2 Evaluate:

a 3×-2 **b** -10×3 **c** -2×-7 **d** 5×-10
e -6×8 **f** 5×-9 **g** -8×11 **h** 3×-11
i 9×-9 **j** -12×-2 **k** 11×-5 **l** -6×-7

3 Determine the missing number in each of the following:

a $-2 \times \square = -2$ **b** $\square \times 5 = -10$ **c** $\square \times 1 = -11$
d $8 \times \square = -32$ **e** $-3 \times \square = 18$ **f** $8 \times \square = -16$
g $-2 \times \square = -8$ **h** $9 \times \square = -9$ **i** $\square \times -7 = -42$
j $\square \times -10 = 30$ **k** $4 \times \square = -12$ **l** $\square \times 12 = -120$

4 Solve the following questions:

a A skydiver falls 70 metres per second for 4 seconds. How many metres does he fall?

b When Tania bought a new bicycle for \$540, she borrowed the money from her parents. She repays them \$70 per week for 6 weeks. How much does Tania still owe her parents?

5 Evaluate:

a $-5 \times 8 \times 5$ **b** $-7 \times 3 \times -3$ **c** $-2 \times 5 \times -2$ **d** $(-3)^3$
e $-8 \times 5 \times -5$ **f** $(-2)^2$ **g** $-7 \times (-1)^2$ **h** $-2 \times 9 \times -5$
i $(-5)^3$ **j** -6×2^2 **k** $-8 \times 2 \times -3$ **l** $(-4)^2 \times 5$

6 Evaluate:

a $(-1)^2$ **b** $(-1)^4$ **c** $(-1)^5$ **d** $(-1)^7$ **e** $(-1)^{10}$

What do you notice?

E DIVIDING NEGATIVE NUMBERS

In this Section we look for rules for the **division** of negative numbers.

We know that $12 \div 4 = 3$, but how do we calculate:

- $12 \div -4$
- $-12 \div 4$
- $-12 \div -4$?

The rules for division are identical to those for multiplication. This is not surprising because multiplication and division are **inverse operations**.

For example, $\div$ by 2 is the same as $\times$ by $\frac{1}{2}$.

RULES FOR DIVISION

(positive) ÷ (positive) = (positive)
(positive) ÷ (negative) = (negative)
(negative) ÷ (positive) = (negative)
(negative) ÷ (negative) = (positive)

Dividing numbers with the **same** signs gives a **positive**. Dividing numbers with **different** signs gives a **negative**.

Example 8 **Self Tutor**

Evaluate:

a $-6 \div 2$ **b** $8 \div -4$

a $-6 \div 2 = -3$ **b** $8 \div -4 = -2$

EXERCISE 3E

1 Evaluate:

a $15 \div 3$
b $15 \div -3$
c $-15 \div 3$
d $-15 \div -3$
e $45 \div 9$
f $-45 \div -9$
g $-45 \div 9$
h $45 \div -9$
i $6 \div 6$
j $6 \div -6$
k $-6 \div 6$
l $-6 \div -6$
m $44 \div 4$
n $-44 \div 4$
o $-44 \div -4$
p $44 \div -4$

2 Determine the missing number in each of the following:

a $12 \div \square = -3$
b $\square \div -2 = 3$
c $-4 \div \square = 1$
d $\square \div 5 = -5$
e $-18 \div \square = 2$
f $\square \div 4 = -3$
g $\square \div -2 = 4$
h $30 \div \square = -6$
i $\square \div -8 = 5$
j $36 \div \square = -4$
k $-15 \div \square = -5$
l $\square \div -4 = 7$
m $72 \div \square = -9$
n $\square \div 10 = -12$
o $\square \div -12 = -12$
p $-96 \div \square = -8$

3 Solve the following questions:

a A company owned equally by seven people has a debt of $350 000. What is each person's share of the debt?

b One night in the Gobi Desert, the temperature drops from 33°C to −12°C in five hours. What is the average temperature change per hour?

The *average* temperature change is the total temperature change divided by the number of hours.

F COMBINED OPERATIONS

The order of operations rules also apply to negative numbers.

- **B**rackets are evaluated first.
- **E**xponents are calculated next.
- **D**ivisions and **M**ultiplications are done next, in the order that they appear.
- **A**dditions and **S**ubtractions are then done, in the order that they appear.

Example 9 **Self Tutor**

Use the correct order of operations rules to calculate:

a $5 + -8 \times 3$ **b** $-5 - 15 \div -5$

a $5 + -8 \times 3$
$= 5 + -24$ {multiplication first}
$= 5 - 24$ {simplify}
$= -19$

b $-5 - 15 \div -5$
$= -5 - -3$ {division first}
$= -5 + 3$ {simplify}
$= -2$

EXERCISE 3F

1 Find, using the order of operations rules:

a $3 + 4 \div -2$ **b** $-1 + -3 \times 2$ **c** $8 \div -2 + 5$
d $-3 \times -2 - 4$ **e** $2 - 6 \div -3$ **f** $-2 \times 4 + -7$
g $7 - 3 \times -3$ **h** $-4 \times -5 - 12$ **i** $3 - 6 \div -6$
j $(-3 + 4) \times -7$ **k** $15 \div (4 - 7)$ **l** $-3 \times (-2 + 5)$

2 Do -3^2 and $(-3)^2$ have the same value? Explain your answer.

3 Min's company makes a \$100 000 profit per month for eight months, and then an \$80 000 loss for each of the next four months. Find her company's total profit or loss.

4 A computer store has the following sales record over a six-week period:

Week 1:	\$388 profit	Week 2:	\$1373 loss	Week 3:	\$179 loss
Week 4:	\$3013 profit	Week 5:	\$832 profit	Week 6:	\$1763 loss.

a What was the store's overall profit or loss for this period?

b What was the store's *average* weekly profit or loss during this period?

5 In indoor cricket, the person batting is penalised 5 runs for each wicket lost.
Josh lost 6 wickets, and scored 17 runs. What was his final score?

G USING YOUR CALCULATOR

Calculators have a $\boxed{(-)}$ or $\boxed{+/-}$ key to specify a negative number.

On most calculators we press this key *before* the number, for example $\boxed{(-)}$ 2.

On some older calculators, however, we press it *after* the number, for example 2 $\boxed{+/-}$.

You will need to check what keys your calculator has and the sequence in which they need to be pressed.

Example 10 **Self Tutor**

Evaluate the following using your calculator:

a $-14 + -71$ **b** $22 - -45$ **c** $-8 \times -4 \div (7 - 11)$

Answers

a Press $\boxed{(-)}$ 14 $\boxed{+}$ $\boxed{(-)}$ 71 $\boxed{\text{ENTER}}$ -85

or Press 14 $\boxed{+/-}$ $\boxed{+}$ 71 $\boxed{+/-}$ $\boxed{=}$

b Press 22 $\boxed{-}$ $\boxed{(-)}$ 45 $\boxed{\text{ENTER}}$ 67

c Press $\boxed{(-)}$ 8 $\boxed{\times}$ $\boxed{(-)}$ 4 $\boxed{\div}$ $\boxed{(}$ 7 $\boxed{-}$ 11 $\boxed{)}$ $\boxed{\text{ENTER}}$ -8

EXERCISE 3G

1 Use your calculator to evaluate:

a $-35 + 61 - 47$ **b** $-26 - -41 + 38$ **c** $-92 - 16 + 57$

d $-12 - -87 - 129$ **e** 38×-25 **f** $-1280 \div 320$

g $-48 \div -12 \times -6$ **h** $-630 \times 8 \div -36$

2 In windy conditions a helicopter falls 30 m, rises 45 m, falls 20 m, rises 10 m, falls 15 m, then rises 12 m. How far is it now above or below its original position?

3 Regina has \$645 in the bank. She withdraws \$423, deposits \$371, deposits \$229, and then withdraws \$738. What is her new bank balance?

4 Abdul wanted to buy a nice car, so he saved €80 per week for 5 years. How much extra money did he need to borrow, to buy a car valued at €26 000?

5 Answer the questions in the **Opening Problem** on page **62**.

KEY WORDS USED IN THIS CHAPTER

- addition
- integer
- negative number
- positive number
- division
- less than
- number line
- subtraction
- greater than
- multiplication
- opposite

REVIEW SET 3A

1 Use a number line to evaluate:

a $4-7$ **b** $-3+6$ **c** 3×-2 **d** -3×-4

2 **a** What must I divide 96 by to get -8?

b Copy and complete: negative $\div$ positive = ...

c Insert $<$ or $>$ to make the following true: $-5 \ \square \ 3$.

3 State the combined effect of:

a borrowing 5 books and returning 2 **b** depositing $78 and withdrawing $88.

4 Evaluate:

a $5 - -3 + -7$ **b** $(-8)^2$ **c** $-20 + 15 \div -3$

5 **a** Arrange in ascending order: 2, -4, 0, -3, -6, 7, 3.

b Find the difference between the greatest and least values in **a**.

6 **a** Use a number line to decrease 2 by 9.

b Insert $<$ or $>$ between 3 and -8 to make a true statement.

c Simplify $12\times(-1)^3$.

7 Roger's business has $12 500 in the bank. He must pay each of his 8 employees a wage of $389 per week for 4 weeks. How much money will be remaining in the bank?

8 Write the opposite of:

a $+9$°C **b** -28 m **c** $+36$ points

9 Which is the greater distance:

A rising from 77 m below sea level to 12 m above sea level, or

B falling from 409 m above sea level to 321 m above sea level?

10 Beck, Cathy, Emily, and Ying agreed to meet at a coffee shop. Beck was 9 minutes late, Cathy was 4 minutes early, Emily was 17 minutes late, and Ying was 10 minutes early.

a Who arrived first?

b Who arrived closest to the agreed time?

c Find the difference between the arrival times of:

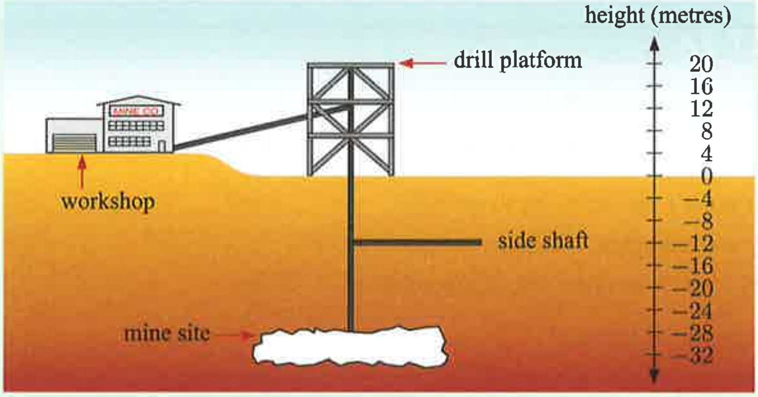
i Beck and Ying **ii** Cathy and Emily

iii Beck and Emily.

11 In a mathematics competition, students are awarded 3 points for a right answer, and penalised 4 points for a wrong answer. Amy gave 6 wrong answers and 24 right answers. Sean gave 14 wrong answers and 16 right answers.

- **a** How many points did each student score?
- **b** By how many points did Amy beat Sean?

12



The illustration shows some important parts of a mine.

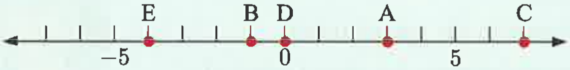
- **a** State the level of:
  - **i** the top of the drill platform
  - **ii** the bottom of the workshop
  - **iii** the side shaft
  - **iv** the top of the mine site.
- **b** How much higher is the drill platform than the side shaft?
- **c** How much lower is the mine site than the workshop?
- **d** Find the difference in height between the mine site and the drill platform.

13 In golf, a player's score is expressed relative to the *par* score for the course. For a par 72 course, a player who completes the course in 75 strokes receives a score of 3 over par, or +3. A player who takes 68 strokes receives a score of 4 under par, or −4.

- **a** For a par 72 course, find the score for a player who completes the round in:
  - **i** 70 strokes
  - **ii** 78 strokes
  - **iii** 67 strokes.
- **b** A tournament is played over 4 rounds.
  - **i** Trevor shoots a score of −3 for each of the rounds. What is his score at the end of the tournament?
  - **ii** Wayne shoots scores of −5, +2, and −4 for the first 3 rounds. What score does he need in the final round to beat Trevor?

REVIEW SET 3B

1 Indicate the position of each point using a number:



2 **a** Copy and complete: negative × negative = ...

b Evaluate -7×-11.

3 **a** Use a number line to increase -15 by 8.

b *Hence* evaluate $3 \times (-15 + 8)$.

4 Evaluate:

a $(-1)^5$ **b** $-12 \div 4 + 13$ **c** $-2 \times -3 \times -4$

5 **a** Arrange in descending order: 5, -4, 0, 1, -1, 3, -6.

b Find the difference between the largest and smallest values.

c Find the sum of the numbers in **a**.

6 Greg wants to lose 12 kg. He initially loses 5 kg, then gains 1 kg per week for 3 weeks, then loses 2 kg.

a Find Greg's overall weight loss or gain.

b How much weight must Greg now lose to reach his goal?

7 If ground level is marked as 0 and the top of a 15 m high oil rig is assigned the integer 15, what integer would be assigned to:

a the top of a 2 m high boundary fence

b the bottom of a mine shaft 129 m below ground level

c the top of a truck 4 m tall?

8 The minimum temperatures for a week in Beijing were:
-5°C, -4°C, -5°C, 2°C, 3°C, 3°C, -1°C.
Calculate the average minimum temperature for that week.

9 A warehouse contains 24 pianos. Each week 3 more are sent to the warehouse from the factory. Over a 4 week period, 1, 6, 3, and 5 pianos are sold. How many pianos are now in the warehouse?

10 A new office building has two lifts. There are two floors of underground carparking, and 9 floors above ground. The lifts can move up and down by 1 floor every 5 seconds.

a Lift 1 is on floor 7 and lift 2 is at B1. How far are the lifts apart?

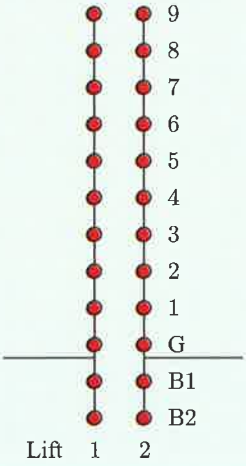
b Lift 1 moves down for 15 seconds. Where is it now?

c Lift 2 moves up for 25 seconds. Where is it now?

d Suppose the lifts are both at floor 3. Lift 1 starts moving up at the same time as lift 2 starts moving down.

i Write an expression for the distance between the lifts after 10 seconds.

ii Will lift 1 reach the top floor before lift 2 reaches the bottom floor?



11 Previous experience has taught Rapunzel that using one's hair to escape is a bad idea. She instead knots bedsheets together, keeping the knots at equal intervals. From her window to the ground is a distance of 25 knots.

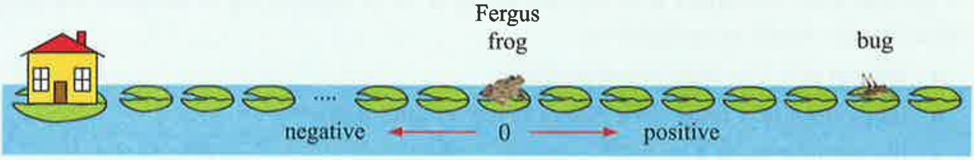
a If her window is 0, what number represents the ground?

b A climbing plant on the side of the tower reaches 11 knots up from the ground. What number represents the top of the plant?

c Rapunzel starts the climb at her window. She climbs down at the rate of 3 knots a minute.

- **i** What is her position after 4 minutes?
- **ii** How far above or below the top of the plant is she?

12



Fergus the Frog is on the lilypad at 0. He has spied a bug which he would like for dinner.

a Write down a number to represent the position of the bug.

b Fergus has forgotten his bow tie for dinner. He jumps back to his house, 2 lilypads at a time. If Fergus makes 4 jumps, what is the position of his house?

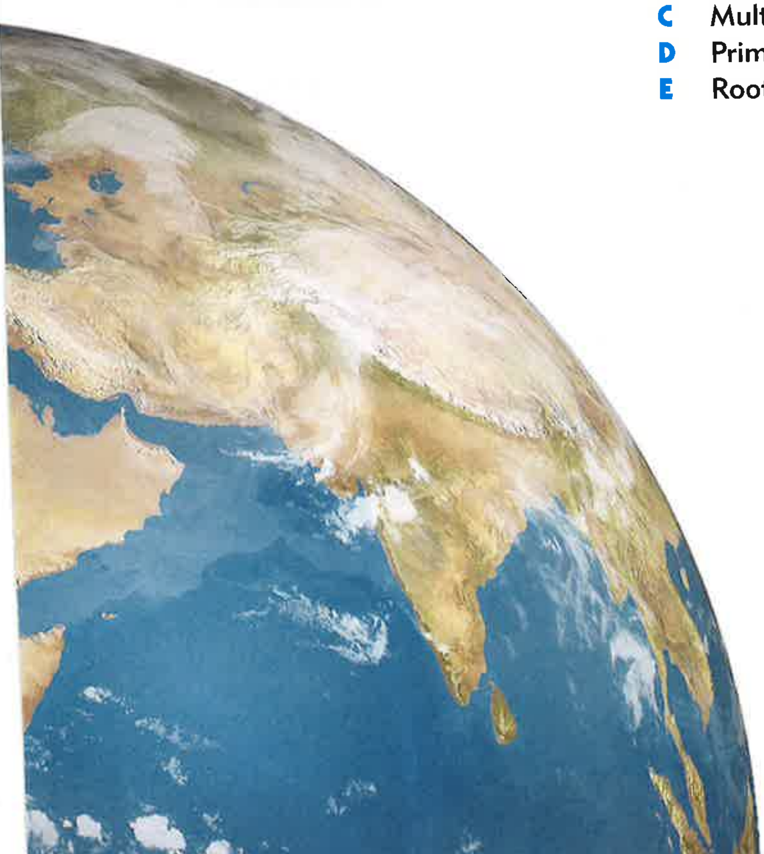
c Meanwhile, the bug has moved one lilypad to the right for each of Fergus' jumps. How far away is the bug now from 0?

d Not wanting to miss his dinner, Fergus chases after the bug. He jumps forward 3 lilypads every time, with the bug fleeing one lilypad for each of Fergus' jumps. How many jumps does Fergus need to catch the bug?

Chapter 4

Properties of numbers

Contents:



OPENING PROBLEM

A school building has 100 lockers. The lockers are lined up in a row, all closed.

Consider the following scenario:

- Eddie goes down the row and opens every locker.
- Terry then goes down the row and closes every *second* locker, starting with locker number 2.
- Nick then goes down the row and *changes the state* of every *third* locker, starting with locker number 3. So, if the locker is closed he opens it, and if the locker is open he closes it.

Things to think about:

a How many lockers does Terry close?
b How many lockers does Nick change?
c How many lockers are touched by both Terry and Nick? Which lockers are they?
d After the three students have passed, how many lockers are left open?

The questions in the **Opening Problem** may appear very difficult and time consuming. However, some knowledge of the properties of numbers can make solving them relatively easy.

A DIVISIBILITY

One number is **divisible** by another if, when we divide, the quotient is a whole number.

For example: 16 is divisible by 2 because $16 \div 2 = 8$.
16 is not divisible by 3 because $16 \div 3 = 5$ with remainder 1.

EVEN AND ODD NUMBERS

A natural number is **even** if it is divisible by 2.
A natural number is **odd** if it is not divisible by 2.

For example: 18 is even because $18 \div 2 = 9$.
23 is odd because $23 \div 2 = 11$ with remainder 1.

EXERCISE 4A.1

1 Decide whether:

a 28 is divisible by 4
b 17 is divisible by 3
c 31 is divisible by 5
d 54 is divisible by 9
e 70 is divisible by 10
f 76 is divisible by 8.

2 Decide whether the following numbers are even or odd:

a 16
b 17
c 45
d 38
e 52
f 89
g 130
h 143

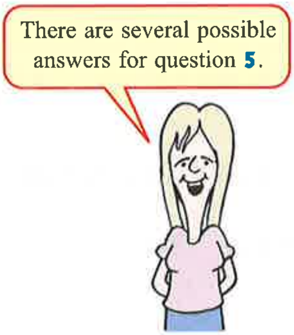
3 Find the number between 90 and 100 which is divisible by 8.

4 Write down the numbers between 20 and 40 which are:

- **a** even and divisible by 3
- **b** odd and divisible by 11
- **c** even and not divisible by 4
- **d** odd and not divisible by 3
- **e** even and square
- **f** odd and cubic.

5 Write the number 60 as:

- **a** the sum of two even numbers
- **b** the sum of two odd numbers
- **c** the product of two even numbers
- **d** the product of an even number and an odd number.



6 Decide whether the following are even or odd:

- **a** the sum of three even numbers
- **b** the sum of three odd numbers
- **c** the product of an even and two odd numbers
- **d** the sum of four odd numbers.

DIVISIBILITY TESTS

We sometimes need to quickly decide whether one number is divisible by another. Obviously this can be done using a calculator provided the number is not too big. However, there are also some simple rules we can use to test for divisibility, without actually doing the division!

For example: Any **even** number is divisible by 2, so its last digit must be 0, 2, 4, 6, or 8.
Any **odd** number is not divisible by 2, so its last digit must be 1, 3, 5, 7, or 9.

INVESTIGATION 1 DIVISIBILITY BY 4 AND 9

One of the joys of mathematics comes from investigating and discovering things for yourself. In this Investigation you should discover rules for divisibility by 4 and by 9.

What to do:

1 Copy and complete the following table. Start with the third column by writing down the last two digits of each number. Then use your calculator to check the numbers for divisibility by 4.

Number	*Divisibility by 4 (Yes/No)*	*Last 2 digits*	*Divisibility by 4 of number formed by last 2 digits (Yes/No)*
81		81	
154		54	
774			
3624			
6957			
9908			

2 Copy and complete: "A natural number is divisible by 4 if ......".

3 Copy and complete the following table:

Number	*Divisibility by 9 (Yes/No)*	*Sum of its digits*
81		$8+1=9$
154		
774		
3624		
6957		
9908		

4 Copy and complete: "A natural number is divisible by 9 if ......".

DIVISIBILITY TESTS FOR NATURAL NUMBERS

Number	**Divisibility Test**
2	If the last digit is even, then the number is divisible by 2.
3	If the sum of the digits is divisible by 3, then the number is divisible by 3.
4	If the number formed by the last *two* digits is divisible by 4, then the original number is divisible by 4.
5	If the last digit is 0 or 5, then the number is divisible by 5.
6	If the number is divisible by both 2 and 3, then it is divisible by 6.
9	If the sum of the digits is divisible by 9, then the number is divisible by 9.
10	If the last digit is 0, then the number is divisible by 10.
11	Add the digits in odd positions. Add the digits in the even positions. Find the difference between your two answers. If the difference is divisible by 11, the original number is divisible by 11.

Example 1 **Self Tutor**

Test for divisibility by 3 and 11:

a 846 **b** 2618

a The sum of the digits of 846 is $8+4+6=18$.
Since 18 is divisible by 3, so is 846.

For the number 846, the sum of the digits in the odd positions is $6+8=14$ and the sum of the digits in the even positions is 4.
The difference is $14-4=10$.
Since 10 is not divisible by 11, 846 is not divisible by 11.

b The sum of the digits of 2618 is $2+6+1+8=17$.
Since 17 is not divisible by 3, 2618 is not divisible by 3.

For the number 2618, the sum of the digits in the odd positions is $1+2=3$ and the sum of the digits in the even positions is $8+6=14$.
The difference is $14-3=11$, which is divisible by 11.
So, 2618 is divisible by 11.

EXERCISE 4A.2

1 Answer *true* or *false* for the following:

a 26 is divisible by 2 **b** 5221 is divisible by 5 **c** 127 is divisible by 3
d 1010 is divisible by 10 **e** 1900 is divisible by 4 **f** 1326 is divisible by 3
g 111 is divisible by 2 **h** 166 is divisible by 9 **i** 9288 is divisible by 9
j 247 is divisible by 11 **k** 5922 is divisible by 6 **l** 5071 is divisible by 11.

2 Determine whether the following are divisible by 3:

a 87 **b** 512 **c** 977 **d** 1002 **e** 56 947 **f** 123 456 789

3 Determine whether the following are divisible by 4:

a 2250 **b** 1024 **c** 30 420 **d** 215 962

4 Determine whether the following are divisible by 9:

a 801 **b** 2763 **c** 3079 **d** 269 730

5 Determine whether the following are divisible by 11:

a 596 **b** 7282 **c** 10 837 **d** 908 281

6 Test the following numbers for divisibility by 2, 3, 4, 5, and 9:

a 250 **b** 3609 **c** 12 345 **d** 14 641

7 A four-digit number has digit form '$a5b1$'. If the number is divisible by 3, what are the possible values of $a+b$?

8 Consider the five-digit number 82 51☐. What digits could replace ☐ so that the number is divisible by:

a 3 **b** 4 **c** 5 **d** 6 **e** 9 **f** 11?

9 **a** Rearrange the digits 1, 4, 5, and 8 to form a number which is divisible by:

i 5 **ii** 4

b Explain why every rearrangement of the digits in **a** will form a number which is divisible by 9.

10 Simone's teacher told her that 1738 is divisible by 11. Simone noticed that reversing the digits of this number gives 8371, which is also divisible by 11.

Explain why, if we are given a number which is divisible by 11, and reverse its digits, the result is also divisible by 11.

ACTIVITY 1 DELECTABLE NUMBERS

A number is called **delectable** if the number formed by its first n digits is always divisible by n.

For example, 4236 is a delectable number because
the number formed by the first digit (4) is divisible by 1,
the number formed by the first 2 digits (42) is divisible by 2,
the number formed by the first 3 digits (423) is divisible by 3,
and the number formed by the first 4 digits (4236) is divisible by 4.

2052 is *not* a delectable number because, although 2 is divisible by 1, and 20 is divisible by 2, 205 is *not* divisible by 3.

1 Which one of the numbers below is a delectable number?

A 461 **B** 5226 **C** 63 **D** 723 **E** 34 265

2 How many three-digit delectable numbers can you make using the digits 1, 2, and 3 once each?

3 Show that it is impossible to make a four-digit delectable number using the digits 1, 2, 3, and 4 once each.

4 **Challenge:**
There is only one nine-digit delectable number which can be made using the digits 1 to 9 once each. Can you find it?
Hint: The first digit is 3.

B FACTORS

The **factors** of a natural number are the natural numbers which divide exactly into it.

For example, the factors of 6 are 1, 2, 3, and 6, since:

$$\begin{aligned} 6 \div 1 &= 6 \\ 6 \div 2 &= 3 \\ 6 \div 3 &= 2 \\ 6 \div 4 &= 1 \text{ remainder } 2 \\ 6 \div 5 &= 1 \text{ remainder } 1 \\ 6 \div 6 &= 1 \end{aligned}$$

When we write a number as a product of factors, we say it is **factorised**.

- 6 may be factorised as a product of two factors in two ways: 1×6 or 2×3.
- 12 has factors 1, 2, 3, 4, 6, 12, and can be factorised as a product of two factors in three ways: 1×12, 2×6, and 3×4.

EXERCISE 4B.1

1 **a** Is 8 a factor of 24? **b** Is 5 a factor of 43?
c Is 7 a factor of 34? **d** Is 9 a factor of 72?

2 **a** List all the factors of 20. **b** Copy and complete: $20 = 4 \times ......$
c Write another pair of factors which multiply to give 20.

3 List all the factors of:
a 8 **b** 16 **c** 30 **d** 36 **e** 44 **f** 56
g 50 **h** 84 **i** 77 **j** 49 **k** 65 **l** 91

4 Copy and complete:
a $24 = 6 \times ......$ **b** $25 = 5 \times ......$ **c** $28 = 4 \times ......$
d $100 = 5 \times ......$ **e** $88 = 11 \times ......$ **f** $88 = 2 \times ......$
g $36 = 2 \times ......$ **h** $36 = 3 \times ......$ **i** $36 = 9 \times ......$
j $49 = 7 \times ......$ **k** $121 = 11 \times ......$ **l** $72 = 6 \times ......$
m $60 = 12 \times ......$ **n** $48 = 12 \times ......$ **o** $96 = 8 \times ......$

5 Write the largest factor (other than itself) of:

a 14 **b** 27 **c** 56 **d** 44 **e** 75 **f** 90

6 What is the smallest number which has factors:

a 2, 3, and 5 **b** 3, 5, and 7 **c** 2, 3, 5, and 7?

7 A number has six factors. Two of its factors are 9 and 21. Find the number.

8 Answer the **Opening Problem** on page **82**.

9 **a** How many different factors do the following numbers have?

i 4 **ii** 9 **iii** 25 **iv** 100

b What numbers have an odd number of factors?

10 Consider again the **Opening Problem** on page **82**. Suppose a fourth student comes along. She changes the state of every fourth locker, starting at locker 4. This process continues for each of the 100 students who use the lockers.

Click on the icon for a demonstration of the process.

DEMO

a How does the number of factors of a locker number determine whether a locker ends up open or closed?

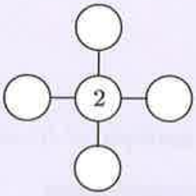
b Write down the numbers of the lockers that will be left open.

PUZZLE — CROSS-PRODUCTS

In each diagram you must put a different number in each circle so that the product of the three numbers going across equals the product of the three numbers going down. In each case, the product is given to you, and you can choose numbers from 1, 2, 3, 4, 5, 6, 7, 8, 9, 10 only.

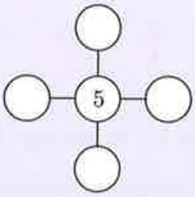
One number is given to you.

a



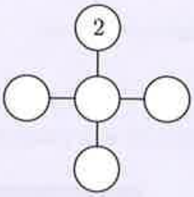
product is 48

b



product is 60

c



product is 30

HIGHEST COMMON FACTOR

The **highest common factor** or **HCF** of two numbers is the largest factor which is common to both of them.

Example 2 **Self Tutor**

Find the HCF of 24 and 40.

The factors of 24 are 1, 2, 3, 4, 6, 8, 12, 24.

The factors of 40 are 1, 2, 4, 5, 8, 10, 20, 40.

$\therefore$ the HCF of 24 and 40 is 8.

EXERCISE 4B.2

1 Find the HCF of:

a 12 and 15	**b** 16 and 20	**c** 21 and 35	**d** 9 and 12
e 16 and 56	**f** 14 and 35	**g** 20 and 36	**h** 18 and 45
i 40 and 46	**j** 44 and 110	**k** 81 and 108	**l** 135 and 360.

2 Ed has two lengths of network cable, one measuring 18 m and the other measuring 12 m. He wishes to cut them into shorter cables which are all the same length. What is the longest length of cable Ed can make in this way?

3 A hardware store sells nails in packets which all contain the same number of nails. Ron bought a total of 320 nails, and Tess bought a total of 200 nails. What is the greatest possible number of nails that could be in each packet?

C MULTIPLES

The **multiples** of any whole number have that number as a factor. They are obtained by multiplying it by 1, then 2, then 3, then 4, and so on.

The multiples of 10 are:	10,	20,	30,	40,	50,	....
	↑	↑	↑	↑	↑	
	10 × 1	10 × 2	10 × 3	10 × 4	10 × 5	
The multiples of 15 are:	15,	30,	45,	60,	75,	....
	↑	↑	↑	↑	↑	
	15 × 1	15 × 2	15 × 3	15 × 4	15 × 5	

The number 30 is a multiple of both 10 and 15, so we say 30 is a **common multiple** of 10 and 15.

Example 3 **Self Tutor**

Find the common multiples of 4 and 6 between 20 and 40.

The multiples of 4 are 4, 8, **12**, 16, 20, **24**, 28, 32, **36**, 40, ...

The multiples of 6 are 6, **12**, 18, **24**, 30, **36**, 42, ...

∴ the common multiples between 20 and 40 are 24 and 36.

EXERCISE 4C.1

1 List the first six multiples of:

a 4 **b** 9 **c** 10 **d** 15 **e** 22 **f** 35

2 Find the:

a seventh multiple of 6

b ninth multiple of 11

c eleventh multiple of 15

d hundredth multiple of 99.

3 Find the:

- **a** smallest multiple of 7 that is greater than 500
- **b** greatest multiple of 12 that is less than 1000.

4 List the numbers from 1 to 30.

- **a** Put a circle around each multiple of 3.
- **b** Put a square around each multiple of 4.
- **c** List the common multiples of 3 and 4 which are less than 30.

5 Use lists of multiples to help answer the following:

- **a** I am an odd multiple of 9. The product of my two digits is also a multiple of 9. Which two numbers could I be?
- **b** I am a square number. I am a multiple of 6 and a factor of 252. What number am I?
- **c** I am a multiple of 7 and a factor of 210. The product of my two digits is odd. What number am I?

6 Find the common multiples of 6 and 10 which are less than 100.

LOWEST COMMON MULTIPLE

The **lowest common multiple** or **LCM** of two numbers is the smallest multiple which is common to both of them.

Example 4 **Self Tutor**

Find the LCM of 6 and 8.

The multiples of 6 are 6, 12, 18, **24**, 30, 36, 42, **48**, 54, 60, ...

The multiples of 8 are 8, 16, **24**, 32, 40, **48**, 56, ...

$\therefore$ the common multiples of 6 and 8 are 24, 48, ...

$\therefore$ the LCM of 6 and 8 is 24.

EXERCISE 4C.2

1 Find the lowest common multiple for each of the following pairs of numbers:

- **a** 4 and 10
- **b** 5 and 15
- **c** 8 and 12
- **d** 12 and 15
- **e** 6 and 10
- **f** 4 and 7
- **g** 8 and 9
- **h** 6 and 14
- **i** 6 and 11
- **j** 5 and 13
- **k** 15 and 25
- **l** 27 and 36

2 Buses arrive at a sports stadium every 8 minutes, and trains arrive every 14 minutes. A bus and a train have just arrived simultaneously. How long will it be before this happens again?

3 A baker bakes the same number of buns each day. On weekdays he sells them in packs of 12, and on weekends he sells them in packs of 13. He always sells complete packs. What is the least possible number of buns he bakes?

INVESTIGATION 2 HCF AND LCM

Given two different positive whole numbers, is there a relationship between the HCF and LCM?

What to do:

1 Copy and complete:

Pair of numbers	*Product of numbers*	*HCF*	*LCM*	*HCF × LCM*
6 and 8	48		24	
6 and 9				
4 and 10				
15 and 25				

2 Use your table to predict the relationship between the HCF and LCM of two different positive whole numbers.

3 **a** Find the HCF of 21 and 28. **b** Hence find their LCM.

4 The product of two numbers is 720 and their HCF is 6. What is their LCM?

D PRIME AND COMPOSITE NUMBERS

Some numbers have only two factors, 1 and the number itself.

For example, the only factors of 7 are 1 and 7, and the only factors of 31 are 1 and 31.

Numbers of this type are called **prime numbers** or **primes**.

A **prime** number is a natural number which has exactly two different factors, 1 and itself.

A **composite** number is a natural number which has more than two factors.

From the definitions of prime and composite numbers we can see that:

The number 1 is neither prime nor composite.

PRIME FACTORS

The number 6 is composite since it has four factors: 1, 2, 3, and 6.

Notice that $6 = 2 \times 3$, and both of these factors are prime numbers.

The number 12 can be written as the product of prime factors $2 \times 2 \times 3$.

All composite numbers can be written as the product of prime number factors in exactly one way (apart from order).

When we write a number as the product of prime factors, it is usual to express it in index form.

For example, we would write $12 = 2^2 \times 3$.

We consider two methods for expressing the composite number 180 as the product of its prime factors:

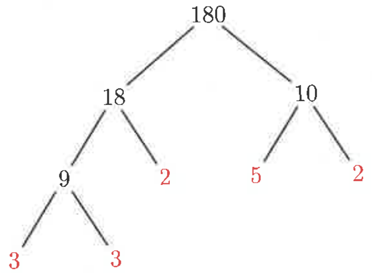
- In the method of **repeated division**, we systematically divide the number by the prime numbers which are its factors, starting with the smallest.

  So, $180 = 2 \times 2 \times 3 \times 3 \times 5$
  $= 2^2 \times 3^2 \times 5$

2	180
2	90
3	45
3	15
5	5
	1

- In the second method we create a **factor tree**. We find a factor pair for the given number, in this case $180 = 18 \times 10$. We use these factors as branches of the tree. We continue the process for each of the branches until we are only left with prime numbers.

  So, $180 = 3 \times 3 \times 2 \times 5 \times 2$
  $= 2^2 \times 3^2 \times 5$



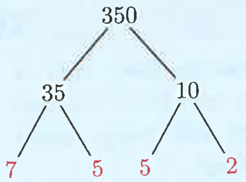
Example 5 **Self Tutor**

Express 350 as the product of prime factors in index form.

Repeated division *or* **Factor tree**

2	350
5	175
5	35
7	7
	1

So, $350 = 2 \times 5 \times 5 \times 7$
$= 2 \times 5^2 \times 7$



So, $350 = 7 \times 5 \times 5 \times 2$
$= 2 \times 5^2 \times 7$

Your factor tree will look different if you begin with a different factor pair. However, this will not affect your final answer.

EXERCISE 4D

1 **a** List all the prime numbers less than 50.
b Is 1 a prime number? Explain your answer.
c Are there any prime numbers which are even?

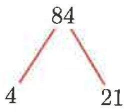
2 Use a list of prime numbers to help you find:
a the smallest odd prime
b the only odd two-digit composite number less than 20
c a prime number which is a factor of 105, 20, and 30.

3 The two digits of a number are the same. Their product is *not* composite. What is the number?

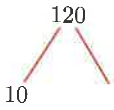
4 Show that the following numbers are composites:
a 6485 **b** 9320 **c** 2222 **d** 4279

5 Copy and complete the following factor trees:

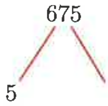
a



b



c



6 Use a factor tree to write as the product of prime factors in index form:

a 24 **b** 70 **c** 63 **d** 72
e 225 **f** 88 **g** 480 **h** 1024

7 Use repeated division to write as the product of prime factors in index form:

a 28 **b** 27 **c** 84
d 160 **e** 216 **f** 528
g 784 **h** 138 **i** 250
j 189 **k** 726 **l** 9625

8 Edward thinks he has found *two* ways of writing the number 16 as the product of prime factors:

$$16 = 2^4 \qquad 16 = 4^2$$

Explain why Edward is wrong.

INVESTIGATION 3 THE SIEVE OF ERATOSTHENES

Eratosthenes (pronounced Er-ra-toss-tha-nees) was a Greek mathematician and geographer who lived between 275 BC and 194 BC. He is credited with many useful mathematical discoveries and calculations.

Eratosthenes was probably the first person to calculate the **circumference** of the Earth, which is the distance around the equator. He did this using the lengths of shadows.

Eratosthenes also found a method for 'sieving' out composite numbers from the numbers from 1 to 100 to leave only the primes.

His method was:

- cross out 1
- cross out all evens except 2
- cross out all multiples of 3, except 3
- cross out all multiples of 5, except 5
- cross out all multiples of 7, except 7
- continue this process using the smallest uncrossed number not already used.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

What to do:

PRINTABLE TABLE OF NUMBERS

1. Print the table of numbers from 1 to 100. Use Eratosthenes' method to discover the primes between 1 and 100.

2. Are there patterns in the way prime numbers occur? Copy the table into your book and count the number of primes in each set of numbers. Is there a pattern?

DEMO

Set of numbers	*Total number of prime numbers*
0 to 9	
10 to 19	
20 to 29	
30 to 39	
40 to 49	
50 to 59	
60 to 69	

ACTIVITY 2 THE 1000 POINT WORD

Rachel decided to allocate points to different words. She first converted each letter into a number using A → 1, B → 2, ..., Z → 26. The product of the numbers was her score for each word.

Using Rachel's method, H E L L O has the value
$8 \times 5 \times 12 \times 12 \times 15 = 86\,400$ points.

What to do:

1. Find the points value for the word:
   a BED **b** MILK **c** JUMP
2. Find the points value for your name. Which of your classmates' names gives the highest points value?
3. Explain why any word containing a 'J' will have a points value ending in 0.
4. Rachel is particularly interested in words which have a value of exactly 1000 points.
   a Explain why it is impossible for such a word to contain the letter 'G'.
   b Make a list of the possible letters that a 1000 point word could contain.
   c Rachel found that B E A D Y is a 1000 point word, since
   $2 \times 5 \times 1 \times 4 \times 25 = 1000$.
   Can you find another one?

E ROOTS

In **Chapter 1** we encountered **square numbers** and **cubic numbers**, which occur when a whole number is squared or cubed. We will now consider the opposite operation, which is to find the **square root** or **cube root** of a whole number.

The **square root** of the number a is the *positive* number which, when squared, gives a.

We write the square root of a as $\sqrt{a}$.

$$\sqrt{a} \times \sqrt{a} = a$$

For example, since $4 = 2^2$, $9 = 3^2$, and $16 = 4^2$,
$\sqrt{4} = 2$, $\sqrt{9} = 3$, and $\sqrt{16} = 4$.

The square roots of 4, 9, and 16 are whole numbers, so 4, 9, and 16 are **perfect squares**.

The square roots of most numbers are not whole numbers.
For example, $\sqrt{2} \approx 1.414\,213\ldots$.

We can find square roots using our calculator. You need to look for the $\sqrt{}$ symbol. You may need to press a key such as `2nd F` or `SHIFT` to access this function.

$(-2)^2 = 4$ also, but the square root of a number is positive, so $\sqrt{4} = 2$ rather than -2.

Example 6 **Self Tutor**

Between which two consecutive integers does $\sqrt{40}$ lie?

The square numbers either side of 40 are 36 and 49.
$\therefore$ $\sqrt{40}$ is between $\sqrt{36}$ and $\sqrt{49}$.
$\therefore$ $\sqrt{40}$ is between 6 and 7.

CUBE ROOTS

The **cube root** of a is the number which, when cubed, gives a.

We write the cube root of a as $\sqrt[3]{a}$.

$$\sqrt[3]{a} \times \sqrt[3]{a} \times \sqrt[3]{a} = a$$

For example, since $8 = 2^3$ and $27 = 3^3$,
$\sqrt[3]{8} = 2$ and $\sqrt[3]{27} = 3$.

Notice that $(-2)^3 = -8$ and so $\sqrt[3]{-8} = -2$

In contrast, we cannot find the *square* root of a negative number.

EXERCISE 4E

1 Find:

a $\sqrt{16}$ **b** $\sqrt{49}$ **c** $\sqrt{81}$ **d** $\sqrt{121}$
e $\sqrt{256}$ **f** $\sqrt{0}$ **g** $\sqrt{1024}$ **h** $\sqrt{1156}$
i $\sqrt{4225}$ **j** $\sqrt{9801}$ **k** $\sqrt{10\,000}$ **l** $\sqrt{14\,400}$

2 Between which two consecutive integers do the following values lie?

a $\sqrt{3}$ **b** $\sqrt{7}$ **c** $\sqrt{30}$ **d** $\sqrt{68}$

Check your answers using a calculator.

3 Find:

a $\sqrt[3]{1}$ **b** $\sqrt[3]{64}$ **c** $\sqrt[3]{125}$ **d** $\sqrt[3]{343}$
e $\sqrt[3]{-1}$ **f** $\sqrt[3]{-27}$ **g** $\sqrt[3]{-64}$ **h** $\sqrt[3]{-1000}$

ACTIVITY 3 GOLDBACH'S "GOLDEN RULES"?

In 1742, **Christian Goldbach** suggested two "golden rules":

- Every even number greater than 4 can be written as the sum of **two odd primes**.
- Every odd number greater than 8 can be written as the sum of **three odd primes**.

What to do:

1 Complete the following table to test Goldbach's first "rule".
The first 15 primes are given to help you: 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47

$6 = 3 + 3$	$8 = 3 + 5$	$10 =$	$12 =$	$14 =$
$16 =$	$18 =$	$20 =$	$22 =$	$24 =$
$26 =$	$28 =$	$30 =$	$32 =$	$34 =$
$36 =$	$38 =$	$40 =$	$42 =$	$44 =$
$46 =$	$48 =$	$50 =$	$52 =$	$54 =$

2 Complete the following table to test Goldbach's second "rule".

$9 = 3 + 3 + 3$	$11 = 3 + 3 + 5$	$13 =$	$15 =$	$17 =$
$19 =$	$21 =$	$23 =$	$25 =$	$27 =$
$29 =$	$31 =$	$33 =$	$35 =$	$37 =$
$39 =$	$41 =$	$43 =$	$45 =$	$47 =$
$49 =$	$51 =$	$53 =$	$55 =$	$57 =$

KEY WORDS USED IN THIS CHAPTER

- composite number
- even number
- factor tree
- lowest common multiple
- odd number
- repeated division
- cube root
- factor
- highest common factor
- multiple
- positive number
- square root
- divisible
- factor pairs
- integer
- negative number
- prime number

REVIEW SET 4A

1 Write the number 36 as:

a the sum of two even numbers
b the sum of two odd numbers
c the product of two even numbers
d the product of an even number and an odd number.

2 Find all possible values of the missing digit □ if:

a 32□ is divisible by 4
b 9□4 is divisible by 3
c 2□7 is divisible by 9.

3 List the factors of:

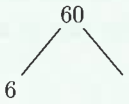
a 72
b 90
c 126

4 List the multiples of 7 which lie between 40 and 60.

5 Show that the following numbers are composites:

a 2950 **b** 1863

6 Complete the factor tree:



7 Find the lowest common multiple of 16 and 24.

8 Use repeated division to write as the product of prime factors in index form:

a 44 **b** 504 **c** 693

9 A wedding cake is $54 \text{ cm} \times 78 \text{ cm}$. The bride and groom want it cut into square pieces of equal size.

a What is the biggest size the pieces could be?

b How many pieces of this size could be cut?

10 Find:

a $\sqrt{196}$ **b** $\sqrt[3]{-125}$

REVIEW SET 4B

1 Decide whether:

a 54 is divisible by 9 **b** 42 is divisible by 4.

2 List the prime numbers between 50 and 70.

3 Determine whether 4536 is divisible by:

a 3 **b** 4 **c** 5 **d** 6

4 Find the largest number which divides exactly into both 63 and 84.

5 Find the greatest multiple of 6 which is less than 70.

6 Find:

a $\sqrt{144}$ **b** $\sqrt[3]{-343}$

7 Find the highest common factor of:

a 14 and 49 **b** 84 and 150.

8 Use a factor tree to write as the product of prime factors in index form:

a 48 **b** 495 **c** 900

9 Suppose the garbage truck visits your house once every 14 days, and the green waste truck visits your house once every 10 days. If they both visit your house today, how long will it be before they are both at your house on the same day again?

10 Between which two consecutive whole numbers does $\sqrt{174}$ lie?

Chapter 5

Fractions

Contents:

OPENING PROBLEM

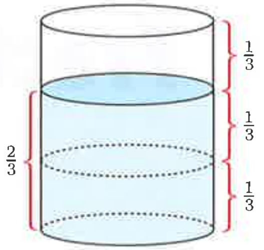
An abalone diver has a daily catch limit. He catches $\frac{1}{6}$ of his limit in the first hour, and $\frac{1}{4}$ of his limit in the second hour.

Things to think about:

- **a** What fraction of his limit has he caught so far?
- **b** What fraction of his limit is he yet to catch?

A **fraction** represents a part of a quantity.

Suppose we divide a container into three equal sections. If two of the sections are filled with water, we say the container is two thirds full.



Two thirds is written as $\frac{2}{3}$

the number of filled parts

the number of equal parts in a whole

There are three thirds in a whole.

Notice that $\frac{1}{3}$ of the container is empty.

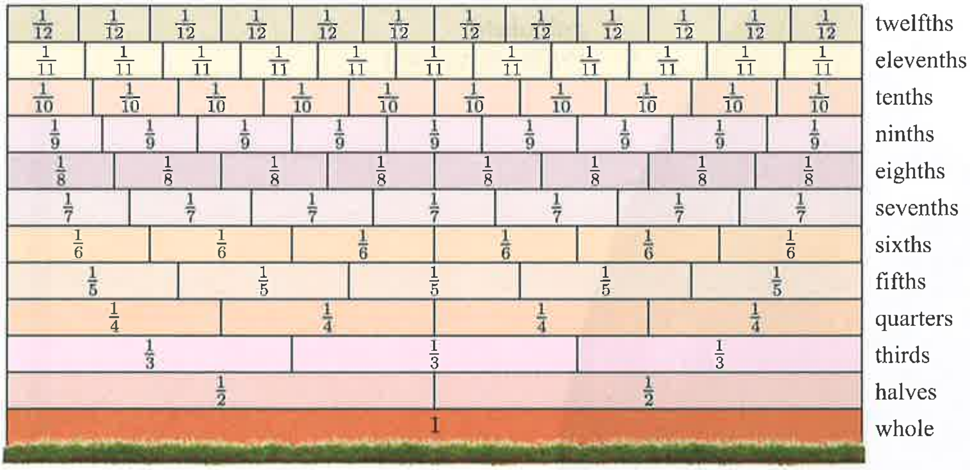
$$\underset{\text{filled part of container}}{\frac{2}{3}} + \underset{\text{empty part of container}}{\frac{1}{3}} = \underbrace{\frac{3}{3}}_{\text{whole container}} = 1$$

DEMO

FRACTION WALL

To illustrate how we can use fractions to divide up a whole quantity, we can use a **fraction wall**.

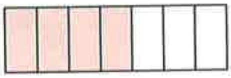
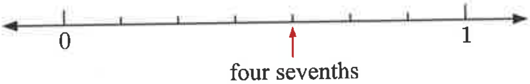
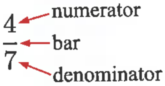
Each layer of the wall is divided into different numbers of equal sized bricks. We can see that one whole can be made up of two halves, or three thirds, or four quarters, or five fifths, and so on.



PRINTABLE FRACTION WALL

A COMMON FRACTIONS

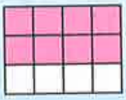
The fraction "four sevenths" can be represented in a number of different ways:

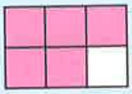
Words	four sevenths
Diagram	as a shaded region *or* as pieces of a pie  
Number line	
Symbolic form	

A fraction written in symbolic form with a bar is called a **common fraction**.

Example 1 🔊 **Self Tutor**

a Draw a diagram to represent $\frac{5}{6}$.

b What fraction is represented by:  ?

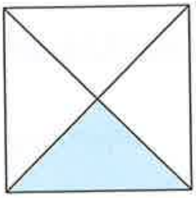
a 

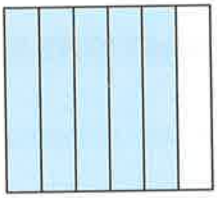
b There are a total of 12 squares, and 8 are shaded.

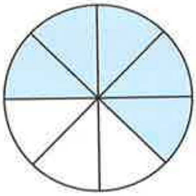
$\therefore$ $\frac{8}{12}$ is shaded.

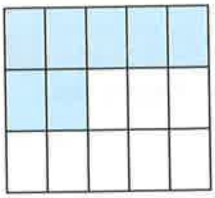
EXERCISE 5A

1 State the fraction represented by the shading:

a 

b 

c 

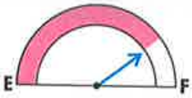
d 

2 Draw a diagram to represent:

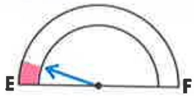
a $\frac{3}{5}$ **b** $\frac{6}{8}$ **c** $\frac{2}{7}$ **d** $\frac{7}{10}$ **e** $\frac{0}{3}$ **f** $\frac{13}{20}$ **g** $\frac{11}{12}$ **h** $\frac{6}{9}$

3 Consider the following fuel gauges:

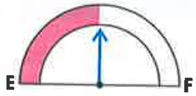
A



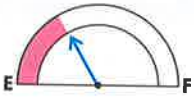
B



C



D



Which gauge shows the tank to be:

a $\frac{1}{3}$ full **b** $\frac{1}{2}$ full **c** $\frac{1}{10}$ full **d** $\frac{4}{5}$ full

e $\frac{9}{10}$ empty **f** $\frac{1}{5}$ empty **g** $\frac{2}{3}$ empty **h** $\frac{1}{2}$ empty?

4 Four bottles of soft drink are left in a refrigerator after a party.

A

B

C

D

Which bottle is:

a $\frac{7}{10}$ full **b** $\frac{1}{8}$ full **c** half full **d** $\frac{19}{20}$ full?

5 There are 23 goldfish in my outside pond. Of these, nine are gold, seven are silver, five are black, and two are white. State what fraction of my goldfish are:

a gold **b** black **c** silver **d** gold or silver.

B FRACTIONS AS DIVISION

When we write a fraction such as $\frac{3}{4}$, the bar indicates division.

So, $\frac{3}{4}$ can be written as $3 \div 4$,

and $3 \div 4$ can be written as $\frac{3}{4}$.

3 ← numerator
— ← bar
4 ← denominator

The numerator 3 is the **dividend**, and the denominator 4 is the **divisor**.

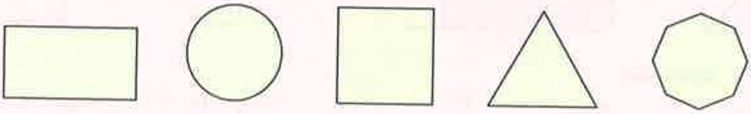
> The division $a \div b$ can be written as the fraction $\frac{a}{b}$.
>
> The fraction $\frac{a}{b}$ can be written as the division $a \div b$.

ACTIVITY 1 FRACTIONS FROM REGULAR SHAPES

What to do:

1 Click on the icon and print the first page. Consider the three sets of these shapes:

PRINTABLE PAGES



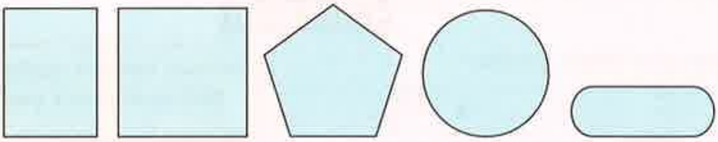
2 For the first set of shapes, divide each shape into two equal parts.
Each part is one half of the whole shape.

3 For the second set, divide each whole shape into three equal parts.
Copy and complete: "Each part is ......"

4 For the third set, divide each whole shape into four equal parts.
Copy and complete: "Each part is ......"

5 Which shape did you find the most difficult to divide equally? Why?

6 Copy and divide each of the following shapes into fifths:



Which shape did you find the most difficult to divide equally? Why?

7 Print out the second page with the six larger shapes. See if you can fold each of the different shapes to create the fractions described in question **2** of **Exercise 5A**. Remember that the portions must all be the same size.

- **a** Which shapes allow you several different ways to fold and create the required fractions?
- **b** Which were the more difficult shapes to create the fractions with?

EXERCISE 5B

1 Write as a fraction:

a $1 \div 2$ **b** $1 \div 5$ **c** $4 \div 7$ **d** $8 \div 9$
e $2 \div 3$ **f** $9 \div 10$ **g** $9 \div 6$ **h** $20 \div 4$

2 Write as a division:

a $\frac{3}{5}$ **b** $\frac{2}{7}$ **c** $\frac{6}{10}$ **d** $\frac{5}{8}$
e $\frac{1}{11}$ **f** $\frac{6}{7}$ **g** $\frac{11}{12}$ **h** $\frac{13}{3}$

Example 2 **Self Tutor**

Write $\frac{14}{7}$ as a division and hence as a whole number.

$$\frac{14}{7} = 14 \div 7$$
$$= 2$$

3 Write as a division and hence as a whole number:

a $\frac{8}{2}$ **b** $\frac{15}{5}$ **c** $\frac{24}{8}$ **d** $\frac{10}{10}$ **e** $\frac{16}{4}$ **f** $\frac{42}{6}$

Example 3 **Self Tutor**

Write as a fraction: **a** $-4 \div 7$ **b** $-8 \div -9$

a $-4 \div 7 = \frac{-4}{7}$ **b** $-8 \div -9 = \frac{-8}{-9}$

4 Write as a fraction:

a $-2 \div 3$ **b** $4 \div -5$ **c** $-6 \div -7$ **d** $10 \div -12$

e $-11 \div 22$ **f** $-23 \div -5$ **g** $16 \div -8$ **h** $-18 \div 2$

Example 4 **Self Tutor**

Write as a division and hence as a whole number:

a $\frac{18}{-6}$ **b** $\frac{-12}{3}$ **c** $\frac{-28}{-4}$

a $\frac{18}{-6}$
$= 18 \div -6$
$= -3$

b $\frac{-12}{3}$
$= -12 \div 3$
$= -4$

c $\frac{-28}{-4}$
$= -28 \div -4$
$= 7$

Division between numbers with **like** signs gives a **positive**.

Division between numbers with **unlike** signs gives a **negative**.

5 Write as a division:

a $\frac{-1}{8}$ **b** $\frac{-4}{-6}$ **c** $\frac{3}{-9}$ **d** $\frac{-10}{-2}$ **e** $\frac{-24}{6}$

6 Write as a division and hence as a whole number:

a $\frac{25}{5}$ **b** $\frac{25}{-5}$ **c** $\frac{-25}{5}$ **d** $\frac{-25}{-5}$

e $\frac{27}{9}$ **f** $\frac{-27}{9}$ **g** $\frac{27}{-9}$ **h** $\frac{-27}{-9}$

7 Write as a division and hence as a whole number:

a $\frac{15}{-3}$ **b** $\frac{-15}{3}$ **c** $\frac{-63}{-7}$ **d** $\frac{63}{-7}$

e $\frac{40}{-10}$ **f** $\frac{-40}{10}$ **g** $\frac{-96}{12}$ **h** $\frac{96}{-12}$

Example 5 **Self Tutor**

Write as a division and hence as a whole number:

a $\frac{28-4}{3 \times 4}$ **b** $\frac{5-7 \times 3}{11-9}$

a $\frac{28-4}{3 \times 4}$
$= \frac{24}{12}$
$= 24 \div 12$
$= 2$

b $\frac{5-7 \times 3}{11-9}$
$= \frac{5-21}{2}$
$= \frac{-16}{2}$
$= -16 \div 2$
$= -8$

The division line of fractions groups both the numerator and the denominator like brackets. Evaluate the numerator and the denominator first, then do the division.

8 Write as a division and hence as a whole number:

a $\dfrac{3+9}{2\times 3}$
b $\dfrac{20-2}{1+5}$
c $\dfrac{6\times 8}{15-3}$
d $\dfrac{5\times 6}{4-7}$
e $\dfrac{17-3}{2\times 3+1}$
f $\dfrac{25-3\times 5}{10-8}$
g $\dfrac{6-5\times 10}{3+8}$
h $\dfrac{3-31}{2\times 3-10}$

C PROPER AND IMPROPER FRACTIONS

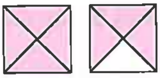
A fraction which has numerator **less** than its denominator is called a **proper fraction**.

A fraction which has numerator **greater** than its denominator is called an **improper fraction**.

For example:

- $\frac{1}{4}$ is a proper fraction

- $\frac{7}{4}$ is an improper fraction.



$$\frac{7}{4}=\frac{4}{4}+\frac{3}{4}=1+\frac{3}{4}$$

When an improper fraction is written as the sum of a whole number and a proper fraction, it is called a **mixed number**.

For example, $\frac{7}{4}$ can be written as the mixed number $1\frac{3}{4}$.

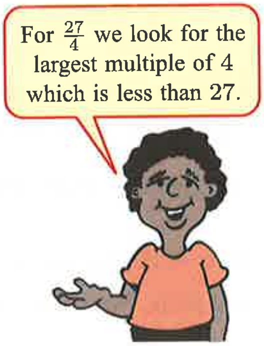
Example 6 Self Tutor

Convert to a mixed number:

a $\dfrac{7}{5}$ **b** $\dfrac{27}{4}$

a $\dfrac{7}{5}=\dfrac{5}{5}+\dfrac{2}{5}$
$=1+\dfrac{2}{5}$
$=1\dfrac{2}{5}$

b $\dfrac{27}{4}=\dfrac{24}{4}+\dfrac{3}{4}$
$=6+\dfrac{3}{4}$
$=6\dfrac{3}{4}$



For $\frac{27}{4}$ we look for the largest multiple of 4 which is less than 27.

We can reverse the steps to convert a mixed number to an improper fraction.

Example 7 Self Tutor

Convert $2\frac{3}{5}$ to an improper fraction.

$2\dfrac{3}{5}=2+\dfrac{3}{5}$
$=\dfrac{10}{5}+\dfrac{3}{5}$
$=\dfrac{13}{5}$

EXERCISE 5C

1 Write as a mixed number:

a $\frac{9}{5}$ **b** $\frac{5}{4}$ **c** $\frac{21}{10}$ **d** $\frac{31}{3}$

e $\frac{23}{6}$ **f** $\frac{25}{7}$ **g** $\frac{64}{9}$ **h** $\frac{103}{10}$

2 Write as an improper fraction:

a $2\frac{1}{2}$ **b** $1\frac{1}{4}$ **c** $2\frac{4}{5}$ **d** $1\frac{3}{8}$

e $2\frac{2}{7}$ **f** $1\frac{3}{10}$ **g** $1\frac{15}{16}$ **h** $4\frac{2}{3}$

3 Millicent High School has 67 students to be put into basketball teams. Each team has eight players in it.

a Write $\frac{67}{8}$ as a mixed number.

b How many complete basketball teams can be made?

4 Ella had 15 m of ribbon which she cut into four equal lengths. Express the length of each ribbon as a mixed number of metres.

D PLACING FRACTIONS ON A NUMBER LINE

Consider the number line:

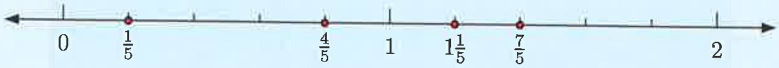
The gaps between the integers are made up of an infinite number of points. Many of these points correspond to **fractions**.

The position of every fraction can be shown on a number line.

For example, to display the fraction $\frac{3}{5}$, we divide the interval from 0 to 1 into 5 equal parts. We mark a dot at the third division going from 0 to 1.

Example 8 **Self Tutor**

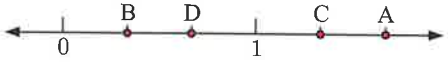
On a number line, show the positions of $\frac{1}{5}$, $\frac{4}{5}$, $\frac{7}{5}$, and $1\frac{1}{5}$.



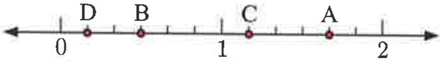
EXERCISE 5D

1 Find the fractions represented by the points A, B, C, and D on the number lines:

a



b



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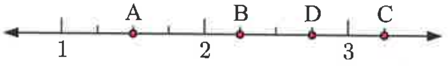


d

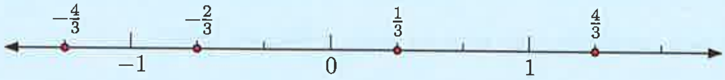
2 Draw a number line to display each set of fractions:

a $\frac{1}{5}, \frac{3}{5}, \frac{2}{5}, \frac{6}{5}$ **b** $\frac{4}{3}, \frac{2}{3}, \frac{7}{3}, \frac{1}{3}$ **c** $\frac{3}{4}, \frac{2}{4}, 1\frac{1}{4}, 1\frac{3}{4}$

d $\frac{7}{10}, \frac{12}{10}, \frac{3}{10}, \frac{9}{10}$ **e** $\frac{1}{7}, \frac{8}{7}, \frac{5}{7}, 1\frac{3}{7}$ **f** $1\frac{5}{6}, \frac{2}{6}, 1\frac{1}{6}, \frac{5}{6}$

Example 9 **Self Tutor**

Draw a number line to display: $-\frac{4}{3}, -\frac{2}{3}, \frac{1}{3}, \frac{4}{3}$.



3 Draw a number line to display:

a $\frac{1}{5}, -\frac{2}{5}, \frac{3}{5}, -\frac{4}{5}$ **b** $\frac{3}{4}, -\frac{1}{4}, \frac{5}{4}, -\frac{3}{4}$ **c** $\frac{2}{3}, -\frac{5}{3}, -\frac{1}{3}, \frac{5}{3}$

4 Draw a number line to display each set of fractions. Hence order them from least to greatest:

a $\frac{2}{4}, -\frac{3}{4}, \frac{1}{4}, -\frac{1}{4}$ **b** $-1\frac{1}{2}, 2\frac{1}{2}, -\frac{6}{2}, \frac{1}{2}$ **c** $\frac{4}{3}, 1\frac{2}{3}, -\frac{2}{3}, -\frac{1}{3}, \frac{2}{3}$

E EQUAL FRACTIONS AND SIMPLIFYING

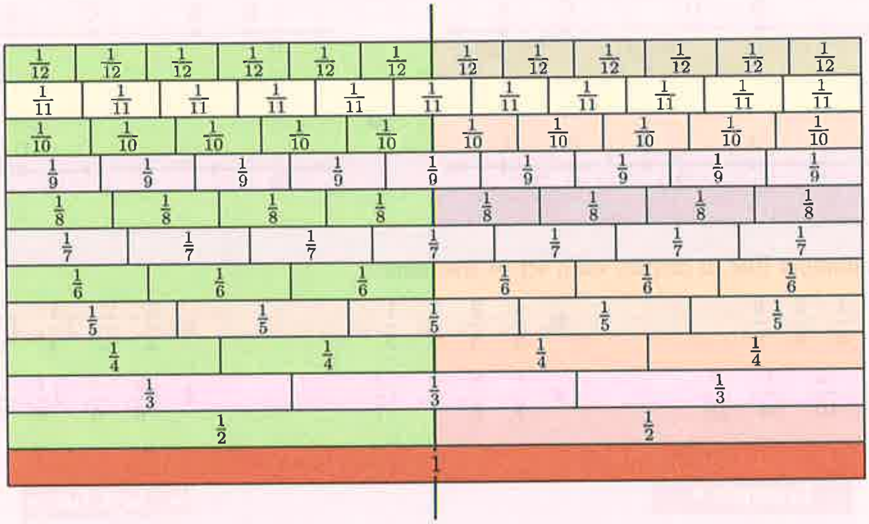
Two fractions are **equal** if they have the same numerical value. They are located at the same point on the number line.

ACTIVITY 2 EQUAL FRACTIONS

From the fraction wall on page **98**, we saw that one whole can be written as $\frac{2}{2}, \frac{3}{3}, \frac{4}{4}, ..., \frac{12}{12}$.

By using the lines that divide each row, we can identify fractions that are equal in size.

For example, by using this fraction wall we can see that $\frac{1}{2}$ can also be written as $\frac{2}{4}$, $\frac{3}{6}$, $\frac{4}{8}$, $\frac{5}{10}$, or $\frac{6}{12}$.



1. Use the lines on the fraction wall to identify fractions that are equal to:

   a $\frac{1}{3}$ **b** $\frac{1}{5}$ **c** $\frac{5}{6}$ **d** $\frac{8}{12}$ **e** $\frac{4}{10}$ **f** $\frac{6}{8}$

2. How can we find equal fractions without using a fraction wall?

3. Find the fractions equal to $\frac{80}{100}$ that appear on the fraction wall.

From the previous **Activity**, we can see that:

We can convert a fraction to an equal fraction by multiplying or dividing both the numerator and denominator by the same non-zero number.

For example, $\underset{\times 2}{\overset{\times 2}{\frac{5}{6} = \frac{10}{12}}}$ and $\underset{\div 4}{\overset{\div 4}{\frac{8}{12} = \frac{2}{3}}}$.

Example 10 **Self Tutor**

Write $\frac{3}{4}$ with denominator 32.

To convert the denominator to 32 we need to multiply by 8.

We must therefore multiply the numerator by 8 also.

$$\underset{\times 8}{\overset{\times 8}{\frac{3}{4} = \frac{24}{32}}}$$

EXERCISE 5E.1

1 Determine whether the following pairs of fractions are equal:

a $\frac{4}{5}, \frac{8}{10}$ **b** $\frac{1}{3}, \frac{3}{8}$ **c** $\frac{6}{15}, \frac{2}{5}$ **d** $\frac{16}{24}, \frac{2}{4}$

2 Which of these fractions is equal to $\frac{3}{4}$?

A $\frac{6}{12}$ **B** $\frac{9}{16}$ **C** $\frac{15}{20}$ **D** $\frac{24}{40}$ **E** $\frac{30}{36}$

3 Use a fraction wall to find two other fractions equal to:

a $\frac{1}{4}$ **b** $\frac{3}{4}$ **c** $\frac{2}{3}$

4 Write with denominator 12:

a $\frac{1}{4}$ **b** $\frac{2}{3}$ **c** $\frac{5}{6}$ **d** $\frac{9}{2}$ **e** $\frac{20}{24}$

5 Write with denominator 20:

a $\frac{1}{5}$ **b** $\frac{3}{4}$ **c** $\frac{13}{10}$ **d** $\frac{26}{40}$ **e** $\frac{36}{80}$

SIMPLIFYING FRACTIONS

We can **simplify** a fraction by removing **common factors** in the numerator and denominator.

When a fraction is written with the smallest possible integer numerator and denominator, we say it is in **lowest terms** or **simplest form**.

We can simplify a fraction by either:

- writing the numerator and denominator as the product of factors and then **cancelling common factors**, or
- dividing the numerator and denominator by their **highest common factor**.

Example 11 **Self Tutor**

Simplify:

a $\frac{7}{21}$ **b** $\frac{36}{84}$ **c** $\frac{100}{90}$

a $\frac{7}{21}$

$= \frac{1 \times \cancel{7}^{1}}{3 \times \cancel{7}_{1}}$

$= \frac{1}{3}$

b $\frac{36}{84}$

$= \frac{36 \div 12}{84 \div 12}$ {HCF of 36 and 84 is 12}

$= \frac{3}{7}$

c $\frac{100}{90}$

$= \frac{10 \times \cancel{10}^{1}}{9 \times \cancel{10}_{1}}$

$= \frac{10}{9}$

EXERCISE 5E.2

1 Write in simplest form:

a $\frac{3}{9}$ **b** $\frac{2}{8}$ **c** $\frac{8}{10}$ **d** $\frac{4}{6}$ **e** $\frac{6}{12}$ **f** $\frac{5}{15}$

2 Write in simplest form:

a $\frac{18}{24}$ **b** $\frac{16}{20}$ **c** $\frac{18}{21}$ **d** $\frac{20}{25}$ **e** $\frac{11}{33}$

f $\frac{21}{30}$ **g** $\frac{14}{35}$ **h** $\frac{15}{27}$ **i** $\frac{32}{50}$ **j** $\frac{36}{96}$

3 Write as an improper fraction in lowest terms:

a $\frac{6}{4}$ **b** $\frac{8}{6}$ **c** $\frac{16}{10}$ **d** $\frac{14}{8}$

e $\frac{20}{12}$ **f** $\frac{35}{30}$ **g** $\frac{40}{36}$ **h** $\frac{48}{44}$

Example 12 **Self Tutor**

Write $\frac{12-2\times 4}{17+3}$ in simplest form.

$$\begin{aligned}\frac{12-2\times 4}{17+3} &= \frac{(12-2\times 4)}{(17+3)}\\ &= \frac{(12-8)}{20}\\ &= \frac{4}{20}\\ &= \frac{1}{5}\end{aligned}$$

The fraction bar acts to group the terms in the numerator and the terms in the denominator. Evaluate the numerator and the denominator first, then do the division.

4 Write in simplest form:

a $\frac{3\times 2}{13-4}$ **b** $\frac{5+3}{8+2\times 2}$ **c** $\frac{6+4}{5+3\times 5}$ **d** $\frac{18+3\times 4}{7+11}$

ONE QUANTITY AS A FRACTION OF ANOTHER

When we compare quantities in the real world, we can write them as fractions with the quantities in the **same units**. We then simplify the fraction into its lowest terms.

Example 13 **Self Tutor**

Write 80 cents as a fraction of $3.00 .

The fraction is $\frac{80 \text{ cents}}{\$3.00}$

$= \frac{80 \text{ cents}}{300 \text{ cents}}$ {$3.00 is 300 cents}

$= \frac{80 \div 20}{300 \div 20}$ {the HCF is 20}

$= \frac{4}{15}$

Give your answers in lowest terms.

EXERCISE 5E.3

1 Write:

a 7 kg as a fraction of 35 kg
b 3 hours as a fraction of 15 hours
c \$27 as a fraction of \$45
d 45° as a fraction of 360°
e 75 m as a fraction of 400 m
f 13 cards as a fraction of 52 cards.

2 Write the first quantity as a fraction of the second quantity. Do not forget to write the quantities in the same units.

a 200 m out of 1 km
b 8 days out of 2 weeks
c 75 cm out of 2 m
d 60 cents out of \$1
e 400 g out of 2 kg
f 8 hours out of 3 days

1 km = 1000 m
1 week = 7 days
1 m = 100 cm
\$1 = 100 cents
1 kg = 1000 g
1 day = 24 hours
1 hour = 60 minutes

3 What fraction of one hour is:

a 20 minutes **b** 15 minutes **c** 24 minutes?

4 What fraction of one day is:

a 3 hours **b** 4 hours **c** 16 hours?

5 Nicky scored 21 marks out of 30 for her mathematics test. What fraction did she get correct?

6 Enrique spent \$3 on a drink and \$5 on a sandwich. What fraction of \$20 did he spend?

7 Jo used 450 g from her 800 g packet of wool to knit a shawl. What fraction of her wool did she use?

F COMPARING FRACTIONS

To compare fractions, we must first write them as equivalent fractions with the same denominator. We usually choose the **lowest common denominator** or **LCD**, which is the lowest common multiple of the original denominators. Once we have done this, we can then compare the numerators.

Example 14 **Self Tutor**

Which is greater, $\frac{4}{5}$ or $\frac{7}{9}$?

The LCM of 5 and 9 is 45.

$\therefore$ the LCD is 45.

$$\frac{4}{5} = \frac{4 \times 9}{5 \times 9} = \frac{36}{45} \quad \text{and} \quad \frac{7}{9} = \frac{7 \times 5}{9 \times 5} = \frac{35}{45}$$

Now $\frac{36}{45} > \frac{35}{45}$, so $\frac{4}{5} > \frac{7}{9}$

Convert the fractions so they have the same lowest common denominator.

EXERCISE 5F

1 Determine which fraction is greater:

a $\frac{1}{3}$ or $\frac{3}{8}$ **b** $\frac{3}{5}$ or $\frac{4}{7}$ **c** $\frac{1}{6}$ or $\frac{2}{11}$

d $\frac{7}{10}$ or $\frac{19}{25}$ **e** $\frac{8}{11}$ or $\frac{3}{4}$ **f** $\frac{7}{12}$ or $\frac{9}{16}$

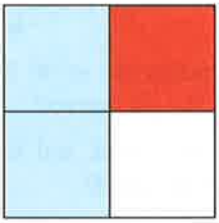
2 Replace ☐ with > or < to make the following statements true:

a $\frac{2}{3}$ ☐ $\frac{3}{4}$ **b** $\frac{3}{5}$ ☐ $\frac{5}{9}$ **c** $\frac{5}{6}$ ☐ $\frac{13}{18}$

d $\frac{3}{11}$ ☐ $\frac{2}{7}$ **e** $\frac{11}{25}$ ☐ $\frac{2}{5}$ **f** $\frac{11}{16}$ ☐ $\frac{7}{10}$

3 Write the fractions $\frac{2}{3}, \frac{5}{8}, \frac{7}{9}$, and $\frac{11}{15}$ in order from smallest to largest.

G ADDING AND SUBTRACTING FRACTIONS



In the diagram opposite, we see that:

$\frac{1}{2}$ or $\frac{2}{4}$ of the square is blue

$\frac{1}{4}$ of the square is red

$\frac{1}{4}$ of the square is unshaded.

The total amount shaded is $\frac{3}{4}$ of the square. So, $\frac{1}{2}+\frac{1}{4}=\frac{2}{4}+\frac{1}{4}=\frac{3}{4}$.

The total amount unshaded is $\frac{1}{4}$ of the square. So, $1-\frac{3}{4}=\frac{4}{4}-\frac{3}{4}=\frac{1}{4}$.

To **add** or **subtract** fractions:

- If necessary, convert the fractions so they have the lowest common denominator.
- Add or subtract the new numerators. The denominator stays the same.

Example 15 **Self Tutor**

Find: **a** $\frac{2}{5}+\frac{1}{4}$ **b** $\frac{2}{3}+\frac{7}{9}$

a
$$\begin{aligned}&\frac{2}{5}+\frac{1}{4}\\&=\frac{2\times 4}{5\times 4}+\frac{1\times 5}{4\times 5} \quad \{\text{LCD}=20\}\\&=\frac{8}{20}+\frac{5}{20}\\&=\frac{13}{20}\end{aligned}$$

b
$$\begin{aligned}&\frac{2}{3}+\frac{7}{9}\\&=\frac{2\times 3}{3\times 3}+\frac{7}{9} \quad \{\text{LCD}=9\}\\&=\frac{6}{9}+\frac{7}{9}\\&=\frac{13}{9}\end{aligned}$$

When adding or subtracting mixed numbers, you should first convert them to improper fractions, and then perform the operation.

Example 16 **Self Tutor**

Find $2\frac{1}{3} - 1\frac{1}{4}$.

$$\begin{aligned} & 2\tfrac{1}{3} - 1\tfrac{1}{4} \\ &= \frac{7}{3} - \frac{5}{4} && \{\text{converting to improper fractions}\} \\ &= \frac{7 \times 4}{3 \times 4} - \frac{5 \times 3}{4 \times 3} && \{\text{LCD} = 12\} \\ &= \frac{28}{12} - \frac{15}{12} \\ &= \frac{28 - 15}{12} \\ &= \frac{13}{12} \\ &= 1\frac{1}{12} \end{aligned}$$

EXERCISE 5G

1 Find:

- **a** $\frac{2}{3} + \frac{1}{3}$
- **b** $\frac{3}{5} - \frac{1}{5}$
- **c** $\frac{4}{7} + \frac{5}{7}$
- **d** $\frac{19}{4} - \frac{3}{4}$
- **e** $\frac{7}{3} - \frac{4}{3}$
- **f** $\frac{1}{6} + \frac{5}{6}$
- **g** $2 + \frac{3}{8} - \frac{7}{8}$
- **h** $1 + \frac{7}{10} - \frac{9}{10}$

2 Find:

- **a** $\frac{1}{4} + \frac{1}{8}$
- **b** $\frac{9}{10} - \frac{3}{5}$
- **c** $\frac{1}{2} - \frac{1}{3}$
- **d** $\frac{3}{4} + \frac{3}{5}$
- **e** $\frac{5}{7} - \frac{2}{3}$
- **f** $\frac{3}{4} - \frac{1}{6}$
- **g** $\frac{7}{6} - \frac{2}{3}$
- **h** $\frac{1}{8} + \frac{5}{12}$
- **i** $\frac{13}{14} - \frac{6}{7}$
- **j** $\frac{3}{15} + \frac{7}{20}$
- **k** $\frac{16}{9} - \frac{1}{6}$
- **l** $\frac{3}{10} - \frac{1}{8}$

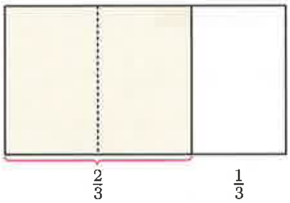
3 Find:

- **a** $2 - 1\frac{1}{4}$
- **b** $3 - \frac{2}{3}$
- **c** $3\frac{2}{3} - 1\frac{1}{2}$
- **d** $1\frac{5}{7} + 1\frac{3}{14}$
- **e** $3\frac{4}{5} - 2\frac{9}{10}$
- **f** $1\frac{1}{9} + 2\frac{5}{6}$
- **g** $4\frac{2}{3} - 3\frac{5}{6}$
- **h** $3\frac{3}{4} + 2\frac{7}{8}$

4 Find:

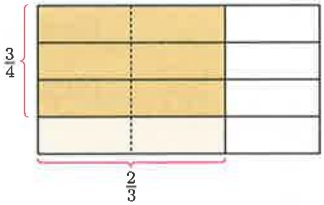
- **a** the sum of $\frac{4}{5}$ and $\frac{5}{7}$
- **b** the number $1\frac{1}{2}$ more than $\frac{3}{8}$
- **c** the number $\frac{5}{6}$ less than $\frac{19}{18}$
- **d** the difference between $4\frac{9}{10}$ and $2\frac{1}{2}$.

H MULTIPLYING FRACTIONS



Philippa has baked a rectangular slab of pizza.

She gives $\frac{2}{3}$ of the pizza to her husband James to take to work for lunch.



James eats $\frac{3}{4}$ of the $\frac{2}{3}$ of the original pizza.

The word "of" means we are multiplying, so James eats $\frac{3}{4} \times \frac{2}{3}$ of the pizza.

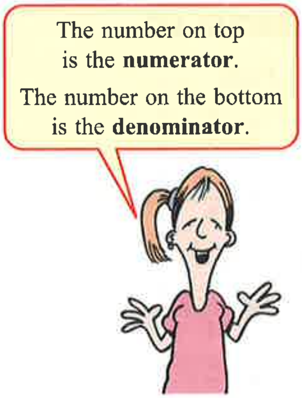
DEMO

To illustrate this, we divide each third of the pizza into 4 equal parts. We now have 12 parts in total. We then shade 3 out of every 4 parts of James' piece. We see that he has eaten 6 of the 12 parts of the original pizza. This is $\frac{6}{12}$ or $\frac{1}{2}$.

$$\text{So,} \quad \frac{3}{4} \times \frac{2}{3} = \frac{6}{12} = \frac{1}{2}.$$

To **multiply** two fractions, we multiply the two numerators to get the new numerator, and multiply the two denominators to get the new denominator.

$$\frac{a}{b} \times \frac{c}{d} = \frac{a \times c}{b \times d}$$



To help make multiplication easier, we can **cancel** any **common factors** in the numerator and denominator *before* we multiply.

For example, for James' pizza, $\frac{3}{4} \times \frac{2}{3} = \frac{{}^1\cancel{3} \times \cancel{2}^1}{{}_2\cancel{4} \times \cancel{3}_1} = \frac{1}{2}$.

Example 17 **Self Tutor**

Find:

a $\frac{2}{11} \times \frac{3}{5}$ **b** $\frac{4}{9} \times \frac{3}{5}$ **c** $\frac{4}{9} \times 1\frac{7}{8}$

a $\frac{2}{11} \times \frac{3}{5}$

$= \frac{2 \times 3}{11 \times 5}$

$= \frac{6}{55}$

b $\frac{4}{9} \times \frac{3}{5}$

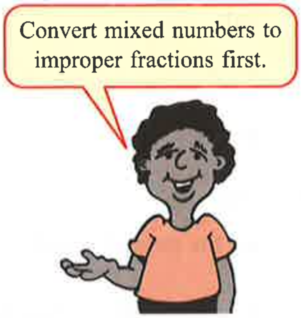
$= \frac{4 \times \cancel{3}^1}{{}_3\cancel{9} \times 5}$

$= \frac{4}{15}$

c $\frac{4}{9} \times 1\frac{7}{8}$

$= \frac{{}^1\cancel{4} \times \cancel{15}^5}{{}_3\cancel{9} \times \cancel{8}_2}$

$= \frac{5}{6}$



Example 18 **Self Tutor**

Find: **a** $\frac{2}{3}$ of 9 **b** $\frac{1}{4}$ of $\frac{2}{5}$

a $\frac{2}{3} \times 9$

$= \frac{2}{3} \times \frac{9}{1}$

$= \frac{2 \times \cancel{9}^{3}}{{}_{1}\cancel{3} \times 1}$

$= 6$

b $\frac{1}{4} \times \frac{2}{5}$

$= \frac{1 \times \cancel{2}^{1}}{{}_{2}\cancel{4} \times 5}$

$= \frac{1}{10}$

EXERCISE 5H

1 Find:

a $\frac{1}{2} \times \frac{1}{3}$ **b** $\frac{1}{2} \times \frac{3}{5}$ **c** $\frac{2}{3} \times \frac{2}{3}$ **d** $\frac{4}{3} \times \frac{1}{5}$

e $\frac{2}{9} \times \frac{2}{3}$ **f** $\frac{1}{6} \times \frac{5}{2}$ **g** $\frac{1}{3}$ of 5 **h** $\frac{5}{3} \times \frac{2}{3}$

i $\frac{3}{4}$ of $\frac{1}{2}$ **j** $1\frac{1}{3} \times \frac{2}{5}$ **k** $\frac{5}{8}$ of 3 **l** $3\frac{1}{2} \times 2\frac{3}{4}$

2 Evaluate, giving your answer in simplest form:

a $\frac{3}{4} \times \frac{1}{3}$ **b** $\frac{1}{2} \times \frac{2}{3}$ **c** $\frac{4}{5} \times \frac{5}{7}$ **d** $\frac{5}{6} \times \frac{2}{3}$

e $\frac{3}{4}$ of 24 **f** $\frac{2}{3} \times \frac{9}{4}$ **g** $\frac{5}{6} \times \frac{3}{10}$ **h** $\frac{3}{8} \times \frac{2}{9}$

i $\frac{3}{11} \times \frac{44}{9}$ **j** $\frac{3}{5}$ of $1\frac{2}{3}$ **k** $\frac{4}{7} \times \frac{21}{16}$ **l** $\frac{9}{7}$ of $1\frac{1}{6}$

3 Find the product of:

a $\frac{3}{8}$ and $\frac{1}{3}$ **b** $5\frac{1}{2}$ and $\frac{3}{11}$ **c** $\frac{5}{7}$ and 28

4 Find:

a $\frac{1}{2} \times \frac{2}{3} \times \frac{1}{2}$ **b** $\frac{1}{3} \times \frac{2}{3} \times \frac{3}{4}$ **c** $\frac{2}{3} \times \frac{1}{4} \times \frac{3}{7}$

d $\frac{2}{3} \times \frac{1}{4} \times \frac{3}{5}$ **e** $\frac{3}{8} \times \frac{4}{3} \times \frac{2}{5}$ **f** $\frac{1}{2} \times \frac{4}{5} \times 3$

5 Find:

a five eighths of 40 **b** four ninths of 45 **c** three fifths of $3\frac{3}{4}$

I RECIPROCALS

Two numbers are **reciprocals** of each other if their product is one.

For any fraction $\frac{a}{b}$, we notice that $\frac{a}{b} \times \frac{b}{a} = 1$.

So, the reciprocal of $\frac{a}{b}$ is $\frac{b}{a}$.

To find the reciprocal of a mixed number, we must first convert it to an improper fraction.

Example 19 Self Tutor

Find the reciprocal of $1\frac{5}{8}$.

$1\frac{5}{8} = \frac{13}{8}$ {converting to an improper fraction}

The reciprocal of $\frac{13}{8}$ is $\frac{8}{13}$.

$\therefore$ the reciprocal of $1\frac{5}{8}$ is $\frac{8}{13}$.

EXERCISE 5I

1 Find the reciprocal of:

a $\frac{3}{4}$ **b** $\frac{2}{3}$ **c** $\frac{5}{6}$ **d** $\frac{4}{7}$ **e** $\frac{8}{3}$ **f** $\frac{18}{5}$

2 Find the reciprocal of:

a $1\frac{1}{2}$ **b** $2\frac{2}{3}$ **c** $2\frac{1}{5}$ **d** $4\frac{3}{4}$ **e** $1\frac{7}{8}$ **f** $5\frac{1}{6}$

3 Find the reciprocal of:

a $-\frac{3}{4}$ **b** $-\frac{1}{3}$ **c** $-\frac{5}{6}$ **d** $-\frac{12}{5}$ **e** $-1\frac{1}{8}$ **f** $-2\frac{4}{5}$

J DIVIDING FRACTIONS

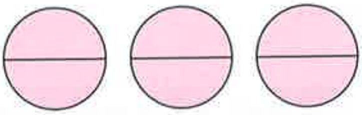
To understand the division of fractions, we need to first understand what it means to divide whole numbers.

To find $6 \div 2$ we ask the question: *How many twos are there in six?*

The answer is 3, so $6 \div 2 = 3$.

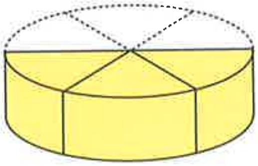
We can divide fractions in the same way.

For example, $3 \div \frac{1}{2}$ may be interpreted as: *How many halves are there in three?*



The answer is 6, so $3 \div \frac{1}{2} = 6$.

However, we know that $3 \times 2 = 6$ also, which suggests that dividing by $\frac{1}{2}$ is the same as multiplying by its *reciprocal*, 2.



Now consider dividing half a cheese equally between 3 people.

Each person would get $\frac{1}{6}$ of the whole,

DEMO

so $\frac{1}{2} \div 3 = \frac{1}{6}$

But $\frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$ also.

This also suggests that dividing by a number is the same as multiplying by its reciprocal.

To **divide** by a number, we multiply by its reciprocal.

Example 20

Self Tutor

Find: **a** $\frac{5}{4} \div \frac{2}{3}$ **b** $1\frac{1}{3} \div 3\frac{1}{2}$

a $\frac{5}{4} \div \frac{2}{3}$

$= \frac{5}{4} \times \frac{3}{2}$ {multiplying by reciprocal}

$= \frac{5 \times 3}{4 \times 2}$

$= \frac{15}{8}$

b $1\frac{1}{3} \div 3\frac{1}{2}$

$= \frac{4}{3} \div \frac{7}{2}$ {converting to improper fractions}

$= \frac{4}{3} \times \frac{2}{7}$ {multiplying by reciprocal}

$= \frac{4 \times 2}{3 \times 7}$

$= \frac{8}{21}$

EXERCISE 5J

1 Find:

a $\frac{3}{4} \div \frac{1}{4}$ **b** $\frac{2}{3} \div \frac{1}{3}$ **c** $\frac{5}{2} \div \frac{3}{2}$ **d** $\frac{8}{5} \div \frac{2}{5}$

2 Find:

a $\frac{2}{3} \div \frac{1}{2}$ **b** $\frac{2}{5} \div \frac{1}{6}$ **c** $\frac{3}{5} \div \frac{2}{3}$ **d** $\frac{2}{5} \div 3$

3 Find:

a $\frac{5}{8} \div 2$ **b** $1\frac{1}{3} \div \frac{1}{2}$ **c** $\frac{3}{10} \div 1\frac{1}{2}$ **d** $\frac{5}{6} \div 3$

e $\frac{4}{5} \div 8$ **f** $2\frac{1}{2} \div 1\frac{3}{4}$ **g** $3\frac{3}{10} \div 1\frac{5}{6}$ **h** $2\frac{3}{4} \div \frac{2}{3}$

4 Find:

a the average of $\frac{3}{4}$ and $\frac{3}{8}$ **b** the quotient of $1\frac{1}{2}$ and $\frac{3}{7}$.

K EVALUATING FRACTIONS USING A CALCULATOR

When we enter operations into a calculator, it automatically uses the BEDMAS rules. However, we need to be careful with more complicated fractions because we need to divide *the whole of the numerator* by *the whole of the denominator*. To make sure the calculator knows what we mean, we insert brackets around the numerator and the denominator.

For example, consider the expression $\frac{5+6}{3-1}$.

If we type in $5\ \boxed{+}\ 6\ \boxed{\div}\ 3\ \boxed{-}\ 1$, the calculator will think we want $5+\frac{6}{3}-1$, and so it will give us the wrong answer.

We need to insert brackets around both the numerator and denominator, giving $\frac{(5+6)}{(3-1)}$.

We type in $\boxed{(}\ 5\ \boxed{+}\ 6\ \boxed{)}\ \boxed{\div}\ \boxed{(}\ 3\ \boxed{-}\ 1\ \boxed{)}$.

Example 21 **Self Tutor**

Find the value of: **a** $\frac{5+7}{6-3}$ **b** $\frac{3\times 6+2}{4\times 2}$

a $\frac{5+7}{6-3}=\frac{(5+7)}{(6-3)}=4$

Calculator: $\boxed{(}\ 5\ \boxed{+}\ 7\ \boxed{)}\ \boxed{\div}\ \boxed{(}\ 6\ \boxed{-}\ 3\ \boxed{)}\ \boxed{=}$

b $\frac{3\times 6+2}{4\times 2}=\frac{(3\times 6+2)}{(4\times 2)}=2\frac{1}{2}$

Calculator: $\boxed{(}\ 3\ \boxed{\times}\ 6\ \boxed{+}\ 2\ \boxed{)}\ \boxed{\div}\ \boxed{(}\ 4\ \boxed{\times}\ 2\ \boxed{)}\ \boxed{=}$

EXERCISE 5K

1 Evaluate the following, then check your answer using a calculator:

a $8+\frac{16}{8}$ **b** $\frac{8+16}{8}$ **c** $9-\frac{6}{12}$ **d** $\frac{9-3}{12}$

e $\frac{15-8}{4+10}$ **f** $15-\frac{8}{4}+10$ **g** $3+\frac{5}{10}+6$ **h** $\frac{3+5}{10+6}$

2 Evaluate using a calculator:

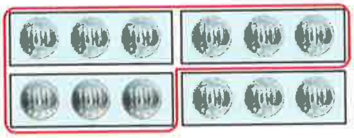
a $\frac{3\times 4+1}{30-4}$ **b** $\frac{45-2\times 3}{15-2}$ **c** $\frac{18\times 6+17}{5\times(8-3)}$

L PROBLEM SOLVING

In this Section we see how fractions are applied to the real world. They can describe a part of a quantity or a group of objects.

For example, $\frac{3}{4}$ of 12 coins is 9 coins

and $\frac{3}{4}\times 12 = \frac{3\times \cancel{12}^{3}}{{}_{1}\cancel{4}\times 1} = 9$



Example 22 Self Tutor

Find $\frac{3}{5}$ of \$85.

$\frac{3}{5}$ of \$85

$= \frac{3}{5}\times \$\frac{85}{1}$

$= \frac{3\times \$\cancel{85}^{17}}{{}_{1}\cancel{5}\times 1}$

$= \$51$

EXERCISE 5L

1 Find: **a** $\frac{2}{3}$ of \$63 **b** $\frac{3}{7}$ of 35 kg.

2 Julie owes Grace $\frac{3}{5}$ of €135. How much does she owe Grace?

3 In a test consisting of 60 questions, Tim answered $\frac{3}{4}$ of the questions correctly. How many questions did Tim answer correctly?

4 The price of a shirt is $\frac{2}{15}$ of the cost of a suit. If the suit costs \$375, find the price of the shirt.

5 Kate cooks $\frac{2}{7}$ of a bag of pasta for dinner. What fraction of the bag of pasta remains?

Example 23 Self Tutor

Rob eats $\frac{1}{3}$ of a watermelon one day and $\frac{3}{8}$ of it the next day.

What fraction of the watermelon remains?

The fraction remaining

$= 1 - \frac{1}{3} - \frac{3}{8}$ {from the whole we subtract the fractions eaten}

$= \frac{24}{24} - \frac{1\times 8}{3\times 8} - \frac{3\times 3}{8\times 3}$ {LCD of 3 and 8 is 24}

$= \frac{24-8-9}{24}$

$= \frac{7}{24}$

6 Workers at an office eat $\frac{3}{5}$ of a cake at morning tea and $\frac{3}{8}$ of it at afternoon tea. What fraction of cake remains?

7 In a basketball game, Jacob gets $\frac{4}{9}$ of the rebounds and Adam gets $\frac{3}{10}$ of the rebounds for their team. What fraction of the rebounds did the rest of the team get?

8 Over 2 successive days Toby paves $\frac{3}{10}$ and $\frac{5}{12}$ of his driveway. What fraction of the driveway must he pave on the third day to finish the job?

Example 24 **Self Tutor**

Carla the cat eats $\frac{2}{3}$ of a tin of cat food for each meal.
How many meals are in 12 tins of cat food?

To answer this, we ask "How many lots of two thirds are there in 12?"

$$\text{Number of meals} = 12 \div \frac{2}{3}$$
$$= \frac{12}{1} \times \frac{3}{2}$$
$$= \frac{{}^{6}\cancel{12} \times 3}{1 \times \cancel{2}_{1}} = 18$$

9 60 kg of pine nuts are poured into packets so that each packet contains $\frac{3}{4}$ kg of pine nuts. How many packets will be filled?

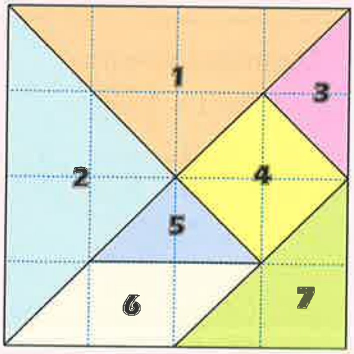
10 3600 L of water are poured into bottles which hold $1\frac{1}{4}$ L each. How many bottles will be filled?

11 John says that his income is now $3\frac{1}{2}$ times what it was 20 years ago. If his current annual income is $63 000, what was his income 20 years ago?

12 Tony's orange tree produces a large number of oranges. He keeps one third of them for himself, and shares the rest between his four children. What fraction of the total number of oranges does each child receive?

ACTIVITY 3 TANGRAMS

A **tangram** is a puzzle consisting of the seven shapes shown.



Using identical square pieces of paper, make two copies of this tangram. Number the pieces on both sheets. Cut one of the sheets into its seven pieces. Use the pieces to help you work out the following:

1 How many triangles like piece **1** would fit into the overall square tangram?

2 What fraction of the tangram is piece **1**?

3 What fraction of piece **1** is piece **3**?

4 What fraction of the tangram is each piece?

TANGRAM

Research: Using the internet, find different shapes which can be made using the pieces in a tangram. Can you make these shapes?

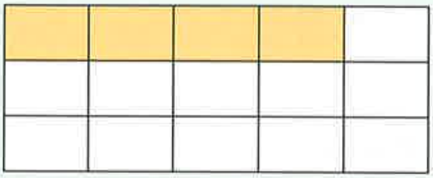
KEY WORDS USED IN THIS CHAPTER

- common fraction
- denominator
- fraction
- fraction wall
- improper fraction
- lowest common denominator
- lowest terms
- mixed number
- numerator
- proper fraction
- reciprocal
- simplest form

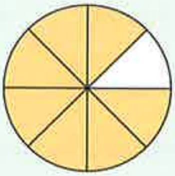
REVIEW SET 5A

1 State the fraction represented by:

a



b



2 Write as a division and hence as a whole number:

a $\frac{20+16}{4\times 3}$ **b** $\frac{5\times 10}{13-11}$ **c** $\frac{5-2\times 13}{3+4}$

3 Write in lowest terms: **a** $\frac{3}{18}$ **b** $\frac{24}{44}$ **c** $\frac{45}{25}$

4 Display using a number line: $-\frac{1}{5}, \frac{2}{5}, 1\frac{1}{5}, \frac{8}{5}$

5 Write as a mixed number: **a** $\frac{29}{3}$ **b** $\frac{38}{5}$ **c** $\frac{53}{7}$

6 Which number is larger, $\frac{3}{8}$ or $\frac{5}{12}$?

7 Find □ such that: **a** $\frac{3}{7}=\frac{\square}{28}$ **b** $\frac{15}{40}=\frac{\square}{8}$ **c** $\frac{1}{3}=\frac{\square}{18}$

8 Find the product of $\frac{5}{11}$ and $1\frac{1}{2}$.

9 Find:

a $\frac{2}{3}+\frac{2}{5}$ **b** $\frac{7}{8}-\frac{2}{3}$ **c** $\frac{3}{5}\div 2\frac{3}{4}$ **d** $1\frac{3}{4}\times 2$

10 Use a calculator to find:

a $\frac{2\times 3+14}{5\times 8}$ **b** $3+\frac{7-2}{5}$

11 Only $\frac{3}{8}$ of a class brought lunch to school yesterday. If there are 24 students in the class, how many brought lunch to school?

12 What fraction of a roll of fabric is left if $\frac{1}{5}$ and $\frac{1}{8}$ of it is used to make dresses?

REVIEW SET 5B

1 Aaron is cooking sausages on a barbecue. Four are chicken, seven are beef, six are pork, and three are lamb. State what fraction of Aaron's sausages are:

a beef **b** chicken **c** beef or lamb.

2 Write $\frac{-28}{7}$ as a division and hence as a whole number.

3 Write as an improper fraction: **a** $1\frac{9}{10}$ **b** $7\frac{1}{5}$.

4 What number is $\frac{3}{7}$ less than $2\frac{1}{2}$?

5 Find the fractions represented by the points A, B, and C:

6 Write as an improper fraction in lowest terms: **a** $\frac{35}{25}$ **b** $\frac{64}{56}$

7 Find:

a $\frac{4}{5} \times \frac{6}{7}$ **b** $\frac{2}{3} + \frac{1}{6}$ **c** $\frac{7}{4} - \frac{4}{5}$ **d** $\frac{3}{8} \div \frac{2}{3}$

e $3\frac{1}{3} + 2\frac{1}{2}$ **f** $\frac{4}{7} \times 2\frac{2}{3}$ **g** $5\frac{2}{9} - 1\frac{5}{6}$ **h** $2\frac{7}{10} \div 3\frac{3}{4}$

8 Find the reciprocal of $3\frac{1}{8}$.

9 Write 30 kg as a fraction of 105 kg.

10 350 kg of plastic is moulded to make garden pots weighing $1\frac{2}{5}$ kg each. How many pots are made?

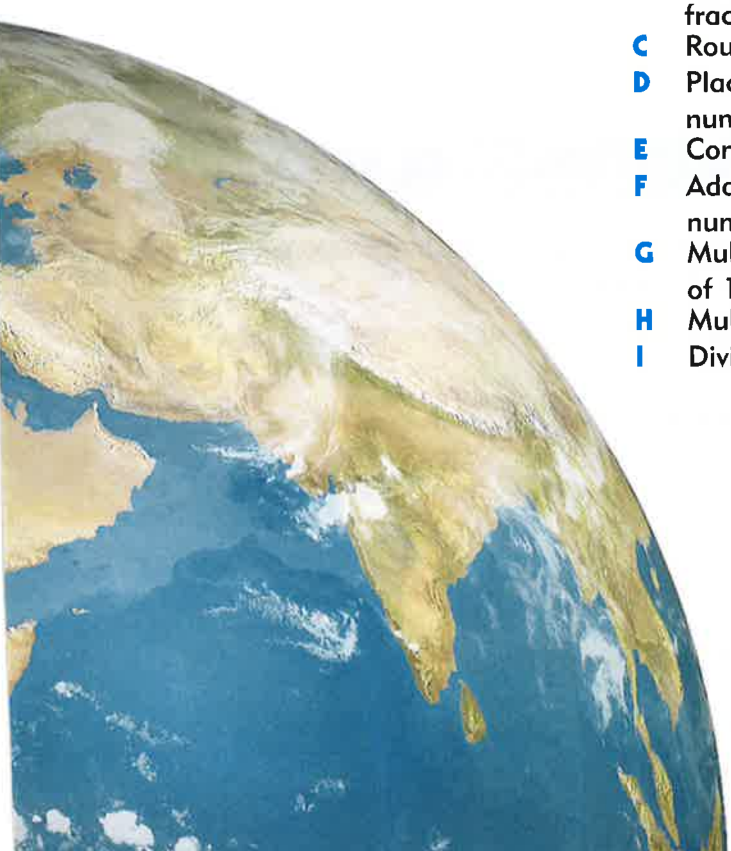
11 Cheryl ate $\frac{1}{5}$ of her block of chocolate yesterday and $\frac{1}{4}$ of it today. What fraction of the block remains?

12 Simplify $\frac{39 \div 3 + 2}{3 \times 2}$.

Chapter 6

Decimal numbers

Contents:



OPENING PROBLEM

The table alongside shows the results of a 400 metre race. Stan was the winner. His time is given, and the other results show how much longer it took each of the remaining competitors to finish. For example, Duncan finished 1.92 seconds after Stan.

	Runner	*Time* (s)
1	Stan	51.47
2	Colin	+0.22
3	Henry	+1.58
4	Duncan	+1.92
5	Bradley	+2.07
6	Kevin	+3.58
7	Tyson	+4.31
8	William	+4.85

Things to think about:

a What was Bradley's time?

b By how much did:

  i Colin beat Henry

  ii Kevin beat Tyson?

Decimal numbers are widely used in everyday life. We see them frequently in situations involving money, and in measurements of length, time, and weight.

ACTIVITY 1 DECIMALS ALL AROUND YOU

In pairs, list examples of where you have seen decimals:

- at home
- in sport
- on the news
- at the shops.
- at school

A PLACE VALUE

The **number system** we use today is a **place value** system using base 10. In this chapter we see how the place value system is extended to include parts of a whole.

If we restrict ourselves to fractions where the denominator is a power of 10, we can use the place value system to represent both whole and fractional numbers.

We introduce a symbol called a **decimal point** to separate the whole number part from the fractional part.

For example: 731.245 represents $700+30+1+\frac{2}{10}+\frac{4}{100}+\frac{5}{1000}$

24.059 represents $20+4+\frac{0}{10}+\frac{5}{100}+\frac{9}{1000}$

When written as a sum like this, we say the number is in **expanded form**.

We normally leave out the $+\frac{0}{10}$ as it has no value. However, it is very important to include the 0 in the decimal number, as it makes sure the 5 and 9 are in the correct place values.

The **place value table** for 731.245 and 24.059 is:

	hundreds	tens	units		tenths	hundredths	thousandths
731.245	7	3	1	.	2	4	5
24.059		2	4	.	0	5	9

When a decimal number does not contain any whole number part, we write a zero in the units place. This gives more emphasis to the decimal point.

For example, we write 0.75 instead of .75 .

Example 1 **Self Tutor**

Write in expanded form: 7.802

$$7.802 = 7 + \frac{8}{10} + \frac{2}{1000}$$

Example 2 **Self Tutor**

Write in decimal form:

a $\frac{3}{10} + \frac{7}{100}$ **b** $4 + \frac{9}{10} + \frac{6}{1000}$

a $\frac{3}{10} + \frac{7}{100} = 0.37$ **b** $4 + \frac{9}{10} + \frac{6}{1000} = 4.906$

EXERCISE 6A

1 Write in expanded form:

a 4.2 **b** 7.53 **c** 9.18 **d** 3.03
e 0.234 **f** 1.059 **g** 5.0061 **h** 0.000 71
i 2.501 **j** 0.0771 **k** 11.912 **l** 0.0101

2 Write in decimal form:

a $\frac{7}{10}$ **b** $\frac{1}{10} + \frac{5}{100}$ **c** $\frac{5}{10} + \frac{4}{100} + \frac{9}{1000}$
d $\frac{3}{100}$ **e** $\frac{1}{10} + \frac{5}{1000}$ **f** $\frac{6}{100} + \frac{7}{1000}$
g $\frac{8}{100} + \frac{4}{1000}$ **h** $\frac{3}{1000} + \frac{9}{10\,000}$ **i** $\frac{6}{10} + \frac{1}{100} + \frac{5}{1000} + \frac{5}{10\,000}$

Example 3 **Self Tutor**

Write in decimal form: $\frac{65}{1000}$

$$\begin{aligned}\frac{65}{1000} &= \frac{60}{1000} + \frac{5}{1000}\\ &= \frac{6}{100} + \frac{5}{1000}\\ &= 0.065\end{aligned}$$

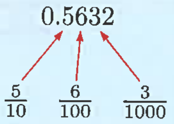
3 Write in decimal form:

a $\frac{71}{100}$ **b** $\frac{13}{100}$ **c** $\frac{54}{100}$ **d** $\frac{267}{1000}$

e $\frac{506}{1000}$ **f** $\frac{97}{1000}$ **g** $\frac{803}{1000}$ **h** $\frac{22}{1000}$

Example 4 Self Tutor

State the value of the digit 3 in 0.5632



$\therefore$ the 3 stands for $\frac{3}{1000}$.

4 State the value of the digit 7 in:

a 1723 **b** 3.7128 **c** 23.07 **d** 88.0672

e 0.8713 **f** 73 066 **g** 81.794 **h** 3182.796

Example 5 Self Tutor

Express $4\frac{137}{1000}$ in decimal form.

$$4\frac{137}{1000} = 4 + \frac{100}{1000} + \frac{30}{1000} + \frac{7}{1000}$$
$$= 4 + \frac{1}{10} + \frac{3}{100} + \frac{7}{1000}$$
$$= 4.137$$

5 Express in decimal form:

a $7\frac{6}{10}$ **b** $3\frac{67}{100}$ **c** $12\frac{17}{100}$ **d** $2\frac{59}{100}$

e $1\frac{461}{1000}$ **f** $6\frac{39}{1000}$ **g** $2\frac{1}{1000}$ **h** $3\frac{7}{10\,000}$

i $5\frac{390}{1000}$ **j** $7\frac{203}{10\,000}$ **k** $\frac{721}{100}$ **l** $\frac{3723}{1000}$

B CONVERTING BETWEEN DECIMALS AND FRACTIONS

Our understanding of decimals and the place value system allows us to convert between decimals and fractions.

CONVERTING DECIMALS TO FRACTIONS

To convert decimals to fractions, we first write the decimal as a fraction where the denominator is a power of 10. We then write the fraction in **simplest form**.

Example 6 Self Tutor

Write as a fraction in simplest form:

a 0.8 **b** 3.88 **c** 0.375

a $0.8 = \frac{8}{10}$
$= \frac{4}{5}$

b $3.88 = 3 + \frac{88}{100}$
$= 3\frac{22}{25}$

c $0.375 = \frac{375}{1000}$
$= \frac{3}{8}$

CONVERTING FRACTIONS TO DECIMALS

Many fractions can be written so that the denominator is a power of 10. To do this we can multiply the numerator and denominator by the same value. It is then easy to write the fraction as a decimal.

Example 7 Self Tutor

Write as a decimal:

a $\frac{7}{20}$ **b** $\frac{41}{250}$

a $\frac{7}{20} = \frac{7 \times 5}{20 \times 5}$ {write with denominator 100}
$= \frac{35}{100}$
$= 0.35$

b $\frac{41}{250} = \frac{41 \times 4}{250 \times 4}$ {write with denominator 1000}
$= \frac{164}{1000}$
$= 0.164$

EXERCISE 6B

1 Write as a fraction in simplest form:

a 0.7 **b** 0.4 **c** 1.1 **d** 2.6
e 0.19 **f** 0.29 **g** 0.25 **h** 0.16
i 0.85 **j** 0.96 **k** 0.15 **l** 0.05
m 0.07 **n** 3.13 **o** 5.08 **p** 7.55

2 Write as a fraction in simplest form:

a 0.101 **b** 0.046 **c** 0.008 **d** 0.205
e 0.125 **f** 0.0004 **g** 0.146 **h** 0.875
i 0.0005 **j** 0.0075 **k** 1.375 **l** 4.076

3 Write as a decimal:

a $\frac{1}{2}$ **b** $\frac{3}{5}$ **c** $\frac{13}{20}$ **d** $\frac{1}{4}$
e $\frac{17}{50}$ **f** $\frac{9}{20}$ **g** $\frac{23}{25}$ **h** $\frac{1}{50}$
i $\frac{31}{250}$ **j** $\frac{103}{200}$ **k** $\frac{14}{25}$ **l** $\frac{207}{500}$
m $\frac{19}{20}$ **n** $\frac{2}{125}$ **o** $\frac{5}{8}$ **p** $\frac{7}{40}$

C ROUNDING DECIMAL NUMBERS

We are often given measurements as decimal numbers. We usually **approximate** the decimal by **rounding off** to a certain number of decimal places.

RULES FOR ROUNDING OFF DECIMAL NUMBERS

The rules for rounding off decimal numbers are the same as those for rounding whole numbers.

- If the digit after the one being rounded is **less than 5** (0, 1, 2, 3, or 4) then we round **down**.
- If the digit after the one being rounded is **5 or more** (5, 6, 7, 8, or 9) then we round **up**.

Example 8 **Self Tutor**

Round 2.117 347 to 3 decimal places.

The digit in the fourth decimal place is 3.

Since 3 is less than 5, we round down.

$\therefore$ $2.117\,347 \approx 2.117$ (to 3 decimal places)

When we round to 3 decimal places, the final answer has 3 digits after the decimal point. We are rounding to the nearest thousandth.

EXERCISE 6C

1 Round 4.769 08 to:

a the nearest whole number
b 1 decimal place
c 2 decimal places
d 3 decimal places.

2 Round 23.0599 to:

a the nearest whole number
b 1 decimal place
c 2 decimal places
d 3 decimal places.

3 Round 8.042 39 to:

a the nearest whole number
b the nearest tenth
c the nearest hundredth
d the nearest ten thousandth.

4 Graham's batting average is calculated as 42.7165. Round this to two decimal places.

5 This year Aludra Gold made a profit of 127.647 million dollars. Round this to the nearest million dollars.

6 Rhys scores an average of 5.234 goals per game for his waterpolo team. Round this to one decimal place.

7 Vicki calculates the interest due on her savings account to be $57.2894. Round this to the nearest cent.

8 Use your calculator to write the following fractions and roots as decimal numbers. Round your answers to 2 decimal places.

a $\frac{8}{23}$
b $\frac{10}{7}$
c $\frac{4}{9}$
d $\frac{613}{32}$
e $\sqrt{3}$
f $\sqrt{43}$
g $\sqrt{74}$
h $\sqrt{106}$

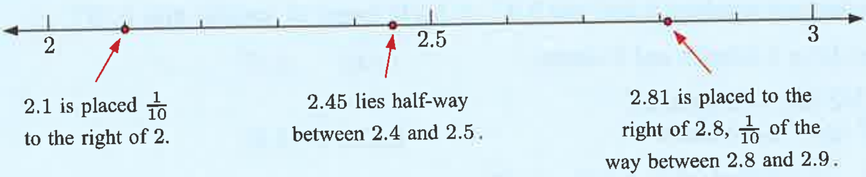
D PLACING DECIMAL NUMBERS ON A NUMBER LINE

When placing decimal numbers on a number line, we generally divide each segment of the number line into **ten equal parts**. Each of these parts is $\frac{1}{10}$ of the segment. If we divide each of the tenths into 10 equal parts, then each part is $\frac{1}{100}$ of the segment.

Example 9 **Self Tutor**

Place the values 2.1, 2.45, and 2.81 on a number line.

We divide a number line from 2 to 3 into ten equal parts.



2.1 is placed $\frac{1}{10}$ to the right of 2.

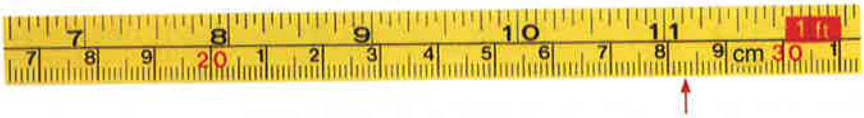
2.45 lies half-way between 2.4 and 2.5.

2.81 is placed to the right of 2.8, $\frac{1}{10}$ of the way between 2.8 and 2.9.

EXERCISE 6D

1 Write down the measurements indicated on the following devices:

a

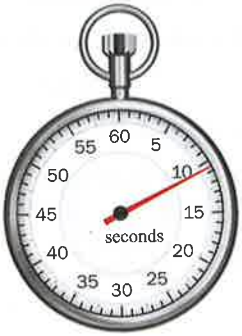


b

c

kilograms

d



2 Write down the value of the number at point A:

a

b

c

d

e

f

3 **a** Place the values 1.4, 1.06, 1.66, and 1.89 on a number line.
b Place the values 0.7, 0.35, 0.82, and 0.11 on a number line.

4 Draw a number line to illustrate the following times: 53.4 seconds, 61.9 seconds, 57.1 seconds, and 63.2 seconds.

E COMPARING DECIMAL NUMBERS

Consider the decimal numbers 5.342 and 5.37. Is 5.342 larger or smaller than 5.37?

Both numbers have 5 wholes and 3 tenths: 5.342 5.37

However, 5.342 has 4 hundredths,
whereas 5.37 has 7 hundredths: 5.342 5.37

Since $4 < 7$, we conclude that $5.342 < 5.37$.

Example 10 **Self Tutor**

Write in order from smallest to largest: 8.66, 8.6, 8.606

We write the numbers with the same number of decimal places:

8.660 8.600 8.606

The numbers each have the same whole number and tenths place:

8.660 8.600 8.606

The digits in the hundredths place are *not* all the same:

8.660 8.600 8.606

So, 8.66 is the largest number.

To order the remaining numbers we look at the thousandths place:

8.600 8.606

So, $8.600 < 8.606$.

From smallest to largest, the numbers are 8.6, 8.606, 8.66.

We can write zeros at the end of decimal numbers without changing the place value of the other digits. This can help us to compare decimal numbers.

EXERCISE 6E

1 Insert >, <, or = between these pairs of numbers:

a 0.339, 0.393	**b** 5.05, 0.55	**c** 0.6, 0.60
d 2.62, 2.6	**e** 0.39, 0.4	**f** 12.121, 21.121
g 0.123, 0.132	**h** $\frac{150}{1000}$, 0.15	**i** 2.4, 2.400
j 0.902, 0.209	**k** 0.008 76, 0.0876	**l** 3.20, 3.201

2 Write in ascending order:

a 1.36, 1.3, 1.036 **b** 8.76, 8.67, 8.6 **c** 0.5, 0.495, 0.052
d 32.7, 32.71, 33.17 **e** 8.066, 7.999, 8.1 **f** 6.304, 6.043, 6.403, 6.34
g 9.1, 9.09, 9.2, 9.009 **h** 0.9, 0.09, 0.99, 0.099

3 Patricia's best four times for a 100 m sprint are 16.98 seconds, 16.91 seconds, 17.19 seconds, and 17.1 seconds. Place these times in order from fastest to slowest.

4 On Monday the Euro could be exchanged for 1.3537 US dollars. For the rest of the week, the exchange figures were: Tuesday 1.3571, Wednesday 1.3602, Thursday 1.3519, and Friday 1.3578.

a Place these exchange rates in order from highest to lowest.
b On which day was the exchange rate: **i** highest **ii** lowest?

F ADDING AND SUBTRACTING DECIMAL NUMBERS

When **adding** or **subtracting** decimal numbers, we write the numbers under one another so the decimal points line up.

When this is done, the digits in each place value will also line up. We then add or subtract as for whole numbers.

Example 11 Self Tutor

Find: 1.76 + 0.961

$$\begin{array}{rr} & 1.760 \\ + & 0.961 \\ & {\scriptstyle 1\ \ 1\ \ } \\ \hline & 2.721 \end{array}$$

We write a 0 on the end of 1.76 so that both numbers have the same number of decimal places.

Example 12 Self Tutor

Find: **a** 3.652 − 2.584 **b** 6 − 0.637

a
$$\begin{array}{rcccccc} & & & & {\scriptstyle 5} & {\scriptstyle 14} & {\scriptstyle 12} \\ & 3 & . & \cancel{6} & \cancel{5} & \cancel{2} \\ - & 2 & . & 5 & 8 & 4 \\ \hline & 1 & . & 0 & 6 & 8 \end{array}$$

b
$$\begin{array}{rcccccc} & {\scriptstyle 5} & & {\scriptstyle 9} & {\scriptstyle 9} & {\scriptstyle 10} \\ & \cancel{6} & . & \cancel{0} & \cancel{0} & \cancel{0} \\ - & 0 & . & 6 & 3 & 7 \\ \hline & 5 & . & 3 & 6 & 3 \end{array}$$

EXERCISE 6F

1 Find:

a 0.2 + 0.7 **b** 0.6 + 0.33 **c** 0.12 + 1.57 **d** 0.17 + 2.36
e 0.9 + 0.23 **f** 13.56 + 6.073 **g** 13.795 + 0.015 **h** 0.071 + 0.477
i 0.0048 + 0.659 **j** 12.66 + 1.302 **k** 0.23 + 0.78 + 3 **l** 0.27 + 3.18 + 1.79

2 Find:

- **a** 2.8 – 0.5
- **b** 3.29 – 1.16
- **c** 3.0299 – 0.0271
- **d** 1.5 – 0.8
- **e** 1.6 – 0.9
- **f** 1 – 0.99
- **g** 3.27 – 1.98
- **h** 1.01 – 0.0026
- **i** 7.2 – 0.65
- **j** 0.083 – 0.0091
- **k** 7.21 – 0.75
- **l** 1.1 – 0.1234

3 Add:

- **a** 29.63, 127.98, and 2.45
- **b** 17.55, 307.2, and 498.92
- **c** 21.38, 279.34, and 10.629
- **d** 9.77, 11.7, 108.54, and 0.28

4 Subtract:

- **a** 7.529 from 19.436
- **b** 15.87 from 21.31
- **c** 12.118 from 18
- **d** 8.135 from 57.2

5 A 2 kg bag of coffee is poured into four smaller bags. The weights of three of the bags are 0.475 kg, 0.81 kg, and 0.59 kg. Find the weight of the fourth bag.

6 A cat jumped 1.8 m from the ground onto a fence, a further 0.95 m onto the garage roof, then another 1.52 m onto the house roof. How high is the house roof above the ground?

7 How much change would be left from a $50 note if you purchased items costing $13.79, $5.25, $23.75, and $3.46?

8

Taxation	£507.90
Superannuation	£153.40
Private Health Cover	£24.62
Union Fees	£14.82

Each fortnight Claire is paid £1356.28 less the deductions given in the table alongside. How much pay does Claire keep each fortnight?

9 Answer the **Opening Problem** on page **122**.

ACTIVITY 2

COSTLESS SHOPPING

In this Activity, you will try to *estimate* how much a selection of items at a supermarket would cost.

What to do:

1. With a friend, nominate 10 items you wish to buy.
2. Each of you should write down estimates for the price of each item.
3. Go to the supermarket together. As you walk through the store, write down the actual prices of each item on your list.
4. Find the difference between your estimate and the actual price for each item.
   - **a** Compare your results with your friend. Who was closer to the actual price for each item?
   - **b** Tally the number of items for which you were closer to the actual price. Who has the better estimates?

c Total the actual prices for **all** items and the estimated prices for **all** items.
  i Find the difference between the totals.
  ii Compare your result with your friend again. Who was closer to the actual total price?

5 Repeat this Activity for other shops, such as electrical or variety stores.

G MULTIPLYING AND DIVIDING BY POWERS OF 10

MULTIPLYING BY POWERS OF 10

Consider multiplying 2.36

- by 100: $2.36 \times 100 = \frac{236}{{}_1\cancel{100}} \times \frac{\cancel{100}^{1}}{1}$
  $= 236$
- by 1000: $2.36 \times 1000 = \frac{236}{{}_1\cancel{100}} \times \frac{\cancel{1000}^{10}}{1}$
  $= 236 \times 10$
  $= 2360$

When we multiply by 100, the decimal point of 2.36 shifts 2 places to the **right**.

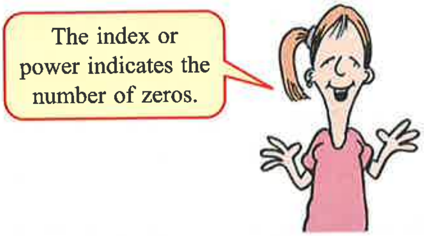
2.36 becomes $236.$

When we multiply by 1000, the decimal point shifts 3 places to the **right**.

2.360 becomes $2360.$

> When multiplying by 10^n we shift the decimal point n places to the **right**.
> The number becomes 10^n times **greater** than it was originally.

Remember $10^1 = 10$
$10^2 = 100$
$10^3 = 1000$
$10^4 = 10\,000$
$\vdots$



Example 13 **Self Tutor**

Find: **a** 9.8×10 **b** 0.0751×100 **c** $13.026 \times 10\,000$

a 9.8×10
$= 9.8 \times 10^1$ {$10 = 10^1$, so shift the decimal point 1 place right.}
$= 98$

b 0.0751×100
$= 0.0751 \times 10^2$ {$100 = 10^2$, so shift the decimal point 2 places right.}
$= 7.51$

c $13.026 \times 10\,000$
$= 13.0260 \times 10^4$ {$10\,000 = 10^4$, so shift the decimal point 4 places right.}
$= 130\,260$

EXERCISE 6G.1

1 **a** Multiply 3.271 by:

  i 10   **ii** 100   **iii** 1000   **iv** 10^7

  b Multiply 7.6 by:

  i 10   **ii** 1000   **iii** 10^4 **iv** 10^6

2 Evaluate:

  a 27×10 **b** 4×100 **c** 2.2×10

  d 16.4×1000 **e** 0.2×10 **f** 0.79×100

  g 8.1×100 **h** 0.5×100 **i** 1.67×10^4

  j 0.036×10^3 **k** $0.007\,61 \times 10^4$ **l** $0.338 \times 100\,000$

DIVIDING BY POWERS OF 10

Consider dividing 3.7

- by 100: $$3.7 \div 100 = \frac{37}{10} \div \frac{100}{1} = \frac{37}{10} \times \frac{1}{100} = \frac{37}{1000} = 0.037$$
- by 1000: $$3.7 \div 1000 = \frac{37}{10} \div \frac{1000}{1} = \frac{37}{10} \times \frac{1}{1000} = \frac{37}{10\,000} = 0.0037$$

When we divide by 100, the decimal point in 3.7 shifts 2 places to the **left**.

$0\overset{\curvearrowleft}{0}\overset{\curvearrowleft}{3}.7$ becomes 0.037.

When we divide by 1000, the decimal point in 3.7 shifts 3 places to the **left**.

$0\overset{\curvearrowleft}{0}\overset{\curvearrowleft}{0}\overset{\curvearrowleft}{3}.7$ becomes 0.0037.

> When dividing by 10^n we shift the decimal point n places to the **left**.
>
> The number becomes 10^n times **less** than it was originally.

Example 14 **Self Tutor**

Find: **a** $0.4 \div 10$ **b** $0.18 \div 1000$

a $0.4 \div 10$

$= \overset{\curvearrowleft}{0}.4 \div 10^1$ {$10 = 10^1$, so shift the decimal point 1 place left.}

$= 0.04$

b $0.18 \div 1000$

$= \overset{\curvearrowleft}{0}\overset{\curvearrowleft}{0}\overset{\curvearrowleft}{0}.18 \div 10^3$ {$1000 = 10^3$, so shift the decimal point 3 places left.}

$= 0.000\,18$

EXERCISE 6G.2

1 **a** Divide 84.6 by:

i 10 **ii** 100 **iii** 1000 **iv** 10^5

b Divide 0.7 by:

i 10 **ii** 1000 **iii** 10 000 **iv** 10^6

2 Write as a decimal number:

a $6 \div 10$ **b** $92 \div 10$ **c** $529 \div 10$ **d** $529 \div 100$

e $529 \div 1000$ **f** $529 \div 10\,000$ **g** $0.3 \div 10$ **h** $0.3 \div 1000$

i $0.97 \div 100$ **j** $0.06 \div 10$ **k** $0.06 \div 100$ **l** $0.022 \div 1000$

m $7.7 \div 10\,000$ **n** $0.2963 \div 100$ **o** $0.0035 \div 100$ **p** $51.6 \div 1000$

H MULTIPLYING DECIMAL NUMBERS

We can explain how decimal numbers are multiplied by first converting the decimals into fractions.

For example, consider the product 4×0.03

$= \frac{4}{1} \times \frac{3}{100}$ {converting to fractions}

$= \frac{12}{100}$ {multiplying fractions}

$= 0.12.$ {converting back to a decimal}

$= 0.12$

So, to find 4×0.03, we multiply the whole numbers 4 and 3, then divide by a power of 10, in this case 100.

Now consider the product 0.4×0.05

$= \frac{4}{10} \times \frac{5}{100}$ {converting to fractions}

$= \frac{20}{1000}$ {multiplying fractions}

$= 0.020.$ {converting back to a decimal}

$= 0.02$

So, to find 0.4×0.05, we multiply the whole numbers 4 and 5, then divide by a power of 10, in this case 1000.

With practise we do not need to convert the decimals to fractions first. We multiply the decimal numbers as though they were whole numbers, then divide by the appropriate power of 10.

Example 15 **Self Tutor**

Find: 0.3×0.07

0.3×0.07

$= (3 \times 7) \div 1000$ {shifting the decimal points a total of 3 places to the right}

$= 21 \div 1000$

$= 0.021.$ {shifting the decimal point 3 places to the left}

$= 0.021$

If the numbers have a whole number part, we can use one figure rounding to estimate the answer. This allows us to check whether the answer we obtain is reasonable.

Example 16 **Self Tutor**

Find 7.9×3.2, and check your answer using a one figure estimate.

$$\begin{aligned} & 7.9 \times 3.2 \\ =\ & (79 \times 32) \div 100 \\ =\ & 2528 \div 100 \\ =\ & 25.28 \\ =\ & 25.28 \end{aligned}$$

$$\begin{array}{r} 7\ 9 \\ \times\ 3\ 2 \\ {}^{2}\ {}^{1} \\ \hline 1\ 5\ 8 \\ 2\ 3\ 7\ 0 \\ {}^{1} \\ \hline 2\ 5\ 2\ 8 \end{array}$$

Check: $7.9 \times 3.2 \approx 8 \times 3 \approx 24$

So, the answer appears reasonable.

EXERCISE 6H

1 Find the value of:

a 7×0.4 **b** 0.8×9 **c** 6×0.5 **d** 0.2×0.4
e 0.6×0.07 **f** 0.03×0.5 **g** 0.03×0.004 **h** 0.009×50
i 80×0.005 **j** 0.0006×40 **k** 30×0.6 **l** 300×0.07
m 2000×0.9 **n** $0.05 \times 50\,000$ **o** $0.4 \times 0.5 \times 0.2$ **p** $0.3 \times 0.7 \times 0.5$

2 Given that $22 \times 471 = 10\,362$, evaluate:

a 2.2×471 **b** 2.2×4.71 **c** 2.2×47.1
d 22×0.471 **e** 0.22×0.471 **f** 2.2×0.471
g 0.22×4.71 **h** 220×0.471 **i** $2.2 \times 0.004\,71$

3 For the following products:

i use one figure rounding to estimate the answer
ii evaluate the product.

a 2×1.8 **b** 3.1×1.9 **c** 8.9×4.2
d 7.3×9.2 **e** 38.6×7.1 **f** 6.23×4.9

4 Evaluate, then check your answer using a calculator:

a 2.8×5.3 **b** 25×0.0004 **c** 0.018×0.23

5 Find the total cost of 12 pens at $1.95 each.

6 Find the total cost of 35 door hinges at €2.50 each.

7 Find the total cost of 2.9 m of chiffon fabric at £5.79 per metre.

8 I am about to bake biscuits for the school fundraiser. I buy 45 kg of flour at $0.69 per kg, and 25 kg of sugar at $1.24 per kg. How much money have I spent in total?

9 Min loads 15 bags of soil improver into her trailer. Each bag has mass 4.5 kg. Find the total mass of the bags.

10 Ron buys 2500 bricks, each weighing 4.3 kg, to build a wall around his courtyard.

  a Find the total mass of the bricks.

  b Ron's truck can only carry 2000 kg at a time. How many truck loads are necessary to transport the bricks?

I DIVIDING DECIMAL NUMBERS

INVESTIGATION DIVISION OF DECIMALS

What to do:

1 Copy and complete the following divisions. Look for a pattern which you can use to help perform the divisions involving decimals.

a $800 \div 200 = \square$, $80 \div 20 = \square$, $8 \div 2 = \square$, $0.8 \div 0.2 = \square$

b $800 \div 20 = \square$, $80 \div 2 = \square$, $8 \div 0.2 = \square$, $0.8 \div 0.02 = \square$

c $80 \div 200 = \square$, $8 \div 20 = \square$, $0.8 \div 2 = \square$, $0.08 \div 0.2 = \square$

2 In each set of divisions, what did you notice about the answers?

3 Did you find that in each set, the division by the smallest *whole* number was the easiest?

From the **Investigation** you should have observed that if both numbers in a division are multiplied or divided by the same power of 10, the result of the division does *not* change. The division is easiest when the divisor is a whole number.

These observations lead to the following rules for division with decimals:

> When **dividing** a **decimal number** by a **whole number**, carry out the division as normal, writing **decimal points under each other**.

Example 17 Self Tutor

Find:

a $32.5 \div 5$ **b** $0.417 \div 3$

a
$$\begin{array}{r} 6\,.\,5 \\ 5\,\overline{)\,3\,2\,.\,{}^{2}5} \end{array}$$
Answer: 6.5

b
$$\begin{array}{r} 0\,.\,1\,3\,9 \\ 3\,\overline{)\,0\,.\,4\,{}^{1}1\,{}^{2}7} \end{array}$$
Answer: 0.139

When **dividing** a **decimal number** by **another decimal number**, write the division as a fraction. Multiply the numerator and denominator by the same power of 10 so the denominator becomes a whole number. Then perform the division.

Example 18

Self Tutor

Find:

a $18 \div 0.06$

b $0.021 \div 1.4$

a

$$\begin{aligned} & 18 \div 0.06 \\ &= \frac{18 \times 100}{0.06 \times 100} \\ &= \frac{1800}{6} \\ &= 300 \end{aligned}$$

b

$$\begin{aligned} & 0.021 \div 1.4 \\ &= \frac{0.021 \times 10}{1.4 \times 10} \\ &= \frac{0.21}{14} \\ &= 0.015 \end{aligned}$$

$$\begin{array}{r} 0.015 \\ 14\,\overline{)\,0.210} \\ \underline{14} \\ 70 \\ \underline{70} \\ 0 \end{array}$$

EXERCISE 6I

1 Find:

a $7.2 \div 2$ **b** $32.7 \div 3$ **c** $80.4 \div 4$ **d** $0.45 \div 5$

e $0.225 \div 9$ **f** $1.82 \div 7$ **g** $46.2 \div 6$ **h** $3.32 \div 4$

i $0.649 \div 11$ **j** $0.0411 \div 3$ **k** $3.84 \div 8$ **l** $0.399 \div 7$

2 Find:

a $0.8 \div 0.4$ **b** $6.3 \div 0.9$ **c** $12 \div 0.4$ **d** $0.35 \div 0.7$

e $0.3 \div 0.15$ **f** $0.25 \div 0.05$ **g** $3.6 \div 0.012$ **h** $2.4 \div 0.003$

i $0.96 \div 0.12$ **j** $10.88 \div 0.17$ **k** $0.052 \div 0.13$ **l** $12.42 \div 0.0023$

3 Find:

a $7.5 \div 5$ **b** $12.1 \div 1.1$ **c** $0.06 \div 0.3$ **d** $0.06 \div 0.003$

e $2.2 \div 0.05$ **f** $0.232 \div 8$ **g** $0.055 \div 2.5$ **h** $0.049 \div 0.07$

4 Erica is painting a wall which has area 10.4 m^2. She can paint an area of 0.4 m^2 each minute. How long will it take her to complete the job?

5 Given that $182 \div 13 = 14$, evaluate:

a $182 \div 1.3$ **b** $18.2 \div 13$ **c** $1.82 \div 1.3$

d $18.2 \div 0.13$ **e** $1.82 \div 130$ **f** $0.182 \div 1.3$

6 Use your calculator to solve the following problems:

a How many 0.15 m lengths of tape can be cut from a roll 37.5 m long?

b If $4300.65 is shared equally among 19 people, how much does each person receive?

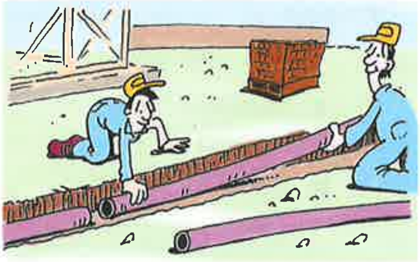
c 5 diamonds have weights 2.738 carats, 1.02 carats, 0.7117 carats, 1.013 carats, and 0.8374 carats. Find:

  i the total weight of the diamonds **ii** the average weight of the diamonds.

d How many £2.75 nut bars can be bought for £77?

e Determine the number of 2.4 m lengths of piping required to construct a 720 m drain.

f How many tins of preserved fruit each costing \$2.55, can be purchased with \$58.65?



Global context

click here

Leap Years

Statement of inquiry:	Decimal numbers are useful for describing natural occurrences.
Global context:	Orientation in space and time
Key concept:	Relationships
Related concepts:	Measurement, Quantity
Objectives:	Knowing and understanding, Applying mathematics in real-life contexts
Approaches to learning:	Thinking, Research

KEY WORDS USED IN THIS CHAPTER

- decimal number
- hundredths
- tenths
- decimal point
- place value
- thousandths
- expanded form
- round

REVIEW SET 6A

1 State the value of the digit:

a 5 in 0.5271 **b** 6 in 47.0461

2 Write:

a 2.1023 in expanded form **b** 0.004 as a fraction in simplest form.

3 Write as a decimal:

a $\frac{4}{5}$ **b** $\frac{9}{25}$ **c** $\frac{13}{200}$

4 Round:

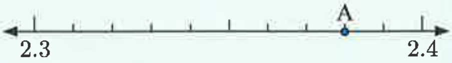
a 28.549 to 2 decimal places **b** 0.4824 to 1 decimal place

c 45.613 to the nearest whole number **d** 0.5385 to 3 decimal places.

5 Write down the value of the number at point A:

a

b



6 Evaluate:

a $0.71 + 0.296$ **b** $9.27 - 3.04$ **c** $8.7 \div 3$

d 0.6×0.8 **e** $14.2 + 8.93$ **f** $63 \div 0.7$

7 Insert $<$, $>$, or $=$ between these pairs of numbers to make a true statement:

a 3.03 and 3.303 **b** 0.514 and 0.541 **c** 2.404 and 2.044

8 **a** Multiply 8.59 by:

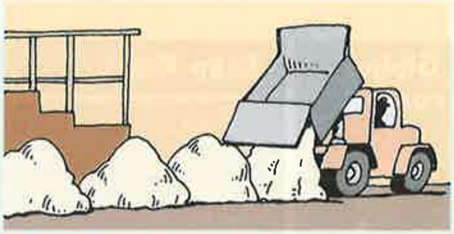
i 100 **ii** 1000 **iii** 10^5

b Divide 67.4 by:

i 10 **ii** 1000 **iii** 10 000

9 A race track is 2.55 km long. How many laps are needed to complete a 153 km race?

10 In one day a truck delivered 48 tonnes of sand to a building site. The first three loads measured 11.25 tonnes, 13.76 tonnes, and 12.82 tonnes. How much sand was delivered in the fourth and final load?



REVIEW SET 6B

1 Write in decimal form:

a $\frac{4}{10}+\frac{3}{100}$ **b** $\frac{7}{10}+\frac{1}{1000}$ **c** $\frac{2}{100}+\frac{8}{10\,000}$

2 Write as a fraction in simplest form:

a 0.86 **b** 2.24 **c** 0.045

3 Scott averages 3.28 steals per game for his hockey team. Round this to 1 decimal place.

4 Given that $28\times 17=476$, find the value of:

a 0.0028×1.7 **b** $0.000\,28\times 170$

5 Place the values 3.7, 3.86, 3.95, and 3.62 on a number line.

6 Evaluate:

a $42.8+19.7$ **b** $7.94-5.16$ **c** 200×0.016

d $11-4.13$ **e** $0.091\div 7$ **f** $0.24\div 0.015$

7 **a** Find 6.7×2.2.

b Check your answer using a one figure estimate.

8 A thermos contains 3.2 litres of tea. How many 0.4 litre cups of tea can be poured from the thermos?

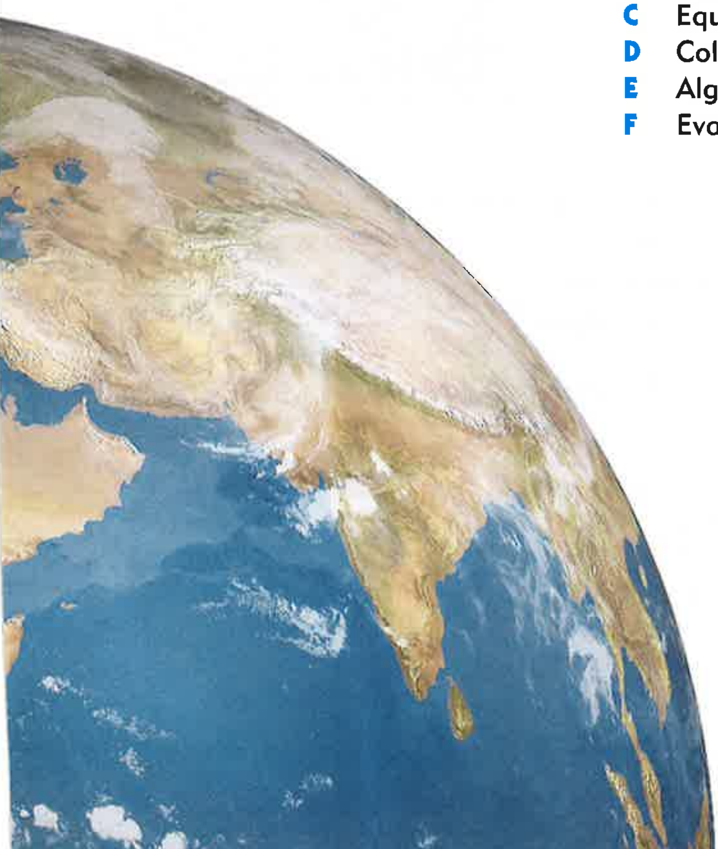
9 A man is 1.3 times as tall as his daughter, who is 136 cm tall. Determine the height of the man.

10 Arrange these numbers from smallest to largest: 3.204, 3.23, 3.023, 3.234

Chapter 7

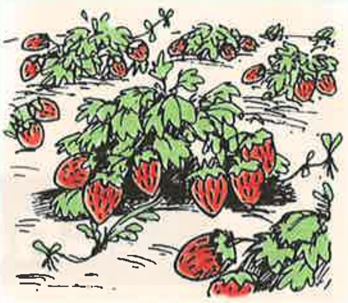
Algebraic expressions

Contents:



OPENING PROBLEM

When Cassi picks strawberries, she places them into n punnets, which are small open boxes. She places 8 strawberries in each punnet, and there are 4 left over.



Things to think about:

- **a** How can we use symbols to write the total number of strawberries Cassi picked?
- **b** Suppose that instead, Cassi places m strawberries in each punnet, and p strawberries are left over. Write an expression for the number of strawberries Cassi picked.

In this chapter we begin our study of **algebra**. In this area of mathematics, we use symbols to represent numbers. We can then write **algebraic expressions** involving the symbols, to represent different quantities.

A WRITING ALGEBRAIC EXPRESSIONS

The symbols used when writing algebraic expressions are called **variables** or **unknowns**.

For example, suppose there are d dogs in a kennel. The variable d represents the unknown number of dogs.

If two more dogs are added to the kennel, there are now $d + 2$ dogs. This is an **algebraic expression** involving d.

Example 1 **Self Tutor**

Brent, Scott, and Takisha are sharing a bag of peanuts. Suppose Brent eats b peanuts, Scott eats 1 less peanut than Brent, and Takisha eats 3 times as many peanuts as Brent.

Write an algebraic expression for the number of peanuts eaten by:

a Scott **b** Takisha.

a Scott eats $b - 1$ peanuts. **b** Takisha eats $3 \times b$ peanuts.

EXERCISE 7A.1

1 Suppose a bus has p passengers. 7 passengers get off at the next stop. How many passengers are now on the bus?

2 Kurt, Robbie, and Leo play in the same hockey team. Last season, Kurt scored g goals, Robbie scored 3 more goals than Kurt, and Leo scored twice as many goals as Kurt. Write an algebraic expression for the number of goals scored by:

a Robbie **b** Leo.

3 A man is n years old.

  a How old was he 12 years ago?

  b The man's wife is 6 years older than he is. How old is she?

  c The man's mother is twice his age. How old is she?

4 A hotel has x floors, with y apartments on each floor. How many apartments does the hotel have in total?

5 My neighbour has x cats. Write an expression for their total number of:

  a tails **b** eyes **c** legs.

Example 2

Self Tutor

In the fridge there are 2 punnets with strawberries in them, plus 4 strawberries left over.

Write an expression for the total number of strawberries if:

  a 6 strawberries have been put in each punnet

  b 9 strawberries have been put in each punnet

  c s strawberries have been put in each punnet.

  a If 6 strawberries have been put in each punnet, then there are $2 \times 6 + 4$ strawberries.

  b If 9 strawberries have been put in each punnet, then there are $2 \times 9 + 4$ strawberries.

  c If s strawberries have been put in each punnet, then there are $2 \times s + 4$ strawberries.

To write more complicated algebraic expressions, it helps to first look at specific number examples.

6 For dessert there are 3 punnets of blueberries, plus an extra 7 blueberries.
Write an expression for the total number of blueberries if there are:

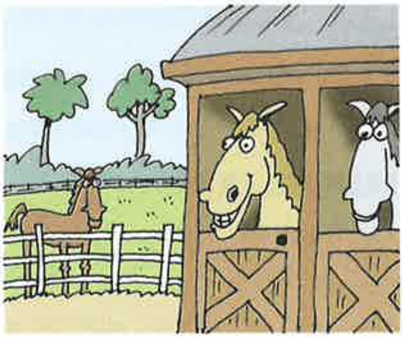
  a 8 blueberries in each punnet

  b 12 blueberries in each punnet

  c b blueberries in each punnet.

7 A farm has 6 paddocks of horses, plus 5 stables with one horse in each.
Write an expression for the number of horses on the farm if each paddock contains:

  a 2 horses **b** 4 horses **c** h horses.



8 Answer the **Opening Problem** on page **140**.

RULES FOR WRITING ALGEBRAIC EXPRESSIONS

Algebraic expressions usually contain a mixture of numbers and symbols. To make it easier to write these expressions, mathematicians have agreed on a set of rules:

PRODUCT NOTATION

- Except when confusion may arise, we leave out × signs between multiplied quantities.
- Numbers are written before variables in any product.

For example, instead of $2 \times a$ or $a \times 2$ we write $2a$.

In a product where there are two or more variables, we write them in **alphabetical order**.

For example, $2 \times b \times a$ is written as $2ab$, and $d \times 3 \times c$ is written as $3cd$.

Example 3 **Self Tutor**

Write in product notation:

a $a \times b$ **b** $b \times 3a$ **c** $2 \times a + b \times 3$ **d** $2 \times (a + b)$

a $a \times b$
$= ab$

b $b \times 3a$
$= 3ab$

c $2 \times a + b \times 3$
$= 2a + 3b$

d $2 \times (a + b)$
$= 2(a + b)$

INDEX NOTATION

Just as $2 \times 2 \times 2 \times 2 = 2^4$, we write $a \times a \times a \times a = a^4$.

In this case, a is the base and 4 is the index or power.

Example 4 **Self Tutor**

Simplify using index notation:

a $2 \times a \times a \times a \times b \times b$ **b** $m \times m - 5 \times n \times n$

a $2 \times a \times a \times a \times b \times b$
$= 2a^3b^2$

b $m \times m - 5 \times n \times n$
$= m^2 - 5n^2$

When a product is written out in full, we say it is in **expanded form**.

For example, a^2b is in index form, and $a \times a \times b$ is in expanded form.

EXERCISE 7A.2

1 Which one of these algebraic expressions is written in correct product notation?

A $7 \times k$ **B** $k7$ **C** $k \times 7$ **D** $7k$

2 Which one of these algebraic expressions is written in correct product notation?

A $x \times 5 \times y$ **B** $5yx$ **C** $5xy$ **D** $5 \times x \times y$ **E** $xy5$

3 Simplify using product notation:

a $3 \times a$
b $a \times 3$
c $5 \times x$
d $x \times 5$
e $6 \times n$
f $c \times d$
g $m \times k$
h $n \times b$
i $x \times y \times 9$
j $x \times y \times z$
k $k \times b \times h$
l $t \times 2 \times s$

4 Simplify using product notation:

a $x \times y + z$
b $3 \times p + 4 \times q$
c $p \times q - r$
d $p - q \times r$
e $u - w \times 7$
f $c \times 4 + d \times 9$
g $e \times f - g \times h$
h $9 - m \times n \times 2$
i $3 \times (d - 3)$
j $4 \times (g + 1)$
k $(x - 5) \times 7$
l $(x - y) \times 2$

5 Write in expanded form:

a x^2
b y^3
c $3x^2$
d $4m^3$
e $8x^3y$
f $5pq^2$
g $c^2 + 4d^3$
h $3v^2 - 5w^2$

6 Simplify using index notation:

a $x \times x$
b $p \times p \times p \times p$
c $4 \times a \times a$
d $b \times b \times b \times 5$
e $3 \times a \times b \times b$
f $f \times f \times g \times g \times g \times h$
g $f \times f + f$
h $w \times w \times w + 7$
i $e \times e \times e - 2 \times e \times e$
j $5 \times a \times a \times a + b \times b$
k $4 \times x \times x \times y \times y + z \times z$
l $5 \times a + a \times a$

B KEY WORDS IN ALGEBRA

Before we look further into algebra, we need to define some *key words*:

A **numeral** is a symbol used to represent a known number.

For example: 5, 0, -7, and $\frac{2}{3}$ are all numerals.

A **variable** is an unknown quantity which we represent by a letter or **pronumeral**.

For example, we could represent:

- the *number of carrots in my garden* by c
- the *speed of a cyclist* by s.

An **expression** is an algebraic form consisting of numerals, variables, and operation signs such as $+$, $-$, $\times$, $\div$, and $\sqrt{}$.

$2x + 5$ and $-3(2x - 1)$ are examples of expressions.

An **equation** is an algebraic statement containing an $=$ sign.

$3x - 7 = 8$ and $\frac{x}{3} = 10$ are examples of equations.

The **terms** of an expression or equation are the algebraic forms separated by $+$ and $-$ signs, the signs being included.

For example:

- the terms of $3x+2y+8$ are $3x$, $2y$, and 8
- the terms of $2x-3y-5$ are $2x$, $-3y$, and -5.

Like terms are terms with exactly the same variable form. They have the same variables to the same powers.

For example:

- $3x$ and $7x$ are like terms
- 8 and 7 are like terms
- $3xy^2$ and $-10xy^2$ are like terms
- $7x$ and 7 are unlike terms
- x and x^2 are unlike terms
- x and xy are unlike terms.

The **constant term** of an expression is the term which does not contain a variable.

For example:

- the constant term in $5x+6$ is 6
- the constant term in $-7+3x^2$ is -7
- there is no constant term in x^2+x.

The **coefficient** of any term is its numerical part, including its sign.

For example:

- the coefficient of p in $2p+4$ is 2
- the coefficient of r in $7-6r$ is -6
- the coefficient of x^2 in x^2+x is 1 since x^2 is $1\times x^2$.

ACTIVITY 1 KEY WORD JUMBLE

Click on the icon and print the activity sheet. It is divided into three sections: key words, definitions, and examples.

Print the sheet and cut out the boxes. Match each key word with its definition and example. Glue the results into your exercise book.

ACTIVITY

EXERCISE 7B

1 Are the following statements true or false? Correct the statements which are false.

a $\dfrac{x+y}{2}$ is an equation.

b $2+a+3b$ has 3 terms.

c $3p+5=1$ is an equation.

d The coefficient of x in $3x+2$ is 3.

e The constant term in $3x+4y-2$ is 2.

f $\dfrac{3}{q}=6$ is an expression.

2 Write down the coefficient of x in:

a $5x$

b $3+4x$

c $6y-5+8x$

d $4x-y+2$

e $5+2y+x$

f $-2x+3y-6$

g $x-7$

h $9-x$

i $10-7x$

3 State the number of terms in each expression of **2**.

4 Consider the expression $3x + 5y - 7 - 2y$.

- **a** How many terms are there in this expression?
- **b** What is the constant term?
- **c** What is the coefficient of the fourth term?
- **d** What are the like terms in this expression?

5 State the like terms in:

a $2x + 3 + 5x + 5$
b $x + y + 5x - y$
c $2x + y^2 + 3x + 2y + 4$
d $3 + q^2 + 7 + 4q^2$
e $2b + 2ab + a$
f $2 + ab + a + 3ab$

C EQUAL ALGEBRAIC EXPRESSIONS

To help us understand better what an algebraic expression means, it is useful to draw pictures.

For example, the algebraic expression $2p + 3$ can be represented by 2 punnets, each containing p strawberries, with 3 strawberries left over.

$2p + 3$

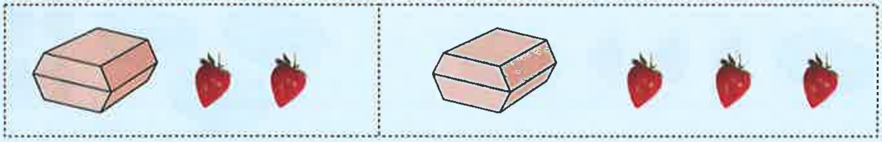
We can use the pictures to help us understand whether two algebraic expressions are **equal**.

Example 5 **Self Tutor**

Suppose we have 2 punnets each containing p strawberries, and 5 strawberries left over.

a Represent each of the following groupings using an algebraic expression:

i



ii

iii

b What equal expressions can be made from these groupings?

a We count the objects in each group and write an expression for each in brackets.

i $(p + 2) + (p + 3)$ **ii** $(2p + 2) + 3$ **iii** $(p + 4) + (p + 1)$

b The total number of strawberries is the same in each case, and is equal to $2p + 5$.

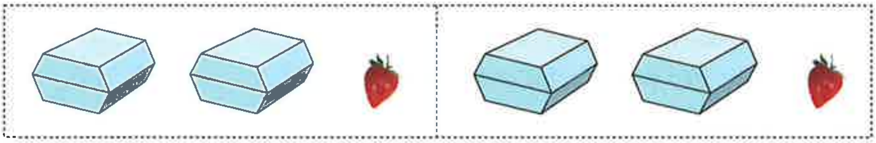
So, $(p + 2) + (p + 3) = (2p + 2) + 3 = (p + 4) + (p + 1) = 2p + 5$

EXERCISE 7C.1

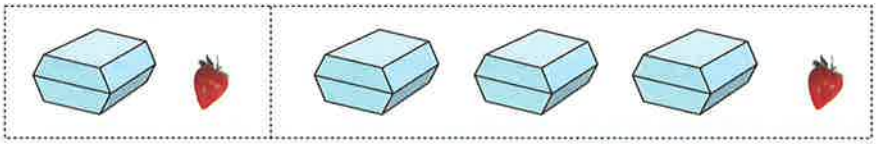
1 Suppose we have 4 punnets each containing p strawberries, and 2 strawberries left over.

a Represent each of the following groupings using an algebraic expression:

i



ii

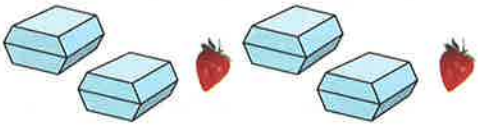


iii

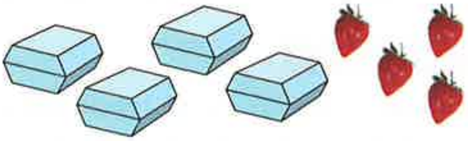
b What equal expressions can be made from these groupings?

2 For each of the following pictures, write *two* different expressions for the total number of strawberries present. Assume there are b strawberries in each punnet.

a

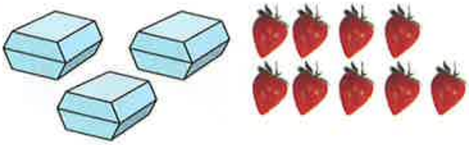


b



c

d



3 Determine which of these are true statements:

a $(b+2)+(b+1)=2b+3$

b $4(b+2)=4b+6$

c $(b+7)+(b+7)=2b+14$

d $(b+2)+(2b+2)=3b+2$

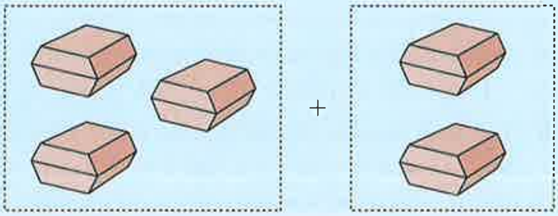
Example 6 **Self Tutor**

By drawing diagrams, write simpler algebraic expressions for:

a $3p+2p$ **b** $(3p+2)+(2p+4)$ **c** $2(2p+3)$

Let p represent the number of strawberries in a punnet.

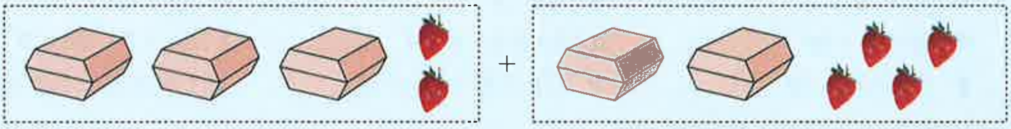
a $3p+2p$ can be represented by:



In total there are 5 punnets with p strawberries in each.

$\therefore\ 3p+2p=5p$

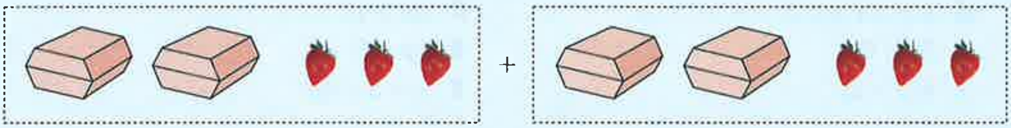
b $(3p+2)+(2p+4)$ can be represented by:



In total there are 5 punnets with p strawberries in each, plus another 6 strawberries.

$\therefore \quad (3p+2)+(2p+4)=5p+6$

c $2(2p+3)$ means 2 lots of $(2p+3)$, or $(2p+3)+(2p+3)$.



In total there are 4 punnets with p strawberries in each, plus another 6 strawberries.

$\therefore \quad 2(2p+3)=4p+6$

4 By drawing diagrams, write simpler algebraic expressions for:

a $3p+p$
b $4p+2p$
c $p+p+3$
d $3p+1+p+4$
e $4(p+3)$
f $p+2+p+5$
g $2(p+2)$
h $3(p+4)$
i $5(p+1)$

Example 7 **Self Tutor**

By drawing diagrams, write simpler algebraic expressions for:

a $5p-2p$
b $4p+3-2p-1$
c $(3p+4)-(p+1)$

Let p represent the number of strawberries in a punnet.

a $5p-2p$ is

which is $3p$.

$\therefore \quad 5p-2p=3p.$

b $4p+3-2p-1$ is

which is $2p+2$.

$\therefore \quad 4p+3-2p-1=2p+2.$

c $(3p+4)-(p+1)$ is

which is $2p+3$.

$\therefore \quad (3p+4)-(p+1)=2p+3.$

5 By drawing diagrams, write simpler algebraic expressions for:

a $5p - 3p$ **b** $4p - p$ **c** $3p - 3p$

d $4p + 3 - 2p$ **e** $3p + 5 - p - 1$ **f** $4p + 5 - 2p - 3$

g $(3p + 2) - (p + 2)$ **h** $(2p + 4) - (p + 3)$ **i** $(5p + 4) - (5p + 3)$

6 Match the equivalent expressions:

a $2(p + 1)$	**A** $5p - 2p$
b $3p$	**B** $2(p + 3)$
c $4(3 + 2p)$	**C** $8p + 16$
d $p + 3 + p + 3$	**D** $8p + 12$
e $5p + 5$	**E** $2p + 2$
f $8(p + 2)$	**F** $p + 5 + 4p$

EXPRESSIONS WITH TWO VARIABLES

We now consider expressions with two different variables. In order for expressions to be equal for *all* values of the variables, the numbers of each of the variables must be the same in all of the expressions.

In the following drawings:

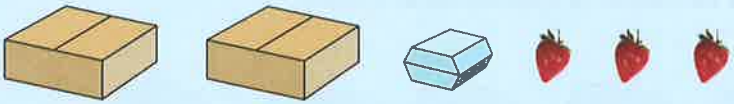
represents 1 strawberry

represents p strawberries in a punnet

represents c strawberries in a crate.

Example 8 **Self Tutor**

Write an algebraic expression for:



There are 2 crates, 1 punnet, and 3 more strawberries.

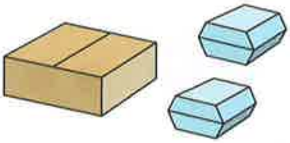
The number of strawberries is therefore $2c + p + 3$.

EXERCISE 7C.2

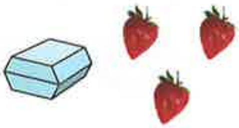
1 Write an algebraic expression for:

a

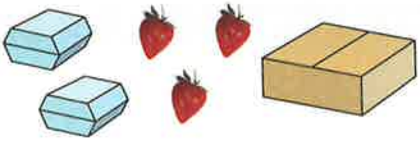
b



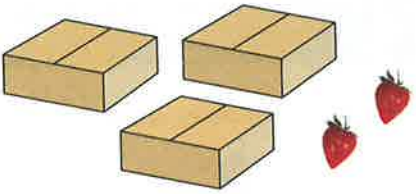
c



d

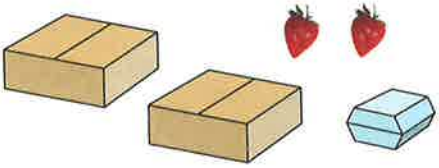


e



f

g



Example 9

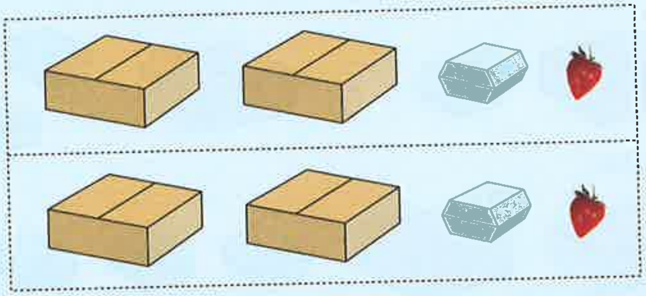
Self Tutor

Illustrate the following algebraic expressions:

$2(2c+p+1)$ and $4c+2p+2$

What can you conclude?

$2(2c+p+1)$ means two lots of $2c+p+1$.



$4c+2p+2$ means:

The numbers of strawberries and each type of container are the same in each case, so no matter what the values of c and p are, the total number of strawberries must be the same.

$\therefore\ 2(2c+p+1) = 4c+2p+2$

2 Illustrate the following pairs of algebraic expressions.
In each case, what can you conclude?

- **a** $2c+2p$ and $2(c+p)$
- **b** $3p+c+2$ and $2+3p+c$
- **c** $3(c+p+2)$ and $3c+3p+6$
- **d** $2(c+2)+2(p+3)$ and $2c+2p+10$

D COLLECTING LIKE TERMS

Consider again the pictures of containers and strawberries we used previously:

represents 1 strawberry

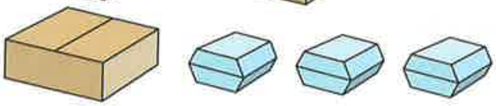
represents p strawberries in a punnet

represents c strawberries in a crate.

In this case,

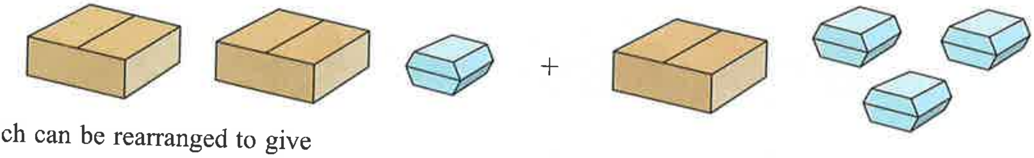
represents $2c + p$ strawberries

and



represents $c + 3p$ strawberries.

Adding these together gives



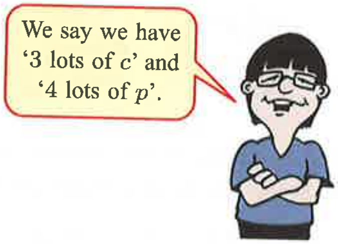
which can be rearranged to give

Using symbols, this means $2c + p + c + 3p = 3c + 4p$.

The algebraic expression on the left has been **simplified** by collecting together the symbols which were the same. This is called **simplifying by collecting like terms**.

$2c$ and c are like terms that add to $3c$,

and p and $3p$ are like terms that add to $4p$.



Example 10

Self Tutor

Simplify:

a $c + c + c + d + d$

b $(n + n) - (m + m + m)$

a $c + c + c + d + d$
$= 3c + 2d$

b $(n + n) - (m + m + m)$
$= 2n - 3m$

Example 11 **Self Tutor**

Simplify by collecting like terms:

a $4x + 3x$ **b** $3x - 2x$ **c** $8x - x$ **d** $3x + 2$

a $4x + 3x = 7x$ $\{4x$ and $3x$ are like terms$\}$

b $3x - 2x = 1x = x$ $\{3x$ and $-2x$ are like terms$\}$

c $8x - x = 7x$ $\{8x - x$ is really $8x - 1x\}$

d $3x + 2$ cannot be simplified as $3x$ and 2 are not like terms.

EXERCISE 7D

1 Simplify:

a $a + a$ **b** $b + b + b$ **c** $a + a + b + b + b$

d $3 + x + x$ **e** $f + f + 2 + f + 1$ **f** $3 - a + 2 + a$

g $(p + p + p) - (q + q)$ **h** $5 - (g + g + g)$ **i** $x + x + x + x + x - 2$

j $1 - (r + r)$ **k** $5 + (z + z + z)$ **l** $2 + m + m + n + 3 + n + n$

2 If possible, simplify by collecting like terms:

a $a + 3a$ **b** $2y + 2y$ **c** $x + y$

d $2x + x + y$ **e** $5b + 7b$ **f** $9r - 6r$

g $4x - 5$ **h** $8n - 2n$ **i** $7p - 5p$

j $3z - 3$ **k** $3c - 3c$ **l** $d + d + d + 3$

m $2 + y + y$ **n** $q + q - q + q$ **o** $4e + 4f$

p $10w - 5w$ **q** $12x - 6$ **r** $p + p + p + p$

s $h + h + h + h$ **t** $2x + 5x + 4x$ **u** $3x + 4y - 2x - y$

Example 12 **Self Tutor**

Simplify by collecting like terms:

a $2x - 4x$ **b** $-3x - x$ **c** $3x + x - 7x$

a $2x - 4x$
$= -2x$

b $-3x - x$
$= -4x$

c $3x + x - 7x$
$= 4x - 7x$
$= -3x$

3 If possible, simplify by collecting like terms:

a $3z - 5z$ **b** $2b - 6b$ **c** $3c - 3$

d $m - 3m$ **e** $x + x - 4x$ **f** $-f - f - f - f$

g $-3y - y - 2y$ **h** $4s + 2s - 6$ **i** $3a + 3ab$

j $4k - 8k$ **k** $4k - 8k + 4$ **l** $-15r - 5$

m $-15r - 5r$ **n** $-t - t - 2t$ **o** $2v - 3v$

p $-3w + 2w - 5$ **q** $x + y - 2x$ **r** $-2y + y - x + 1$

Example 13 **Self Tutor**

Simplify by collecting like terms:

a $3a+b+4a+2b+1$ **b** $3a+6b-a-2b$

a $3a+b+4a+2b+1$
$=3a+4a+b+2b+1$
$=7a+3b+1$

b $3a+6b-a-2b$
$=3a-a+6b-2b$
$=2a+4b$

4 Simplify by collecting like terms:

a $6x+2x+5y+y$ **b** $p+2p+5q+3q$ **c** $4a+3b+3a+3b$
d $c+d-c+d$ **e** $4v+3-5v-7$ **f** $5x+4z-x-2z$
g $g+2h-g+3h$ **h** $5r+5+t-r$ **i** $8x-12y+4x-y$
j $a^2b+ab^2-2a^2b-ab^2$ **k** $3xy-x^2y-xy+4x^2y$ **l** $2ab+a^2b-ab^2-3a^2b$

5 Pat says that $4-(p+p+p)$ is the same as $4-p+p+p$.

a By replacing p by 3 in both expressions, show that Pat is wrong.
b Simplify each expression algebraically.

E ALGEBRAIC PRODUCTS

Algebraic products are products which contain at least one variable.

$5\times 3k$ and $x^2\times 4y$ are examples of algebraic products.

To simplify algebraic products we:

- calculate the coefficient of the final product by multiplying all the numbers
- simplify the variables using index notation where appropriate.

Example 14 **Self Tutor**

Simplify:

a $6a\times 3b$ **b** $4x^2\times 5x$

a $6a\times 3b$
$=6\times a\times 3\times b$
$=18\times a\times b$
$=18ab$

b $4x^2\times 5x$
$=4\times x\times x\times 5\times x$
$=20\times x^3$ {3 factors of x}
$=20x^3$

EXERCISE 7E

1 Simplify:

a $4c\times 2$ **b** $5\times 6x$ **c** $3y\times 7$ **d** $6\times 4t$
e $9p\times 5q$ **f** $4m\times 7n$ **g** $8t\times 6c$ **h** $11z\times 9y$

2 Simplify:

- **a** $3x \times x$
- **b** $y \times 5y$
- **c** $2a \times 7a$
- **d** $8m \times 4m$
- **e** $6z \times 9z$
- **f** $x^2 \times 4x$
- **g** $(7x)^2$
- **h** $x \times 5x^2$
- **i** $3n \times 4n^2$
- **j** $7y^2 \times 6y$
- **k** $10k \times 8k^2$
- **l** $(2x)^2 \times x$
- **m** $5x \times xy$
- **n** $a \times 3ab$
- **o** $8x \times 5xy$
- **p** $9x^2y \times 7y$

ACTIVITY 2 "THINK OF A NUMBER" GAMES

What to do:

- Play in pairs, calling the players A and B.
- Player A chooses a number while player B calls out the steps in each game.
- Choose a few different values in each game and take it in turns to be player A or B.
- Discuss the results. Letting the number be x, write algebraic expressions to describe the steps in each game.

Game 1

Step 1: Think of a number.
Step 2: Double it.
Step 3: Add 7.
Step 4: Take away 1.
Step 5: Divide by 2.
Step 6: Subtract the original number.
Step 7: State the output.

Game 2

Step 1: Think of a number.
Step 2: Add nine.
Step 3: Multiply by two.
Step 4: Subtract eighteen.
Step 5: Divide by two.
Step 6: Subtract the original number.
Step 7: State the output.

F EVALUATING ALGEBRAIC EXPRESSIONS

If we know the values of all of the variables in an algebraic expression, we can find the value of the algebraic expression.

To **evaluate** an algebraic expression, we **substitute** the given numbers for the variables, and then calculate the result.

Example 15 **Self Tutor**

Evaluate $3c + 7$ when:

a $c = 4$ **b** $c = 10$

a If $c = 4$ then

$$\begin{aligned} &3c + 7 \\ &= 3 \times 4 + 7 \\ &= 12 + 7 \\ &= 19 \end{aligned}$$

b If $c = 10$ then

$$\begin{aligned} &3c + 7 \\ &= 3 \times 10 + 7 \\ &= 30 + 7 \\ &= 37 \end{aligned}$$

EXERCISE 7F.1

1 Evaluate:

a $2z+7$ when $z=1$

b $3y-1$ when $y=4$

c $2x+1$ when $x=4$

d $2+9d$ when $d=2$

e $19-3q$ when $q=3$

f $12-4a$ when $a=2$

g $3(k+1)$ when $k=6$

h $5(j-2)$ when $j=7$

2 Suppose we have 3 bags each containing p potatoes, and 10 potatoes left over.

a Write an expression for the total number of potatoes.

b Find the total number of potatoes present if:

i $p=5$ **ii** $p=12$ **iii** $p=25$

Example 16 **Self Tutor**

If $x=3$ and $y=2$, evaluate:

a $2x-y$ **b** $3(3y+5x)$

a

$$\begin{aligned} &2x-y \\ &=2\times 3-2 \\ &=6-2 \\ &=4 \end{aligned}$$

b

$$\begin{aligned} &3(3y+5x) \\ &=3(3\times 2+5\times 3) \\ &=3(6+15) \\ &=3\times 21 \\ &=63 \end{aligned}$$

3 If $x=3$ and $y=4$, find the values of:

a $2x$ **b** $x+3y$ **c** $3x+2y$ **d** $4(x+y)$

e $6x-4y$ **f** $5y-3x$ **g** $3(x-y)$ **h** $2(x+2y)$

i $3(4x-3y)$ **j** $6(y-x)-5$ **k** $3(2x-5y)$ **l** $7+2y-6x$

4 A group of x adults and y children attend a concert. Each adult ticket costs €40, and each child ticket costs €15.

a Write an expression to represent the total cost for the group.

b Find the total cost if:

i $x=2$ and $y=2$ **ii** $x=3$ and $y=4$

iii $x=4$ and $y=7$

5 If $p=4$, $q=2$, and $r=5$, find the value of:

a $q+r$ **b** pqr **c** p^2 **d** pq

e $r-q$ **f** $3q-p$ **g** $2r-3p$ **h** $3q^2$

i $3(p+3q)$ **j** $9(2q-p)$ **k** $2(pr-11)$ **l** $pq+qr$

6 If $a = 2$, $b = 0$, and $c = 3$, evaluate:

a $a + 4c$
b $4a + 5c$
c $4(a - c)$
d $4a - 4c$
e c^2
f $4c^2$
g $7b - 6$
h $2(a + 7c)$
i $ab + bc$
j $8(2b - 3c)$
k abc
l $6(c - 5a)$

NEGATIVE SUBSTITUTION

Variables do not always take positive values. They can also be assigned negative values.

To avoid confusion with signs, we usually write negative substitutions in brackets.

Example 17

Self Tutor

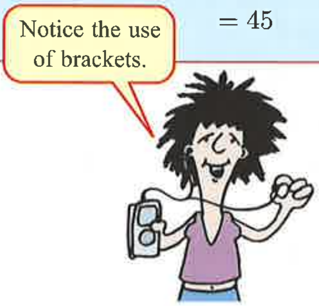
If $x = 5$ and $y = -4$, find the value of:

a $2x + 3y$
b $2(x - y)$
c $x^2 - xy$

a
$$\begin{aligned} &2x + 3y \\ &= 2 \times 5 + 3 \times (-4) \\ &= 10 + -12 \\ &= 10 - 12 \\ &= -2 \end{aligned}$$

b
$$\begin{aligned} &2(x - y) \\ &= 2(5 - (-4)) \\ &= 2(5 + 4) \\ &= 2 \times 9 \\ &= 18 \end{aligned}$$

c
$$\begin{aligned} &x^2 - xy \\ &= 5^2 - 5 \times (-4) \\ &= 25 - (-20) \\ &= 25 + 20 \\ &= 45 \end{aligned}$$



EXERCISE 7F.2

1 If $w = -1$, $x = 2$, and $y = -4$, evaluate:

a $x - w$
b $4xy$
c $y - w$
d $3(y + w)$
e wxy
f $w + y - x$
g $y - x^2$
h $wx + xy$
i $3(x - 2w)$
j $x + wy$
k $2(x + y)$
l $2w^2 - 3y$

2 If $f = 2$, $g = 8$, and $h = -5$, find the value of:

a $f - 2g$
b $\dfrac{gh}{f}$
c $f^2 - h$
d $3h + 5f$
e $4(g - h)$
f $f(g + h)$
g $3(2g - 3h)$
h $g(h - 3f)$

KEY WORDS USED IN THIS CHAPTER

- algebra
- algebraic product
- coefficient
- constant term
- equation
- evaluate
- expanded form
- expression
- like terms
- numeral
- pronumeral
- simplify
- substitution
- unknown
- variable

REVIEW SET 7A

1 Suppose a peach tree has p peaches, and 8 peaches are picked from the tree. Write an expression for the number of peaches that are still on the tree.

2 Which one of these algebraic expressions is written in correct product notation?

A $4ba$ **B** $a \times b \times 4$ **C** $ab4$ **D** $4 \times a \times b$ **E** $4ab$

3 **a** How many terms are in the expression $4x^3 + 6x^2 + 4x + 1$?

b State the coefficient of y in $2x^2 + 3xy - 7y$.

4 Suppose we have 2 punnets with strawberries in them, plus 5 strawberries left over.

Write an expression for the total number of strawberries if Mario has put in each punnet:

a 4 strawberries **b** 7 strawberries **c** s strawberries.

5 Find the like terms in the following:

a $2x^2 - 4x + 3 + 3x^2 - 6x$ **b** $5a - 3b + a - 2b + 1$

c $3c - c^2 + 3c + 2$ **d** $2e + ef - 4e + 2ef + 2f$

6 Simplify, if possible, by collecting like terms:

a $7x + 5x + 5$ **b** $2p + 1 + 3p - 3$

c $3x - 5y + 5x + 5y$ **d** $-7c + 5d^2 + cd + 4c$

7 Write in index form:

a $k \times k \times k$ **b** $5 \times m \times m \times n$ **c** $7 \times a \times a + a \times b \times b$

8 Evaluate:

a $4x + 3$ when $x = 6$ **b** $7(m - 5)$ when $m = 9$

9 Simplify:

a $5p \times 7$ **b** $2f \times 4f$ **c** $9s \times 4s^2$

10 If $a = -2$, $b = 5$, and $c = -3$, evaluate:

a abc **b** $5a - 7b$ **c** $3(2c - 3b)$

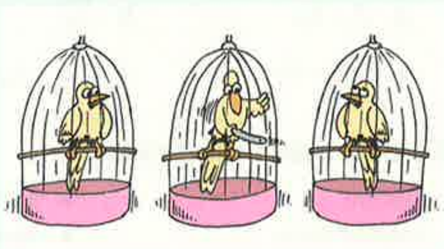
REVIEW SET 7B

1 There are 3 cages which contain some birds. How many birds are present in total if each cage contains:

a 3 birds, and there are 2 flying nearby

b b birds, and there are 2 flying nearby

c b birds, and there are f flying nearby?



2 Write in expanded form:

a m^3 **b** $7t^2$ **c** $6xy^3$ **d** $4p^2 - q^2$

3 For the expression $6x^2 - 2xy + 2y - 3y^2$, state the coefficient of y^2.

4 For the expression $6x - 6y + 2x + y + 1$, state:

- **a** the constant term
- **b** the like terms.

5 Simplify using product notation:

- **a** $x \times 7$
- **b** $(c - d) \times 2$
- **c** $q \times 8 \times p$

6 Simplify:

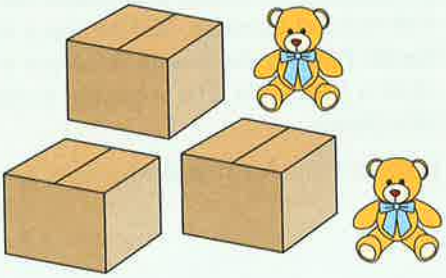
- **a** $8c - 2c$
- **b** $7a + 6 - 2a - 1$
- **c** $12q - 8 - 8q$
- **d** $12x - 5 - 7x + 3$

7 Simplify:

- **a** $8a \times 3b$
- **b** $y^2 \times 9y$
- **c** $4mn \times 11n^2$

8 3 boxes of teddy bears are delivered to a toy store, plus an additional 2 bears.

- **a** How many teddy bears are there in total if each box contains t bears?
- **b** Find the total number of teddy bears if:
  - **i** $t = 3$
  - **ii** $t = 5$
  - **iii** $t = 12$



9 Determine which of these are true statements:

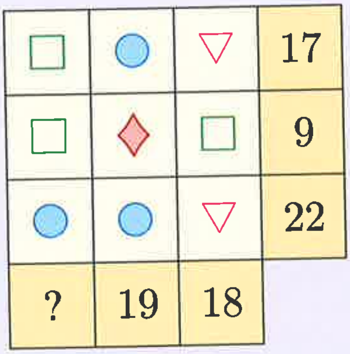
- **a** $(b + 4) + (b + 5) = b + 9$
- **b** $(b + 1) + (2b + 6) = 3b + 7$

10 If $a = -2$, $b = -3$, and $c = 5$, find the value of:

- **a** c^2
- **b** $2b - 5a$
- **c** $3a^2 + 4b - c$

PUZZLE

Each of the 4 different symbols represents a number. The sum for each row and column is given, except for one value.



□	●	▽	17
□	◆	□	9
●	●	▽	22
?	19	18	

1 **a** Decide which of the options below shows the correct value for each symbol.

A □ = 4, ◆ = 8, ● = 6, ▽ = 7
B □ = 4, ◆ = 5, ● = 7, ▽ = 6
C □ = 2, ◆ = 5, ● = 7, ▽ = 8
D □ = 6, ◆ = 3, ● = 8, ▽ = 2

b Hence, find the unknown value in the puzzle.

2 Is there a way to find the unknown value, without finding the value of each symbol?

HISTORICAL NOTE

SOFYA KOVALEVSKAYA (1850 - 1891)

Sofya was born in Moscow in 1850. As a girl she studied three languages, music, art, and mathematics. She was a very fast learner, but unfortunately many universities refused to allow women to study mathematics and science. She therefore moved to Heidelberg University in Germany, but even there she had to dress in an unfeminine manner to be accepted by the male-dominated mathematics department.

In 1874 she obtained a degree at the University of Gottingen, but because she was a woman she was unable to obtain work as a teacher or researcher at any university.

In 1888 she went to Paris to continue her study. Following the death of her husband and a battle with illness, she wrote hundreds of important mathematical formulae. She became a lecturer at the University of Stockholm, and was awarded the Bordin Prize by the French Academy of Sciences for her research on *The rotation of a solid body about a fixed point*. Her work was so impressive the prizemoney was increased from 3000 francs to 5000 francs.

Unfortunately, Sofya died two years later at the age of 41, after being struck down by influenza.

Chapter 8

Percentage

Contents:

OPENING PROBLEM

Mason is in Year 7, running for school captain. His father Peter is running for mayor of the local council. Mason received 63 votes, while Peter received 2576 votes.

Things to think about:

- **a** "Since Peter received more votes, Peter performed better than Mason." Is this a fair conclusion?
- **b** What is a fairer way to compare their performances?
- **c** Peter received 32% of the total vote for council mayor. What does this mean?
- **d** 105 students voted in the election for school captain. What percentage of the total vote did Mason receive?
- **e** Who do you think performed better?

A UNDERSTANDING PERCENTAGES

We use **percentages** to compare a portion with a whole amount. The whole amount is represented by 100%, which has the value 1.

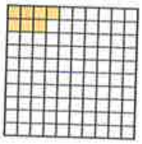
Percentages are commonly used to describe interest rates, sale prices, test results, inflation, changes in profit levels, employment levels, and much more.

% reads **per cent**, which means '**in every hundred**'.

If an object is divided into one hundred equal parts, then each part is called 1 per cent or 1%.

Thus, $\frac{1}{100} = 1\%$

$\frac{7}{100} = 7\%$



$\frac{100}{100} = 100\%$

So, a percentage is like a fraction which has denominator 100.

$$x\% = \frac{x}{100}$$

Example 1

Self Tutor

a Write 56% as a fraction with denominator 100.

b Write $\frac{78}{100}$ as a percentage.

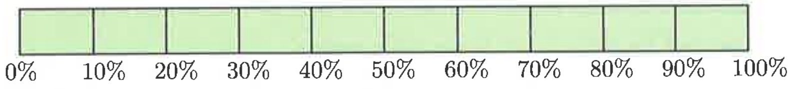
a $56\% = \frac{56}{100}$

b $\frac{78}{100} = 78\%$

It is very common to estimate percentages. It is important you are able to imagine the portion that a particular percentage describes.

We often hear statements like: "contains 25% real juice"
"80% of the students passed their examination"
"the unemployment rate is 5%"
"40% off sale!"

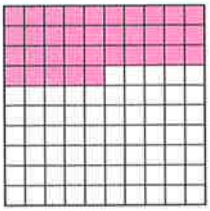
We need to remember that 100% describes a whole, and the whole is made up of 10 lots of 10%.



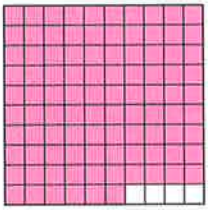
EXERCISE 8A

1 What percentage is represented by the following shaded diagrams?

a


b


c


d


2 Write as a fraction with denominator 100:

a 13% **b** 37% **c** 6% **d** 92% **e** 79%

3 Write as a percentage:

a $\frac{17}{100}$ **b** $\frac{38}{100}$ **c** $\frac{90}{100}$ **d** $\frac{125}{100}$ **e** $\frac{1}{100}$

4 Computers use a shaded bar to show the progress when downloading a file.
Which progress bar shows the download to be:

a 50% complete
b 7% complete
c 40% complete
d 95% complete?

A

Downloading...

B

Downloading...

C

Downloading...

D

Downloading...

5 The members of the Brown family each have their own shampoo bottle. Whose bottle is:

a 75% full **b** 20% full
c 5% full **d** 90% full?

ACTIVITY 1 EVERYDAY USE OF PERCENTAGE

What to do:

1 Read the following examples of everyday percentage use:

- 25% of the homes in my street have roses growing in the front garden.
- 65% of students at a school voted for a greater variety of lollies in the school canteen.
- 27% of primary school age children do not eat fresh fruit and vegetables.
- The netball goal shooter had a 68% accuracy rate for the whole season.
- Sarah improved by 10% in her times table tests.
- Last year, over 52% of 5 - 14 year old children played sport outside school hours.
- The number of children attending the cinema during the school holidays has dropped 12% since last year.
- The humidity at 9 am was 46%, and at 3 pm it was 88%.

2 For each of the above examples, suggest:

  a how the percentage may have been calculated

  b why this information might be useful.

3 Discuss your answers with your class.

DISCUSSION

Is it possible to have a percentage greater than 100%?

What would a percentage greater than 100% mean?

In what situations might it be reasonable to have a percentage more than 100%?

B INTERCHANGING NUMBER FORMS

Percentages, fractions, and decimals can all be used to describe a part of a whole. It is important that you can convert between these number forms.

CONVERTING FRACTIONS INTO PERCENTAGES

Many fractions can be converted to percentage form by first writing the fraction with denominator 100.

Example 2

Self Tutor

Write as a percentage:

a $\frac{1}{4}$ **b** $\frac{3}{5}$ **c** $\frac{71}{200}$

a $\frac{1}{4}$

$= \frac{1 \times 25}{4 \times 25}$

$= \frac{25}{100}$

$= 25\%$

b $\frac{3}{5}$

$= \frac{3 \times 20}{5 \times 20}$

$= \frac{60}{100}$

$= 60\%$

c $\frac{71}{200}$

$= \frac{71 \div 2}{200 \div 2}$

$= \frac{35.5}{100}$

$= 35.5\%$

Not all fractions can be easily written with denominator 100, so they cannot be easily expressed as a percentage using this method. Since we need to find out how many parts out of 100 the fraction represents, an alternative method is to multiply the fraction by 100%. Since $100\% = \frac{100}{100} = 1$, multiplying by 100% is the same as multiplying by 1. We are therefore not changing the value of the number.

Example 3

Self Tutor

Write as a percentage:

a $\frac{3}{40}$ **b** $\frac{7}{8}$ **c** $\frac{2}{3}$

a $\frac{3}{40}$

$= \frac{3}{40} \times 100\%$

$= \frac{30\cancel{0}}{4\cancel{0}}\%$

$= 7.5\%$

b $\frac{7}{8}$

$= \frac{7}{8} \times 100\%$

$= \frac{700}{8}\%$

$= 87.5\%$

c $\frac{2}{3}$

$= \frac{2}{3} \times 100\%$

$= \frac{200}{3}\%$

$= 66\frac{2}{3}\%$

EXERCISE 8B.1

1 Write as a percentage:

a $\frac{7}{10}$ **b** $\frac{9}{25}$ **c** $\frac{11}{20}$ **d** $\frac{1}{2}$ **e** $\frac{2}{5}$

f $\frac{3}{4}$ **g** $\frac{33}{50}$ **h** $\frac{41}{200}$ **i** $\frac{341}{1000}$ **j** $\frac{709}{1000}$

2 Write as a percentage by multiplying by 100%:

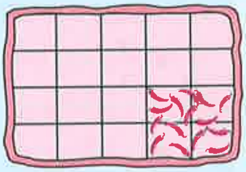
a $\frac{9}{40}$ **b** $\frac{3}{8}$ **c** $\frac{7}{80}$ **d** $\frac{21}{250}$ **e** $\frac{1}{3}$

3 Use your calculator to write the following as a percentage, rounding your answer to 1 decimal place:

a $\frac{1}{6}$ **b** $\frac{5}{7}$ **c** $\frac{4}{9}$ **d** $\frac{10}{13}$ **e** $\frac{18}{37}$

Example 4 **Self Tutor**

A rectangular pizza is cut into 20 equal pieces as shown. Chilli has been put on 4 pieces.



a What fraction of the pizza *does not* have chilli?

b What percentage of the pizza *does not* have chilli?

c What percentage of the pizza has chilli?

a There are 20 pieces and 16 do not have chilli.

$\therefore \quad \frac{16}{20} = \frac{4}{5}$ does not have chilli.

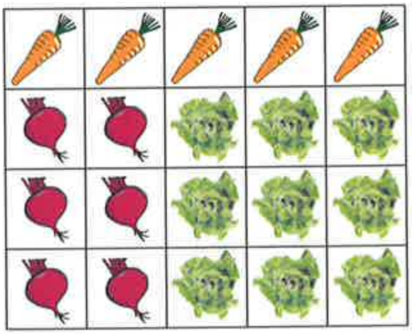
b $\frac{4}{5} = \frac{4 \times 20}{5 \times 20} = \frac{80}{100} = 80\%$

$\therefore \quad 80\%$ does not have chilli.

c If 80% does not have chilli, then 20% does have chilli. $\{80\% + 20\% = 100\%\}$

The percentage with chilli
+ the percentage without chilli
= 100%.

4 A garden plot is planted with beetroot, carrots, and lettuces as shown.



a What fraction of the plot has lettuces?

b What percentage of the plot has carrots?

c What percentage of the plot *does not* have beetroot?

5

A restaurant has 80 seats. 50 are occupied by diners.

a What fraction of the seats are occupied?

b What percentage of the seats are occupied?

c What percentage of the seats are unoccupied?

CONVERTING PERCENTAGES INTO FRACTIONS

To convert a percentage into a fraction, we first write it as a fraction with denominator 100. We then write the fraction in simplest form.

Example 5 **Self Tutor**

Express as a fraction in lowest terms:

a 30% **b** 76% **c** 120%

a 30%
$= \frac{30}{100}$
$= \frac{30 \div 10}{100 \div 10}$ {HCF is 10}
$= \frac{3}{10}$

b 76%
$= \frac{76}{100}$
$= \frac{76 \div 4}{100 \div 4}$ {HCF is 4}
$= \frac{19}{25}$

c 120%
$= \frac{120}{100}$
$= \frac{120 \div 20}{100 \div 20}$ {HCF is 20}
$= \frac{6}{5}$

EXERCISE 8B.2

1 Express as a fraction in lowest terms:

a 75% **b** 6% **c** 15% **d** 20%
e 100% **f** 55% **g** 150% **h** 8%
i 88% **j** 700% **k** 62% **l** 245%

2 84% of the students at a school are right-handed. What fraction of the students are:

a right-handed **b** left-handed?

CONVERTING DECIMALS INTO PERCENTAGES

To convert a decimal into a percentage, we **multiply by 100%.**

Example 6 **Self Tutor**

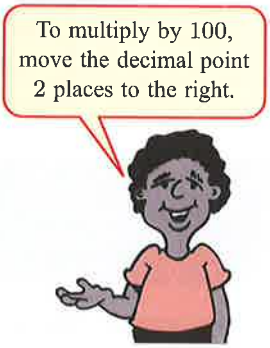
Write as a percentage:

a 0.28 **b** 0.064 **c** 2.7

a 0.28
= 0.28 × 100%
= 28%

b 0.064
= 0.064 × 100%
= 6.4%

c 2.7
= 2.70 × 100%
= 270%



EXERCISE 8B.3

1 Write as a percentage:

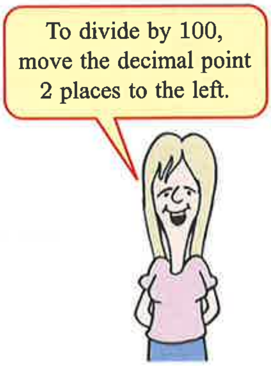
a 0.38 **b** 0.93 **c** 0.15 **d** 0.317
e 0.546 **f** 0.802 **g** 0.07 **h** 1.58

2 Write as a percentage:

a 0.9 **b** 0.004 **c** 0.059 **d** 0.4073
e 1.6 **f** 4.2 **g** 3 **h** 0.0026

CONVERTING PERCENTAGES INTO DECIMALS

To convert a percentage into a decimal, we **divide by 100%**.



Example 7 **Self Tutor**

Express as a decimal:

a 31% **b** 150%

a 31%
$= 0\overset{\curvearrowleft\curvearrowleft}{31}.\ \%$
$= 0.31$

b 150%
$= 1\overset{\curvearrowleft\curvearrowleft}{50}.\ \%$
$= 1.5$

EXERCISE 8B.4

1 Write as a decimal:

a 89%	**b** 60%	**c** 18%	**d** 8%
e 49.5%	**f** 125%	**g** 200%	**h** 38.01%
i 37.5%	**j** 0.002%	**k** 129.8%	**l** 77.7%

2 Copy and complete this table of common conversions:

Percentage	*Fraction*	*Decimal*
100%		
75%		
50%		
25%		
20%		

Percentage	*Fraction*	*Decimal*
10%		
5%		
1%		
$33\frac{1}{3}\%$		
$66\frac{2}{3}\%$		

C ONE QUANTITY AS A PERCENTAGE OF ANOTHER

Percentages are often used to **compare quantities**. It is therefore useful to be able to express one quantity as a percentage of another.

We may only compare quantities which have something in common. For example, expressing "5 bicycles as a percentage of 45 cars" is *not possible*. However, expressing "5 bicycles as a percentage of 45 *vehicles*" does make sense, because bicycles are a type of vehicle. We say we can only compare "**like with like**".

Also, when we compare quantities we must make sure they have the **same units**.

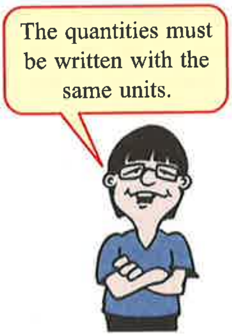
For example, if we are asked to express "35 cm as a percentage of 7 m" we would normally convert the larger unit to the smaller one. So, we would find "35 cm as a percentage of 700 cm".

To express one quantity as a percentage of another, we first write them as a fraction and then convert the fraction to a percentage.

Example 8 **Self Tutor**

Express 45 minutes as a percentage of 3 hours.

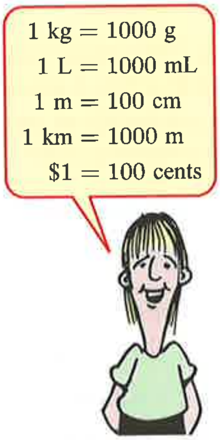
$$\begin{aligned}\frac{45 \text{ minutes}}{3 \text{ hours}} &= \frac{45 \text{ minutes}}{3 \times 60 \text{ minutes}} && \{\text{write as a fraction with the same units}\}\\ &= \frac{45}{180} \times 100\% && \{100\% = 1\}\\ &= 25\%\end{aligned}$$



EXERCISE 8C

1 Express the first quantity as a percentage of the second:

- **a** 2 cm, 5 cm
- **b** 13 kg, 20 kg
- **c** 1 L, 10 L
- **d** \$3, \$6
- **e** 4 km, 10 km
- **f** 6 m, 8 m
- **g** 36 minutes, 40 minutes
- **h** 200 kg, 250 kg
- **i** 75 mL, 375 mL
- **j** 24 minutes, 1 hour
- **k** 450 g, 1 kg
- **l** 800 m, 2 km
- **m** 3 months, 1 year
- **n** 25 cm, 0.5 m
- **o** 800 mL, 4 L
- **p** 300 m, 1.5 km
- **q** 70 cents, \$4.20
- **r** 14 days, 3 weeks



Example 9 **Self Tutor**

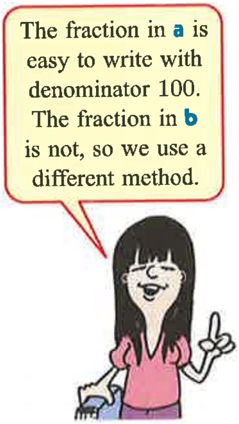
Express as a percentage:

a A test mark of 17 out of a possible 25.

b Out of 1600 vehicles sold in a month, 250 of them were vans.

a
$$\begin{aligned}&\frac{17 \text{ marks}}{25 \text{ marks}}\\ &= \frac{17 \times 4}{25 \times 4}\\ &= \frac{68}{100}\\ &= 68\%\end{aligned}$$

b
$$\begin{aligned}&\frac{250 \text{ vans}}{1600 \text{ vehicles}} && \{\text{vans are vehicles, so units the same}\}\\ &= \frac{250}{1600} \times 100\% && \{100\% = 1\}\\ &= \frac{25\,000}{1600}\%\\ &= 15.625\%\end{aligned}$$



2 Express as a percentage:

- **a** 43 marks out of a possible 50 marks
- **b** 470 chocolate bars sold out of 1000 made
- **c** 160 hybrid vehicles out of 400 cars sold
- **d** 75 marks out of a possible 120 marks
- **e** a darts player makes 135 out of a possible 180 points.

3 Express as a percentage:

- **a** 49 out of 70
- **b** 440 mL out of 2 L
- **c** \$18 out of \$300
- **d** 84 cm out of 2 m
- **e** 550 g out of 1 kg
- **f** 18 hours out of 1 day.

4 The 25 students of a class were each given a calculator at the start of the year. By the end of the year, six of the students had lost their calculators, and five of the students had broken theirs. What percentage of the class had:

a lost their calculators
b broken their calculators
c lost *or* broken their calculators?

5 A television station is required to show at least 55% local programming between 6 am and midnight each day.
The station averages 10 hours of local programming during these times. Has the station met the requirement?

6 Of the 1500 trains leaving a train station one day, 1290 left within 5 minutes of the scheduled time.
What percentage of the trains left within 5 minutes of the scheduled time?

7 The table below shows the forest area and total land area of various countries.

	Forest area (km^2)	*Land area* (km^2)	*Forest as % of land area*
Bangladesh	14 394	130 170	11.1%
Colombia	603 980	1 109 500	54.4%
Finland	221 570	303 890	
Indonesia	937 470	1 811 570	
Madagascar	124 960	581 540	
Niger	11 916	1 266 700	
Philippines	77 198	298 170	
Spain	183 493	498 800	

a Use a calculator to complete the table. Round the percentages to 1 decimal place.
b Which country has the highest percentage of its land covered by forest?

INVESTIGATION SCORING SPREADSHEET

The spreadsheet in this Investigation is used to record basketball scores. It will calculate each player's score as a percentage of the team's total score.

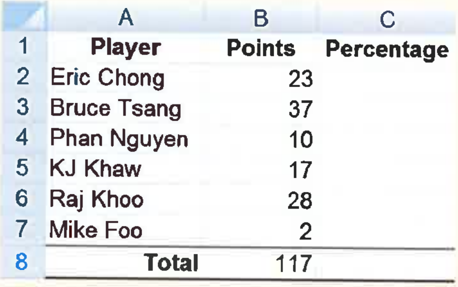
In a basketball game, players had the following scores:

Eric Chong	23 points
Bruce Tsang	37 points
Phan Nguyen	10 points
KJ Khaw	17 points
Raj Khoo	28 points
Mike Foo	2 points

SPREADSHEET

What to do:

1. Open the spreadsheet and enter the data as shown.
2. Find the total score by entering the formula =sum(B2:B7) in cell B8.
3. In cell C2, enter the formula =B2/B8 * 100. Fill this formula down to C7. Use the 'Decrease Decimal' icon .00 →.0 on the toolbar to set the number of decimal places to 2.
4. Discuss with your class why we used the $ signs in the formula in **3**.



	A	B	C
1	**Player**	**Points**	**Percentage**
2	Eric Chong	23	
3	Bruce Tsang	37	
4	Phan Nguyen	10	
5	KJ Khaw	17	
6	Raj Khoo	28	
7	Mike Foo	2	
8	**Total**	117	

5. Activate cell B8 and fill the formula across to C8. What do you notice?
6. In over-time, Eric scored an extra 7 points and Mike scored 4 more points. Change the points on your spreadsheet. How has each player's percentage changed?

ACTIVITY 2 PERCENTAGES AROUND US

What to do:

In this Activity you will be asked to calculate percentages of various things at home and at school.

In each case:
- think about the method you will use to find the percentage
- consider whether to count every item or use a sample
- estimate the percentage, then use your method to calculate it.

1. In your home, what percentage of:
   - **a** rooms have tiled floors
   - **b** electrical goods are used every day
   - **c** groceries in your fridge were grown or made in your country?
2. Of all the students in your school, what percentage regularly buy their lunch from the school canteen?
3. What percentage of your classmates:
   - **a** were born overseas
   - **b** have at least one parent born overseas?
4. From a normal school week, what percentage of lesson time is spent on each subject?

D FINDING A PERCENTAGE OF A QUANTITY

To find a **percentage of a quantity**, we first convert the percentage to a **decimal**, then **multiply** to find the required amount.

For example, 25% of €20

$= 0.25 \times €20$ $\quad \{25\% = \frac{25}{100} = 0.25\}$

$= €5$

Example 10 **Self Tutor**

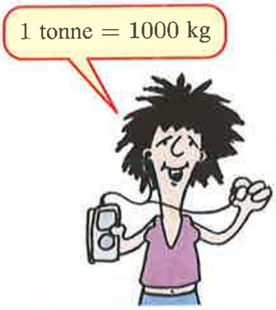
Find:

a 35% of \$5000 **b** 12.4% of 6 m (in cm)

a 35% of \$5000
= 0.35 × \$5000
= \$1750

b 12.4% of 6 m
= 0.124 × 600 cm
= 74.4 cm

EXERCISE 8D



1 Find:

- **a** 25% of 36
- **b** 10% of 70
- **c** 20% of 45
- **d** 36% of £4200
- **e** 125% of \$600
- **f** 95% of 5 tonnes
- **g** 3.8% of 100 m
- **h** 112% of 5000 mL
- **i** 15% of 1 hour (in minutes)
- **j** 29% of 1 tonne (in kg)

2 Carly has 20 sweets. She gives 35% of them to her sister. How many sweets does Carly give away?

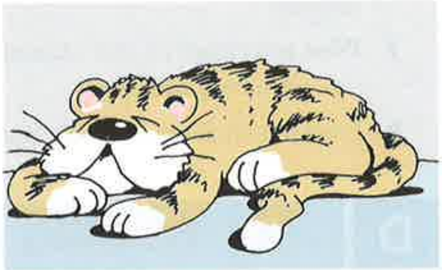
3 88% of seeds in a packet are expected to germinate. If the packet contains 150 seeds, how many are expected to germinate?

4 In Formula One racing, a driver must complete at least 90% of the race distance in order to be classified. How many laps of a 60 lap race must a driver complete to be classified?

5 In a class of 20 students, 70% are boys, 85% are 12 years old, 10% are 13 years old, and 35% catch the bus to school. Calculate the number of students who:

- **a** are boys
- **b** do not catch the bus to school
- **c** are 12 years old
- **d** are either 12 or 13 years old.

6 A kitten will reach 70% of its adult body weight after six months. If a cat weighs 4.5 kg when fully grown, estimate how much it weighed when it was six months old.



7 Fran's Fruit Juice contains 60% orange juice. How much orange juice is contained in a:

- **a** 300 mL glass
- **b** 2 litre bottle of Fran's Fruit Juice?

8 

An orchard produces 150 tonnes of apples in one season. 30 tonnes of the apples are second grade, 8% are unfit for sale, and the rest are first grade. First grade apples sell for \$1640 per tonne, and second grade for \$1250 per tonne. Find:

- **a** the weight of apples unfit for sale
- **b** the weight of first grade apples
- **c** the total value of the apple harvest.

E PERCENTAGE INCREASE OR DECREASE

There are many situations where quantities are either increased or decreased by a certain percentage.

For example:

- class sizes are increased by 10%
- the price of goods increases by 4%
- a man on a diet reduces his weight by 8%
- a store has a 25% discount sale.

We will examine *two methods* for dealing with **percentage increase** or **percentage decrease**.

METHOD 1: WITH TWO STEPS

Using this method we
- find the size of the increase or decrease

then
- apply this change to the original quantity by addition or subtraction.

Example 11 **Self Tutor**

a Increase $5000 by 20%. **b** Decrease $80 000 by 12%.

a
20% of $5000
= 0.2 × $5000
= $1000
∴ the new amount
= $5000 + $1000
= $6000

b
12% of $80 000
= 0.12 × $80 000
= $9600
∴ the new amount
= $80 000 − $9600
= $70 400

EXERCISE 8E.1

1 Perform these operations using two steps:

a increase 160 kg by 10% **b** decrease 12 km by 25% **c** increase $28 500 by 5%

d decrease 650 mL by 40% **e** increase 200 L by 2.5% **f** decrease £450 by 18%.

2 At the end of last year, Arthur's international student group had 65 members. It has increased this year by 20%.

a Find the number of new students who have joined the group.

b How many students are now in the group?

METHOD 2: WITH ONE STEP USING A "MULTIPLIER"

The original amount is 100% of the quantity.

- If we **increase** an amount by 20%, then this is **added**, and in total we have
  100% + 20% = 120% of the original.
  So, to **increase** an amount by 20% in **one step**, we multiply the original by 120%.
  The value 120% or 1.2 is called the **multiplier**.

- If we **decrease** an amount by 20%, then this is **subtracted**, and we are left with $100\% - 20\% = 80\%$ of the original.
  So, to **decrease** an amount by 20% in **one step**, we multiply the original by 80%.
  The value 80% or 0.8 is called the **multiplier**.

Example 12 **Self Tutor**

a Increase \$5000 by 20%.

b Decrease \$80 000 by 12%.

a To increase by 20%, we multiply by $100\% + 20\% = 120\%$.

$\therefore$ the new amount
$= 120\%$ of \$5000
$= 1.2 \times \$5000$
$= \$6000$

b To decrease by 12%, we multiply by $100\% - 12\% = 88\%$.

$\therefore$ the new amount
$= 88\%$ of \$80 000
$= 0.88 \times \$80\,000$
$= \$70\,400$

EXERCISE 8E.2

For a decrease, the multiplier is less than 1.

For an increase, the multiplier is greater than 1.

1 Find the multiplier for:
  - **a** an increase of 5%
  - **b** a decrease of 6%
  - **c** an increase of 12%
  - **d** a decrease of 25%
  - **e** a decrease of 49%
  - **f** an increase of 34%.

2 Perform these operations using a multiplier:
  - **a** increase €750 by 12%
  - **b** decrease 145 kg by 8%
  - **c** decrease 800 m by 16%
  - **d** increase 800 L by 65%
  - **e** increase 128 km by 9%
  - **f** decrease \$850 000 by 32%.

3 A bakery reported a 12% decrease in the sale of bread this year.
If the bakery sold 7000 loaves of bread last year, how many loaves did it sell this year?

4 Rita currently pays £250 rent per week, but has been told her rent will increase by 6%. How much rent will she pay after the increase?

5 The price of a bus ticket in March was \$5. The price rose by 10% in May, then by another 10% in November. Find the cost of a bus ticket in November.

6 When increasing a quantity by 10% and then decreasing the result by 10%, the overall result is to decrease the original quantity by 1%. Use multipliers to explain why this occurs.

7 In 2010, farmer Chris produced 1000 tonnes of wheat. The table below describes the percentage change in his wheat harvest each year in comparison with the previous year.

2011	2012	2013
↑ 20%	↓ 37.5%	↑ 44%

  - **a** How much wheat did Chris produce in 2013?
  - **b** In which year did Chris produce:
    - **i** the most wheat
    - **ii** the least wheat?

8 The attendance at council meetings has dropped by 15% each month for the last 3 months. Find the overall percentage decrease in attendance over this period.

F FINDING A PERCENTAGE CHANGE

If we increase or decrease a quantity, the **change** in the quantity is the final amount minus the original amount.

change = final amount − original amount

For example:

- if €50 increases to €60, the change is €60 − €50 = €10
- if €50 decreases to €42, the change is €42 − €50 = −€8

A **positive** change means an **increase**.
A **negative** change means a **decrease**.

PERCENTAGE CHANGE

If we compare the change to the original amount and express this as a percentage, then we have calculated the **percentage change**.

$$\textbf{percentage change} = \frac{\textbf{change}}{\textbf{original amount}} \times 100\%$$

Example 13 Self Tutor

Find the percentage increase or decrease in the change from:

a \$120 to \$150 **b** 400 m to 250 m.

a change
= final amount − original amount
= \$150 − \$120
= \$30

∴ percentage change

$= \dfrac{\text{change}}{\text{original amount}} \times 100\%$

$= \dfrac{30}{120} \times 100\%$

= 25%

∴ there is a 25% increase.

b change
= final amount − original amount
= 250 m − 400 m
= −150 m

∴ percentage change

$= \dfrac{\text{change}}{\text{original amount}} \times 100\%$

$= \dfrac{-150}{400} \times 100\%$

= −37.5%

∴ there is a 37.5% decrease.

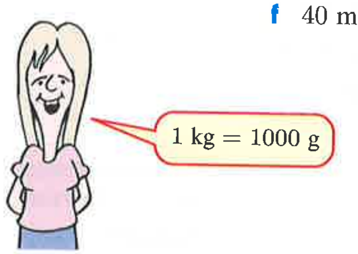
EXERCISE 8F

1 Describe the change if:

- **a** 120 cm grows to 156 cm
- **b** 95 kg drops to 76 kg
- **c** £495 increases to £550
- **d** 750 mL decreases to 650 mL
- **e** my ride to school now takes 12 minutes whereas before it took 15 minutes
- **f** I used to pay \$45 to fill my car with petrol, but now I pay \$52.

2 Find the percentage increase or decrease for the following changes:

- **a** 35 cm to 42 cm
- **b** 15 tonnes to 24 tonnes
- **c** 150 kg to 126 kg
- **d** €196 to €147
- **e** 1.2 kg to 900 g
- **f** 40 minutes to 1 hour.



3 In a clearance sale, the price of a refrigerator was reduced from \$1800 to \$1260. What was the percentage decrease in price?

4 At the start of term, Hermione took 4 minutes and 35 seconds to swim 10 laps of the school pool. By the end of term, it took her only 4 minutes and 13 seconds. Find the percentage decrease in Hermione's swim time.

5 The average height of trees in a park was 1.7 m. Three years later, the average height had increased to 2.65 m. State the percentage growth of the trees over this period.

G BUSINESS APPLICATIONS

Percentages are used every day for a variety of problems involving **money**. For example, we use percentages to describe **profit**, **loss**, and **discount**.

PROFIT AND LOSS

In order to run a successful business, shopkeepers must sell their products at prices that are greater than what they paid for them. When they achieve this, they make a **profit**.

However, sometimes products must be sold at prices less than those for which they were bought. In this case the shopkeeper makes a **loss**.

- The **cost price** is the price at which the business buys a product.
- The **selling price** is the price at which the business sells a product.
- **Profit** or **loss** = selling price − cost price.
- A **profit** occurs if the selling price is greater than the cost price.
- A **loss** occurs if the selling price is less than the cost price.

Example 14 🔊 **Self Tutor**

Find the profit or loss that occurs when:

a an electronics store buys a television for \$180 and sells it for \$295

b a car yard buys a car for \$900 and sells it for \$650.

a selling price − cost price
= \$295 − \$180
= \$115
So, a profit of \$115 is made.

b selling price − cost price
= \$650 − \$900
= −\$250
So, a loss of \$250 is made.

A **positive** value means a **profit**.
A **negative** value means a **loss**.

PERCENTAGE PROFIT OR LOSS

Knowing that you made a profit of \$2 on an item is not always very useful information by itself. \$2 is a very small profit on an expensive item like a washing machine, but it is a very large profit on a cheap item like a chocolate bar.

It is therefore often more useful to express the profit or loss as a **percentage of the cost price**.

- $\textbf{percentage profit} = \dfrac{\textbf{profit}}{\textbf{cost price}} \times 100\%$
- $\textbf{percentage loss} = \dfrac{\textbf{loss}}{\textbf{cost price}} \times 100\%$

Example 15 🔊 **Self Tutor**

A microscope was purchased for \$600 and sold two months later for \$450.
Find the profit or loss as a percentage of the cost price.

selling price − cost price
= \$450 − \$600
= −\$150

So, a loss of \$150 was made.

$$\text{Percentage loss} = \frac{\text{loss}}{\text{cost price}} \times 100\%$$
$$= \frac{150}{600} \times 100\%$$
$$= 25\%$$

EXERCISE 8G.1

1 Copy and complete:

	Cost price	*Selling price*	*Profit or loss?*	*How much profit or loss?*
a	\$35	\$65		
b	£225	£160		
c	€520	€670		
d	¥26 500	¥33 000		

2 Copy and complete:

	Cost price	*Selling price*	*Profit or loss?*
a	€56		€20 profit
b	$420		$75 loss
c		£580	£195 profit
d		€200	€65 loss

3 For each of the following transactions, find:

i the profit or loss **ii** the percentage profit or loss (rounded to 1 decimal place).

- **a** Janet bought a toy car set for $45 and sold it for $65.
- **b** Dave bought a motorbike for £7000 and sold it for £5850.
- **c** I bought a kettle for €38 and sold it for €15.
- **d** Lana sold a stereo which cost her $190, for $250.
- **e** Robert sold a chainsaw which cost him £240, for £330.

4 A boat was purchased for €26 000 and three years later was sold for €19 000. Find the loss as a percentage of the cost price, rounding your answer to 1 decimal place.

5 A retailer buys a lounge suite for $680 and sells it for $1120. Find the profit as a percentage of the cost price, rounding your answer to 1 decimal place.

6 A used car salesman buys a car for £13 500, and then sells it three days later for £18 000.

- **a** Find the profit made on the sale of the car.
- **b** Express this profit as a percentage of the cost price.

7 A department store purchased 15 handbags for a total of €825. The store then sold them for €90 per handbag. Calculate the total profit, and express this profit as a percentage of the cost price. Round your answer to 1 decimal place.

8 A sweets shop buys 1600 lollipops for 20 cents each. Only 910 of the lollipops are sold for 35 cents each.

- **a** Determine whether the shop made a profit or loss.
- **b** Express the profit or loss as a percentage of the cost price. Round your answer to 3 decimal places.

Example 16 **Self Tutor**

Find the selling price for goods purchased for $150 and sold at a 20% profit.

For a 20% profit we must increase $150 by 20%.

Two step method: *or* *Using the multiplier:*

Profit = 20% of $150
= 0.2 × $150
= $30

So, the selling price
= $150 + $30
= $180

To increase by 20%, we multiply by 120%.

So, the selling price = 120% of $150
= 1.2 × $150
= $180

9 Find the selling price of goods bought for:

- **a** \$600 and sold for a 30% profit
- **b** €450 and sold for a 20% loss
- **c** \$3500 and sold for a 15% loss
- **d** ¥25 000 and sold for a 12% profit.

10 Deanna bought a bedroom suite for \$2400 and sold it for a profit of 8%. At what price did she sell the bedroom suite?

11 Aaron bought a wardrobe for £200, but was forced to sell it at a 30% loss. At what price did he sell the wardrobe?

DISCOUNT

In order to attract customers or to clear old stock, many businesses reduce the price of items from the **marked price** shown on the price tag.

The amount of money by which the marked price of the item is reduced is called **discount**.

selling price = marked price − discount

Example 17 **Self Tutor**

A dishwasher has a marked price of \$495. During a sale, a \$120 discount is offered. Find the sale price of the dishwasher.

selling price = marked price − discount
= \$495 − \$120
= \$375

So, the dishwasher is on sale at \$375.

Discount is often stated as a percentage of the marked price. It is thus a **percentage decrease**.

Example 18 **Self Tutor**

The marked price of a video projector is \$1060. If a 22% discount is offered, find the actual selling price.

We have to decrease \$1060 by 22%.

To decrease by 22% we multiply by 100% − 22% = 78% {using the multiplier method}

So, selling price = 78% of marked price
= 78% of \$1060
= 0.78 × \$1060 {"of" indicates ×}
= \$826.80

EXERCISE 8G.2

1 Find the selling price of:

- **a** a lamp with a marked price of $49, if a $15 discount is offered
- **b** a cordless drill which has been discounted by £80 from its marked price of £329.

2 The marked price of a billiard table is $2450, and an 8% discount is offered. Find the actual selling price.

3 A television marketing company advertises exercise equipment for €180. It offers a 12% discount for the first 50 callers. What is the actual selling price if you are one of the first 50 callers?

4 A toaster with a marked price of £50 has been discounted to £42.

- **a** Find the size of the discount.
- **b** Express the discount as a percentage of the marked price.

5 A department store is offering the following discounts:

- 5% off for customers who spend more than $100
- 7.5% off for customers who spend more than $200
- 10% off for customers who spend more than $500.

Find the final price paid by a customer who buys:

- **a** a television set marked at $180
- **b** a washing machine marked at $320
- **c** a dishwasher marked at $890.

6 Copy and complete:

	Marked price	*Discount*	*Selling price*	*Discount as a % of marked price*
a	$160	$40		
b	£500			34%
c	$2.40			15%
d		$0.75	$3.40	
e	€252		€163	

Global context

click here

Elections

Statement of inquiry:	Mathematics can help us analyse the fairness of different voting systems.
Global context:	Fairness and development
Key concept:	Logic
Related concepts:	Generalisation, Justification
Objectives:	Communicating, Applying mathematics in real-life contexts
Approaches to learning:	Communicating, Social

KEY WORDS USED IN THIS CHAPTER

- change
- fraction
- percentage
- cost price
- loss
- profit
- decimal
- marked price
- selling price
- denominator
- multiplier

REVIEW SET 8A

1 Of the bottles shown below, which one is 35% full?

A

B

C

D

E

2 Express as a fraction in lowest terms:

a 29% **b** 74% **c** 45% **d** 190%

3 Write as a percentage:

a 0.56 **b** 0.239 **c** 2.6 **d** 0.0071

4 Express:

a 27 cm as a percentage of 50 cm **b** 6 hours as a percentage of one day

c 409 mL as a percentage of 1 L.

5 13 of the 20 houses on a street have burglar alarms. What percentage of the houses have burglar alarms?

6 In a school of 800 students, 67% have brown hair, 24% have black hair, 2% have red hair, and the remainder have fair hair. Find:

a the percentage of fair-haired students **b** the number of brown-haired students.

7 15% of the crowd at an international rugby tournament were Kenyan supporters. There were 81 000 spectators at the tournament. How many supported Kenya?

8 Find the multiplier for:

a an increase of 45% **b** a decrease of 75% **c** an increase of 9%.

9 Julie bought a laptop for €600, then sold it three years later for €438.

a Find Julie's profit or loss.

b Find the profit or loss as a percentage of the cost price.

10 Find the selling price of a book that is bought for $60 and sold at a 30% profit.

REVIEW SET 8B

1 Which of the pie portions below represents:

a 75%

b 20%

c $33\frac{1}{3}\%$ of the pie?

A

B

C

2 Write as a percentage:

a $\frac{17}{25}$ **b** $\frac{19}{20}$ **c** $\frac{55}{200}$ **d** $\frac{1}{8}$

3 Write as a decimal:

a 47% **b** 6% **c** 92.7% **d** 165%

4 Find:

a 28% of 350 m **b** 17% of 680 kg

5 Student council meetings can only proceed if 70% of the council is present. The council consists of 40 students, but only 25 students are present. Will the meeting proceed?

6 A fruit punch contains 35% orange juice, 25% mango juice, 35% soda water, and the rest is passionfruit pulp.

a What percentage of the fruit punch is passionfruit pulp?

b In a 5 L bowl of punch, how much is: **i** soda water **ii** mango juice?

c Express the amount of soda water as a percentage of the amount of passionfruit pulp.

7 **a** Increase $500 by 25%. **b** Decrease 40 km by 10%.

8 A tour company has organised a bus tour around France. So far, 12 Australian tourists, 7 Canadian tourists, and 6 American tourists have booked.

a What percentage of the tourists are Australian?

b A new booking has confirmed 4 New Zealand tourists will be on the tour.

i Find the percentage increase in tourists.

ii What percentage of the tourists are Canadian? Round your answer to 1 decimal place.

9 A guitar was bought for $250 and sold for $295. Find the profit as a percentage of the cost price.

10 A pair of jeans has a marked price of $80. If a 25% discount is offered, find the actual selling price.

Chapter 9

Equations

Contents:

OPENING PROBLEM

Ray is at the Royal Show with five good friends. They decide to all go on the MegaTwister, one of the new rides at the Show. It costs \$8 per ride. Ray's mum has given them \$30 to help pay for one ride for each of them. They will each have to pay some extra money as well.

Suppose each of the six friends pays $\$x$ extra.

Things to think about:

- **a** Explain why the total amount paid by the six friends is $(6x + 30)$ dollars.
- **b** Explain why $6x + 30 = 6 \times 8$.
- **c** Given the equation $6x + 30 = 48$, how can we find the exact value of x?
- **d** How much does each of the friends pay?

In this chapter we look at **algebraic equations** and methods used to solve them.

A EQUATIONS

An **equation** is a mathematical sentence which indicates that two expressions have the same value. An equation always contains an **equal sign** $=$.

An equation has a **left hand side (LHS)** and a **right hand side (RHS)**, and these are separated by the equal sign.

For example, a simple equation is: $\underset{\textbf{LHS}}{\underset{\uparrow}{3 \times 5}} \quad \underset{\textbf{equals}}{\underset{\uparrow}{=}} \quad \underset{\textbf{RHS}}{\underset{\uparrow}{7 + 8}}$.

The LHS $= 3 \times 5 = 15$ and the RHS $= 7 + 8 = 15$. Since LHS $=$ RHS, the equation is true.

Example 1 **Self Tutor**

Decide whether these equations are true or false:

a $4 + 5 = 11 - 2$ **b** $2 \times 4 = 20 \div 2$

a LHS $= 4 + 5 = 9$
RHS $= 11 - 2 = 9$
Since LHS $=$ RHS, the equation is true.

b LHS $= 2 \times 4 = 8$
RHS $= 20 \div 2 = 10$
Since LHS $\neq$ RHS, the equation is false.

Most of the equations we will look at contain an **unknown** such as x. We call these **algebraic equations**.

A **linear equation** contains one unknown which is raised to the power 1.

For example, $2x - 1 = 9$ is an algebraic equation, and is also a linear equation.

If we replace the x with a number, the resulting equation will either be true or false.

Example 2 **Self Tutor**

a Is the equation $x - 3 = 12 \div 4$ true or false when $x = 5$?

b Is the equation $7 \times 2 = x + 6$ true or false when $x = 8$?

a If $x = 5$, $\text{LHS} = 5 - 3 = 2$
and $\text{RHS} = 12 \div 4 = 3$
$\text{LHS} \neq \text{RHS}$, so the equation is false.

b If $x = 8$, $\text{LHS} = 7 \times 2 = 14$
and $\text{RHS} = 8 + 6 = 14$
$\text{LHS} = \text{RHS}$, so the equation is true.

EXERCISE 9A

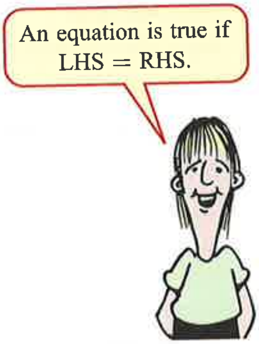
1 Decide whether these equations are true or false:

a $5 \times 3 = 15$
b $28 \div 4 = 6$
c $7 + 5 = 2 \times 6$
d $22 - 11 = 5 \times 2$
e $24 \div 3 = 17 - 4$
f $13 + 19 = 4 \times 8$

2 **a** Is $x \div 3 = 15 - 8$ true or false when $x = 21$?
b Is $x + 9 = 20 \div 2$ true or false when $x = 4$?
c Is $16 - 7 = x + 2$ true or false when $x = 5$?
d Is $14 \div 2 = 23 - x$ true or false when $x = 14$?
e Is $3x = 11 + 7$ true or false when $x = 6$?
f Is $30 \div 6 = x - 6$ true or false when $x = 11$?

3 Is the equation $3x + 4 = 10$ true or false when:

a $x = 1$
b $x = 2$
c $x = 3$?



B SOLVING SIMPLE EQUATIONS

When we **solve** an equation, we find the value of the unknown which makes the equation true.

This value is called the **solution** of the equation.

Consider the equation $6x + 30 = 48$.

If we replace x with different numbers, most of them will make the equation false.

For example:

If $x = 1$, the LHS is $6 \times 1 + 30 = 6 + 30 = 36$ ✗
If $x = 5$, the LHS is $6 \times 5 + 30 = 30 + 30 = 60$ ✗
If $x = 3$, the LHS is $6 \times 3 + 30 = 18 + 30 = 48$ ✓

So, if $x = 3$, then the $\text{LHS} = \text{RHS}$, and the equation is true.

We say $x = 3$ is the **solution** of the equation $6x + 30 = 48$.

Example 3 **Self Tutor**

The solution to $2x + 9 = 23$ is one of the numbers $\{5, 6, 7, 8\}$. Find the solution.

If $x = 5$, LHS $= 2 \times 5 + 9 = 10 + 9 = 19$ ✗
If $x = 6$, LHS $= 2 \times 6 + 9 = 12 + 9 = 21$ ✗
If $x = 7$, LHS $= 2 \times 7 + 9 = 14 + 9 = 23$ ✓

So, the solution to $2x + 9 = 23$ is $x = 7$.

SOLVING BY GUESS, CHECK, AND IMPROVE

One method of solving simple equations is to try different values for the unknown until the correct solution is found.

We start with an initial **guess**, and then **check** whether our guess is correct. If it is not, we decide whether our guess is too low or too high, and **improve** our guess.

Example 4 **Self Tutor**

Solve the equation $4x - 13 = 35$ using *guess, check, and improve.*

We start with an initial guess of $x = 5$:

x	$4x - 13$	
5	7	← much too low
9	23	← getting closer
11	31	← almost
12	35	LHS = RHS ✓

So, the solution is $x = 12$.

EXERCISE 9B

1 One of the numbers in the brackets is the solution to the equation. Find the solution.

- **a** $x + 7 = 13$ $\{4, 5, 6, 7\}$
- **b** $21 - x = 9$ $\{10, 12, 14, 16\}$
- **c** $2x + 5 = 17$ $\{2, 4, 6, 8\}$
- **d** $3x - 2 = 25$ $\{8, 9, 10, 11\}$
- **e** $16 - 2x = 6$ $\{2, 3, 4, 5\}$
- **f** $5x + 3 = 38$ $\{6, 7, 8, 9\}$

2 Find the solution using *guess, check, and improve*:

- **a** $2x + 3 = 17$
- **b** $5x - 4 = 41$
- **c** $3x - 15 = 18$
- **d** $4 + 11x = 81$
- **e** $33 - 2x = 19$
- **f** $50 - 3x = 26$

3 Hassan and Yaasin are trying to solve the equation $4x + 10 = 46$.

- **a** Hassan guesses that the solution is $x = 3$. Is his guess too low or too high?
- **b** Yaasin guesses that the solution is $x = 12$. Is his guess too low or too high?
- **c** Find the solution to the equation.

ACTIVITY 1 GUESS, CHECK, AND IMPROVE

What to do:

1. Click on the icon to run the software.
2. For the equation given, enter the value of x which you think is the solution. The value of the left hand side of the equation will be calculated. You will be told if your guess was correct, or if it gave a value that was too large or too small.
3. Keep guessing until you get the correct answer.
4. Compete with your class to see who is the fastest to correctly solve 10 equations.

ACTIVITY

DISCUSSION GUESS, CHECK, AND IMPROVE

Discuss in groups of 2 or 3:

- What strategies do you use to decide on your initial guess?
- What are the problems with using guess, check, and improve to solve equations?

C MAINTAINING BALANCE

The **balance** of an equation is like the balance of a set of scales. Changing one side of the equation without doing the same thing to the other side will upset the balance.

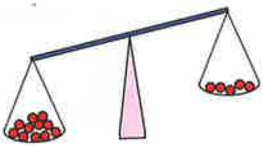
For example, the scales opposite represent the equation $5 = 5$.

If 6 is added to both sides, we obtain another true equation:

$$5 + 6 = 5 + 6$$

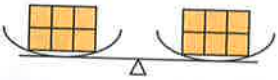
If 6 was added to one side only, then the scales would be unbalanced, and the equation would become false:

$$5 + 6 \neq 5$$

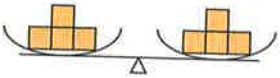


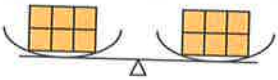
To maintain the balance, whatever is done on one side of the equal sign must also be done on the other side.

Imagine a set of scales with six identical blocks on each side. The scale is **balanced**.

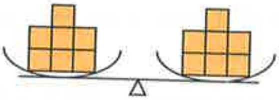


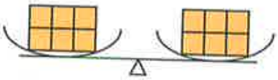
If we subtract 2 blocks from each side we get:



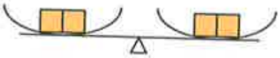


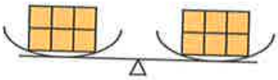
If we add 1 block to each side we get:



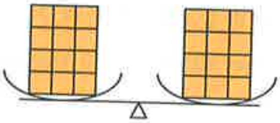


If we divide the number of blocks on each side by 3, we get:





If we multiply the number of blocks on each side by 2, we get:

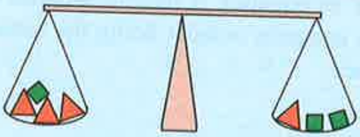


Notice that the scales are still balanced in each case!

Example 5

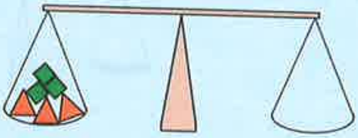
Self Tutor

This set of scales is perfectly balanced:

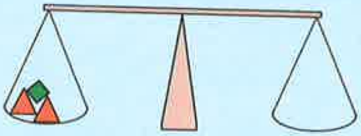


Complete the right hand side of the scales below, so that the balance is maintained:

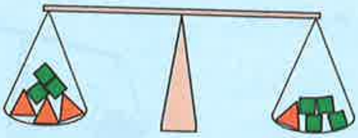
a



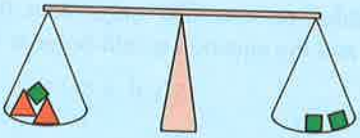
b



a 2 ■ have been added to the LHS, so we must also add 2 ■ to the RHS.



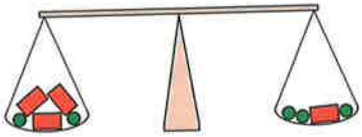
b 1 ▲ has been taken from the LHS, so we must also take 1 ▲ from the RHS.



EXERCISE 9C.1

PRINTABLE SCALES

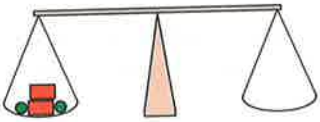
1 This set of scales is perfectly balanced:



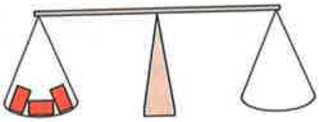
Complete the right hand side of the scales below, so that the balance is maintained:

a

b

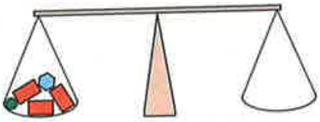


c

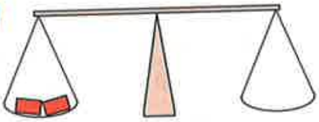


d

e



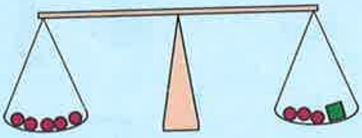
f



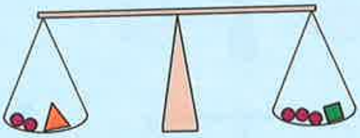
Example 6 **Self Tutor**

These scales are perfectly balanced. Find the relationship between the objects:

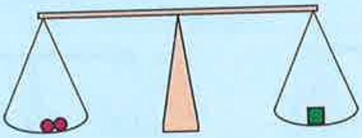
a



b

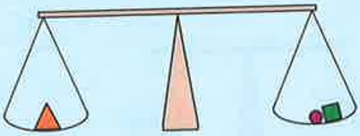


a By taking 3 ● from both sides, we get:



So, 1 🟩 is equal to 2 ●

b By taking 2 ● from both sides, we get:

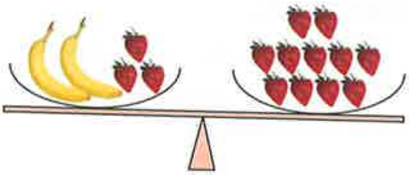


1 ▲ is equal to 1 ● plus 1 🟩

So, ▲ = ● + 🟩

We take objects away until each object type appears on only one side of the scales.

2 This set of scales is balanced with two bananas and three strawberries on one side, and 11 strawberries on the other.

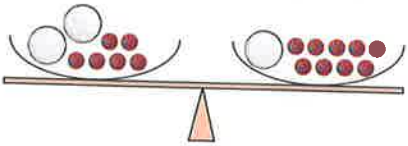


a Three strawberries are taken from the left side. What must be done to the right side to keep the scales balanced?

b There are now two bananas on the left hand side. How many strawberries balance their weight?

c How heavy is one banana in terms of strawberries?

3 This set of scales is balanced with two golf balls and six marbles on the left, and one golf ball and nine marbles on the right.



a Six marbles are taken from the left side. What must be done to the right side to keep the scales balanced?

b The golf ball on the right side is then removed. What must now be done to the left side to keep the scales balanced?

c Redraw the scales if both **a** and **b** occur, maintaining balance at each step.

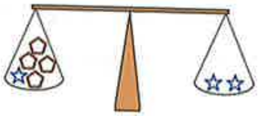
d How heavy is one golf ball in terms of marbles?

4 These scales are perfectly balanced. Find the relationship between the objects.

a

b

c



d

e

f

g

h

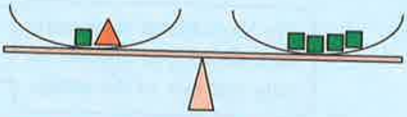
i

Example 7

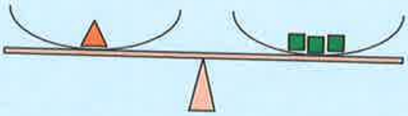
Self Tutor

This set of scales is balanced.

Find ▲ if ■ = 4.



By taking 1 ■ from both sides, we get:



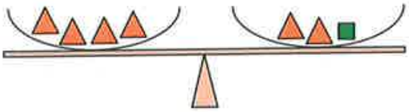
So, ▲ = 3 × ■

= 3 × 4 {since ■ = 4}

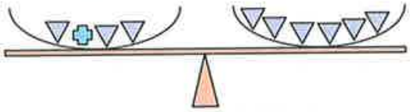
= 12

5 These scales are perfectly balanced.

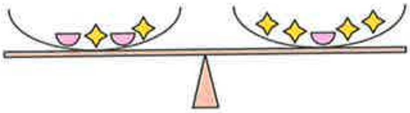
a Find ■ if ▲ = 3.



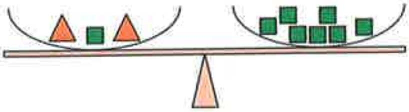
b Find ✚ if ▽ = 5.



c Find ✧ if ◡ = 4.



d Find ▲ if ■ = 8.



Check your answer by finding the total amount on each side of the scales.

BALANCING EQUATIONS

The rules for balancing scales can also be used to balance equations.

The **balance** of an equation will be maintained if we:

- add the same amount to both sides
- subtract the same amount from both sides
- multiply both sides by the same amount
- divide both sides by the same amount.

If the balance of an equation is maintained, we will not change the solution(s) of the equation.

Example 8 **Self Tutor**

Consider the equation $x + 5 = 10$. What equation results when we perform the following on both sides of the equation:

a add 3
b subtract 3
c divide by 2
d multiply by 4?

a
$$x + 5 = 10$$
$$\therefore\ x + 5 + 3 = 10 + 3$$
$$\therefore\ x + 8 = 13$$

b
$$x + 5 = 10$$
$$\therefore\ x + 5 - 3 = 10 - 3$$
$$\therefore\ x + 2 = 7$$

c
$$x + 5 = 10$$
$$\therefore\ \frac{x+5}{2} = \frac{10}{2}$$
$$\therefore\ \frac{x+5}{2} = 5$$

d
$$x + 5 = 10$$
$$\therefore\ 4(x + 5) = 4 \times 10$$
$$\therefore\ 4(x + 5) = 40$$

∴ means *therefore*. We use it to show that the next line of work follows from the previous line.

EXERCISE 9C.2

1 Find the equation which results from *adding*:

a 2 to both sides of $x+1=5$
b 3 to both sides of $x+2=6$
c 4 to both sides of $x-4=7$
d 6 to both sides of $2x-6=8$
e 6 to both sides of $1-x=2$
f 5 to both sides of $3-2x=1$

2 Find the equation which results from *subtracting*:

a 3 from both sides of $x+4=12$
b 4 from both sides of $x+4=7$
c 2 from both sides of $2-x=11$
d 5 from both sides of $2x+5=3$
e 4 from both sides of $5x+4=2$
f 3 from both sides of $2x-1=5$

3 Find the equation which results from *multiplying* both sides of:

a $x=2$ by 2
b $2x=3$ by 4
c $\dfrac{x}{3}=6$ by 3
d $x+1=3$ by 2
e $\dfrac{x-1}{3}=2$ by 3
f $\dfrac{2-x}{2}=-3$ by 2

4 Find the equation which results from *dividing* both sides of:

a $2x=4$ by 2
b $2(x+1)=8$ by 2
c $3x+9=6$ by 3
d $4x-4=8$ by 4
e $5x=8$ by 5
f $3(x-2)=18$ by 3
g $-x=7$ by -1
h $-2(1-x)=-12$ by -2

D INVERSE OPERATIONS

Imagine starting with $50 in your pocket. You find $5, and then buy a drink for $5. You are left with the original $50 you had in your pocket.

This can be illustrated by a **flowchart** such as:

Adding 5 and subtracting 5 have the opposite effect. One undoes the other.

We say that

addition and subtraction are **inverse operations.**

Now imagine you start with $50, and your friend gives you the same amount. Your money is now doubled. If you decide to give half to your brother, you will be back to your original $50.

We can again illustrate the process by a flowchart:

Multiplying by 2 and dividing by 2 undo each other.

We say that

multiplication and division are **inverse operations.**

SOLVING EQUATIONS

We can solve simple equations using *inverse operations*. However, we must remember to maintain the *balance* by performing the same operation on *both sides*.

For example, consider $x+3=7$, where 3 has been added to x.

$$x+3=7$$
$$\therefore\ x+3-3=7-3 \quad \{\text{subtracting 3 is the inverse of adding 3}\}$$
$$\therefore\ x=4 \quad \{\text{simplifying}\}$$

By performing the correct inverse operation, we are left with x by itself on one side of the equation. The solution is on the other side.

EXERCISE 9D

1 State the inverse of each of the following operations:

a $+\,4$ **b** $\times\,2$ **c** $\div\,3$

d $-\,1$ **e** $-\,\frac{1}{3}$ **f** $\div\,\frac{1}{2}$

g $+\,\frac{3}{4}$ **h** $\times\,7$ **i** $-\,11$

2 Simplify:

a $x-6+6$ **b** $x\times 2\div 2$ **c** $x+4-4$

d $5x\div 5$ **e** $\frac{3x}{3}$ **f** $\frac{x}{4}\times 4$

g $7+x-7$ **h** $\frac{2}{3}x\div\frac{2}{3}$ **i** $\frac{3x}{2}\times 2$

Example 9 **Self Tutor**

Solve for x using a suitable inverse operation: $x+5=11$

$$x+5=11$$
$$\therefore\ x+5-5=11-5 \quad \{\text{The inverse of } +5 \text{ is } -5\text{, so we take 5 from both sides.}\}$$
$$\therefore\ x=6$$

3 Find x using an inverse operation:

a $x+2=9$ **b** $x+6=15$ **c** $x+9=20$

d $x+5=0$ **e** $x+4=1$ **f** $x+7=-3$

g $5+x=13$ **h** $2+x=11$ **i** $x+15=11$

j $x+9=-8$ **k** $x+0.5=0.7$ **l** $\frac{1}{3}+x=1$

Example 10

Self Tutor

Solve for y using a suitable inverse operation: $y-6=-2$

$$y-6=-2$$

$$\therefore\ y-6+6=-2+6$$ {The inverse of -6 is $+6$, so we add 6 to both sides.}

$$\therefore\ y=4$$

4 Find y using an inverse operation:

a $y-4=5$
b $y-8=17$
c $y-7=0$
d $y-12=-2$
e $y-6=-13$
f $y-3=-3$
g $y-8=25$
h $y-1.3=2.6$
i $y-\frac{1}{2}=4$

Example 11

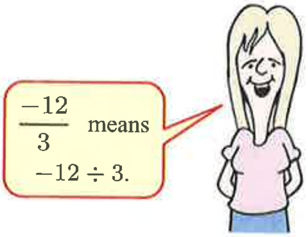
Self Tutor

Solve for t using a suitable inverse operation: $3t=-12$

$$3t=-12$$

$$\therefore\ \frac{3t}{3}=\frac{-12}{3}$$ {The inverse of $\times 3$ is $\div 3$, so we divide both sides by 3.}

$$\therefore\ t=-4$$



5 Find a using an inverse operation:

a $2a=10$
b $3a=12$
c $2a=16$
d $5a=20$
e $4a=-16$
f $6a=30$
g $5a=-35$
h $8a=64$
i $10a=7$

Example 12

Self Tutor

Solve for d using a suitable inverse operation: $\frac{d}{7}=8$

$$\frac{d}{7}=8$$

$$\therefore\ \frac{d}{7}\times 7=8\times 7$$ {The inverse of $\div 7$ is $\times 7$, so we multiply both sides by 7.}

$$\therefore\ d=56$$

6 Find x using an inverse operation:

a $\frac{x}{3}=2$
b $\frac{x}{2}=6$
c $\frac{x}{4}=7$
d $\frac{x}{5}=-1$
e $\frac{x}{3}=18$
f $\frac{x}{11}=-2$
g $\frac{x}{6}=2$
h $\frac{x}{-2}=5$
i $\frac{x}{-3}=-6$

7 Find the unknown using a suitable inverse operation:

- **a** $3 + a = 11$
- **b** $b - 17 = -4$
- **c** $8c = 48$
- **d** $d \div 4 = 11$
- **e** $e + 19 = 12$
- **f** $2f = -10$
- **g** $g - 17 = 24$
- **h** $\frac{h}{7} = 3$
- **i** $5 + i = -3$
- **j** $6j = -42$
- **k** $k - \frac{1}{2} = 1$
- **l** $4l = -8$
- **m** $16 + m = 12$
- **n** $\frac{n}{-4} = -2$
- **o** $\square - 9 = -14$
- **p** $p + 0.4 = 0.9$
- **q** $\frac{q}{-10} = 5$
- **r** $-3r = -15$
- **s** $s - \frac{2}{5} = 0$
- **t** $\frac{t}{7} = -4$
- **u** $9u = 5$

ACTIVITY 2 INVERSE OPERATIONS

Click on the icon to practise solving equations using a single inverse operation.

INVERSE OPERATIONS

E ALGEBRAIC FLOWCHARTS

To solve more complicated equations, we must understand how expressions are **built up**. We can study this using an **algebraic flowchart**.

Example 13 Self Tutor

Complete the following flowcharts:

a $\boxed{x} \xrightarrow{\times 2} \boxed{} \xrightarrow{+ 5} \boxed{}$

b $\boxed{x} \xrightarrow{+ 5} \boxed{} \xrightarrow{\times 2} \boxed{}$

c $\boxed{x} \xrightarrow{- 3} \boxed{} \xrightarrow{\div 2} \boxed{}$

d $\boxed{x} \xrightarrow{\div 2} \boxed{} \xrightarrow{- 3} \boxed{}$

---

a $\boxed{x} \xrightarrow{\times 2} \boxed{2x} \xrightarrow{+ 5} \boxed{2x + 5}$

b $\boxed{x} \xrightarrow{+ 5} \boxed{x + 5} \xrightarrow{\times 2} \boxed{2(x + 5)}$

c $\boxed{x} \xrightarrow{- 3} \boxed{x - 3} \xrightarrow{\div 2} \boxed{\frac{x - 3}{2}}$

d $\boxed{x} \xrightarrow{\div 2} \boxed{\frac{x}{2}} \xrightarrow{- 3} \boxed{\frac{x}{2} - 3}$

By reversing the flowchart with **inverse operations**, we will be able to *undo* the expression and find the value of the unknown.

Example 14

Self Tutor

Use a flowchart to show how $5x+2$ is 'built up'.
Reverse it to 'undo' the expression.

Build up: $\boxed{x} \xrightarrow{\times\ 5} \boxed{5x} \xrightarrow{+\ 2} \boxed{5x+2}$

Undoing: $\boxed{5x+2} \xrightarrow{-\ 2} \boxed{5x} \xrightarrow{\div\ 5} \boxed{x}$

To undo the expression, perform the **inverse operations** in the **reverse order**.

Example 15

Self Tutor

Use a flowchart to show how $\dfrac{x+3}{2}$ is 'built up'.

Reverse it to 'undo' the expression.

Build up: $\boxed{x} \xrightarrow{+\ 3} \boxed{x+3} \xrightarrow{\div\ 2} \boxed{\dfrac{x+3}{2}}$

Undoing: $\boxed{\dfrac{x+3}{2}} \xrightarrow{\times\ 2} \boxed{x+3} \xrightarrow{-\ 3} \boxed{x}$

EXERCISE 9E

1 Copy and complete the following flowcharts:

a $\boxed{x} \xrightarrow{\times\ 2} \boxed{\quad} \xrightarrow{-\ 3} \boxed{\quad}$

b $\boxed{x} \xrightarrow{+\ 1} \boxed{\quad} \xrightarrow{\times\ 4} \boxed{\quad}$

c $\boxed{x} \xrightarrow{-\ 2} \boxed{\quad} \xrightarrow{\div\ 3} \boxed{\quad}$

d $\boxed{x} \xrightarrow{\div\ 2} \boxed{\quad} \xrightarrow{+\ 5} \boxed{\quad}$

2 Copy and complete the following flowcharts by inserting the missing operations:

a $\boxed{x} \xrightarrow{?} \boxed{3x} \xrightarrow{?} \boxed{3x+2}$

b $\boxed{x} \xrightarrow{?} \boxed{\dfrac{x}{3}} \xrightarrow{?} \boxed{\dfrac{x}{3}-1}$

c $\boxed{x} \xrightarrow{?} \boxed{x-7} \xrightarrow{?} \boxed{\dfrac{x-7}{2}}$

d $\boxed{x} \xrightarrow{?} \boxed{x+4} \xrightarrow{?} \boxed{2(x+4)}$

3 Use flowcharts to show how to 'build up' and then 'undo' the following expressions:

a $2x+4$ **b** $3x-1$ **c** $4x+3$ **d** $5x-12$

4 Use flowcharts to show how to 'build up' and then 'undo' the following expressions:

a $\dfrac{x}{3}-1$ **b** $\dfrac{x-1}{3}$ **c** $\dfrac{x+5}{3}$ **d** $\dfrac{x}{3}+5$

e $3x+8$ **f** $3(x+8)$ **g** $2(x-6)$ **h** $2x-6$

F SOLVING EQUATIONS

So far we have solved simple equations by:

- guess, check, and improve
- using one inverse operation.

When we are given an equation which contains a *built up* expression, a systematic approach is to use **inverse operations** to undo the build-up of the expression in the **reverse order**. This allows us to write the unknown by itself.

For example, to solve $5x + 7 = 17$, we look at how $5x + 7$ is built up from x:

$$\begin{tikzcd} \boxed{x} \arrow[r, shift left, "\times 5"] & \boxed{5x} \arrow[l, shift left, "\div 5"] \arrow[r, shift left, "+7"] & \boxed{5x+7} \arrow[l, shift left, "-7"] \end{tikzcd}$$

To obtain $5x + 7$ from x, we $\times$ 5, then $+$ 7.

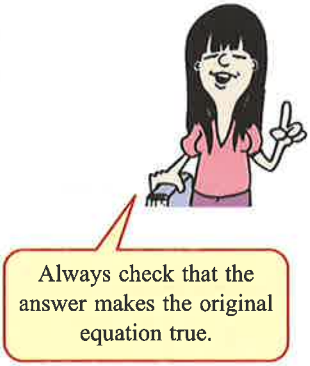
So, to isolate x from $5x + 7$, we $-$ 7 and then $\div$ 5.

Example 16 **Self Tutor**

Find x: $5x + 7 = 17$

$$\begin{aligned} 5x + 7 &= 17 \\ \therefore\ 5x + 7 - 7 &= 17 - 7 \quad \text{\{subtracting 7 from both sides\}} \\ \therefore\ 5x &= 10 \\ \therefore\ \frac{5x}{5} &= \frac{10}{5} \quad \text{\{dividing both sides by 5\}} \\ \therefore\ x &= 2 \end{aligned}$$

Check: LHS $= 5 \times 2 + 7 = 10 + 7 = 17 =$ RHS ✓



Example 17 **Self Tutor**

Solve for x: $\frac{x}{3} - 4 = -3$

$$\begin{aligned} \frac{x}{3} - 4 &= -3 \\ \therefore\ \frac{x}{3} - 4 + 4 &= -3 + 4 \quad \text{\{adding 4 to both sides\}} \\ \therefore\ \frac{x}{3} &= 1 \\ \therefore\ \frac{x}{3} \times 3 &= 1 \times 3 \quad \text{\{multiplying both sides by 3\}} \\ \therefore\ x &= 3 \end{aligned}$$

Check: LHS $= \frac{3}{3} - 4 = 1 - 4 = -3 =$ RHS ✓

EXERCISE 9F

1 Find x:

a $2x-3=7$	**b** $3x+5=8$	**c** $5x+9=24$
d $4x-16=0$	**e** $2x-4=-10$	**f** $3x-1=0$
g $10x-2=18$	**h** $6x+5=10$	**i** $2x+11=11$
j $11x-3=41$	**k** $7x-5=-4$	**l** $4x+15=17$

2 Solve for x:

a $\frac{x}{2}+1=3$	**b** $\frac{x}{3}-2=5$	**c** $\frac{x}{5}+7=1$
d $\frac{x}{4}-1=-6$	**e** $\frac{x}{2}-3=-3$	**f** $\frac{x}{10}+6=-2$

Example 18 **Self Tutor**

Solve for x: $\frac{x-3}{7}=-3$

$$\frac{x-3}{7}=-3$$

$$\therefore\ 7\left(\frac{x-3}{7}\right)=7\times-3 \qquad \{\text{multiplying both sides by } 7\}$$

$$\therefore\ x-3=-21$$

$$\therefore\ x-3+3=-21+3 \qquad \{\text{adding 3 to both sides}\}$$

$$\therefore\ x=-18$$

Check: LHS $=\frac{-18-3}{7}=\frac{-21}{7}=-3=$ RHS ✓

3 Solve for x:

a $\frac{x-2}{2}=5$	**b** $\frac{x+3}{3}=2$	**c** $\frac{x-5}{4}=-1$	**d** $\frac{x+4}{3}=12$
e $\frac{x-8}{-2}=3$	**f** $\frac{x+6}{3}=0$	**g** $\frac{x-7}{-3}=2$	**h** $\frac{x+4}{-4}=-1$

Example 19 **Self Tutor**

Solve for x: $3(x-4)=39$

$$3(x-4)=39$$

$$\therefore\ \frac{3(x-4)}{3}=\frac{39}{3} \qquad \{\text{dividing both sides by } 3\}$$

$$\therefore\ x-4=13$$

$$\therefore\ x-4+4=13+4 \qquad \{\text{adding 4 to both sides}\}$$

$$\therefore\ x=17$$

Check: LHS $=3(17-4)=3\times13=39=$ RHS ✓

4 Solve for x:

a $3(x-1)=12$

b $4(x+2)=24$

c $5(x-3)=25$

d $2(x+7)=8$

e $7(x-1)=0$

f $12(x-2)=24$

g $3(x-2)=9$

h $5(x+2)=-15$

i $10(x+4)=60$

j $6(x+3)=42$

k $6(x-1)=-18$

l $3(x-2)=2$

5 Solve for x:

a $7x-3=18$

b $\frac{x}{2}-1=4$

c $2(x+5)=20$

d $\frac{x-3}{4}=5$

e $\frac{x}{2}=-4$

f $3(x-6)=0$

g $\frac{x}{3}+2=-1$

h $3x+7=-5$

i $\frac{x+1}{3}=-2$

j $4x-6=14$

k $\frac{x+4}{-5}=2$

l $2(x+7)=14$

m $\frac{x+2}{4}=-\frac{1}{2}$

n $3x+15=0$

o $9(x-2)=-63$

p $\frac{x-11}{-2}=8$

q $6(x+1)=-54$

r $\frac{x}{5}-1=7$

G EQUATIONS WITH A REPEATED VARIABLE

In many equations, the variable appears more than once. In this course we consider some simple cases where the variable appears more than once on the LHS.

To solve these equations, we begin by simplifying the LHS and then isolating the unknown.

Example 20 **Self Tutor**

Solve for x: $3x+2x=65$

$$\begin{aligned} 3x+2x &= 65 \\ \therefore\quad 5x &= 65 && \{\text{collecting like terms}\} \\ \therefore\quad \frac{5x}{5} &= \frac{65}{5} && \{\text{dividing both sides by } 5\} \\ \therefore\quad x &= 13 \end{aligned}$$

EXERCISE 9G

1 Solve for x:

a $2x+x=18$

b $3x-x=24$

c $5x+2x=-63$

d $4x-x-24=0$

e $3x-4+5x=0$

f $9x-5-3x=13$

Example 21 **Self Tutor**

Solve for x: $x + (x + 10) + 50 = 180$

$$\begin{aligned} x + (x+10) + 50 &= 180 \\ \therefore\ x + x + 10 + 50 &= 180 && \{\text{expanding brackets}\} \\ \therefore\ 2x + 60 &= 180 && \{\text{collecting like terms}\} \\ \therefore\ 2x + 60 - 60 &= 180 - 60 && \{\text{subtracting 60 from both sides}\} \\ \therefore\ 2x &= 120 \\ \therefore\ \frac{2x}{2} &= \frac{120}{2} && \{\text{dividing both sides by 2}\} \\ \therefore\ x &= 60 \end{aligned}$$

2 Solve for x:

a $x + (x + 40) = 180$

b $x + (x - 30) + 50 = 180$

c $x + (x + 20) + (x + 40) = 180$

d $x + (x + 50) + (x - 20) = 180$

DISCUSSION

The equations in this chapter have exactly one solution.

Can you write down an equation which has:

- no solutions
- two solutions
- infinitely many solutions?

H WORD PROBLEMS

Most real life problems are described using sentences rather than symbols. Before we can solve these problems, we must first write the problem as an equation.

Example 22 **Self Tutor**

When a number multiplied by 3 is added to 5, the result is 23.
Write this as an algebraic equation.

Suppose we represent the number with the letter n.

Start with a number	n
multiply it by 3	$3n$
add 5	$3n + 5$
the result is 23	$3n + 5 = 23$

The algebraic equation which represents the problem is $3n + 5 = 23$.

EXERCISE 9H.1

1 Starting with the number n, write each of the following as an algebraic equation:

- **a** When a number is multiplied by 5, the result is 30.
- **b** When 10 is added to a number, the result is 23.
- **c** When a number is divided by 4, and then 6 is added, the result is 8.
- **d** When a number is subtracted from 11, and the result is divided by 3, the answer is 2.

2 Write each of these as an equation.

- **a** Paul has x coins, then his father gives him 12 more. Paul now has 27 coins.
- **b** Filipa had $\$x$ in her purse. She then spent $150 at a mall. She now has $80.
- **c** Derren is x years old. His daughter Fiona, who is one third his age, is 12 years old.
- **d** A plant was initially x cm high. In a week it doubled its height, and the next week it grew another 10 cm. It was then 31 cm high.

SOLVING WORD PROBLEMS

To solve word problems, we follow these steps:

Step 1: Let x be the unknown quantity to be found.
Step 2: Write an equation using the information in the question.
Step 3: Solve the equation.
Step 4: Write the answer to the question in a sentence.

Example 23 **Self Tutor**

I am thinking of a number. If I subtract 7 from the number, the result is 11. What is the number?

Let x be the number.

When 7 is subtracted from the number, the result is 11.

$$\therefore \quad x - 7 = 11$$

$$\therefore \quad x - 7 + 7 = 11 + 7 \qquad \{\text{adding 7 to both sides}\}$$

$$\therefore \quad x = 18$$

The number is 18.

Check: If I subtract 7 from 18, the result is $18 - 7 = 11$ ✓

EXERCISE 9H.2

1 I am thinking of a number. When 9 is added to the number, the result is 15. What is the number?

2 I am thinking of a number. When the number is tripled, the result is 4 more than 11. What is the number?

3 I am thinking of a number. When the number is halved, and then 8 is added, the result is 13. What is the number?

Example 24 **Self Tutor**

Callum has a collection of badges. His aunt gives him 9 more badges. Then, while Callum is on holidays, he collects enough to double his collection. He now has 132 badges in total. How many badges did Callum have to start with?

Let x be the number of badges Callum had to start with.

He is given 9 by his aunt, so the number is now $x+9$.

Callum doubles his collection, so he now has $2(x+9)$.

So, $2(x+9)=132$

$$\therefore \frac{2(x+9)}{2}=\frac{132}{2} \quad \{\text{dividing both sides by } 2\}$$

$$\therefore x+9=66$$

$$\therefore x+9-9=66-9 \quad \{\text{subtracting 9 from both sides}\}$$

$$\therefore x=57$$

Callum had 57 badges to start with.

4 Ethan has a box of chocolates. He ate 7 chocolates, and now there are 17 chocolates left. How many chocolates did Ethan have to start with?

5 A school band has 15 musicians and some singers. The members of the band are split into 6 groups of 4 students. How many singers are in the band?

6 At a kitchenware store, Yvonne bought a bowl for $15, as well as 4 plates. The total cost was $35. How much did each plate cost?

7 Nicholas went to the state car museum, where he saw cars and three motorbikes. In total there were 134 wheels in the museum. How many cars did Nicholas see?

8 At a birthday party, Hessa was given some balloons. Her friend Afra was given 9 balloons. The two girls decided to share their balloons equally, and now they each have 7 balloons. How many balloons was Hessa given?

9 David bought some boxes of ice blocks. There were six ice blocks in every box. When he opened the freezer he realised that his children had found them and had already eaten 3. If there are now 21 ice blocks left, how many boxes did David buy?

10 Each day a salesman is paid £80 plus one tenth of the value of the sales he makes for the day. On one day the salesman is paid £200. What value of sales did he make that day?

KEY WORDS USED IN THIS CHAPTER

- algebraic equation
- equation
- isolate
- unknown
- balance
- flowchart
- solution
- build up
- inverse operation
- undo

REVIEW SET 9A

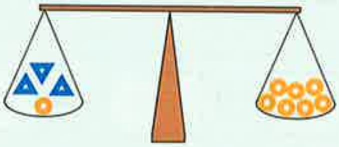
1 Decide whether these equations are true or false:

a $4 \times 7 = 30 - 2$ **b** $10 \div 2 = 14 - 7$ **c** $5 + 13 = 6 \times 3$

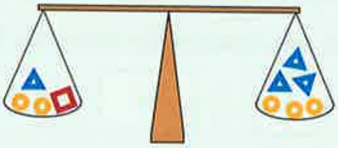
2 One of the numbers $\{6, 7, 8, 9\}$ is the solution to the equation $3x - 11 = 10$. Find the solution.

3 The following scales are perfectly balanced. Find the relationship between the objects:

a



b



4 State the inverse of the following operations:

a $-\ 7$ **b** $\div\ 8$ **c** $+\ 13$ **d** $\times\ \frac{1}{2}$

5 Find x using an inverse operation:

a $x + 4 = 17$ **b** $x - 8 = 2$ **c** $3x = -15$ **d** $\frac{x}{-2} = -12$

6 Copy and complete the following flowcharts:

a $\boxed{x} \xrightarrow{?} \boxed{2x} \xrightarrow{?} \boxed{2x - 9}$ **b** $\boxed{x} \xrightarrow{\div 3} \boxed{} \xrightarrow{+ 4} \boxed{}$

7 Use a flowchart to show how the following expressions are built up from x:

a $3(x - 4)$ **b** $\frac{x + 2}{3}$

8 Solve for x:

a $2x - 9 = -5$ **b** $\frac{x+4}{9} = -1$ **c** $\frac{x}{3} - 6 = 3$

d $\frac{x}{5} + 12 = 10$ **e** $2x + 7 = 15$ **f** $2(x - 8) = 28$

9 Charlotte saves the same amount of money each week. She starts with €12, and after 4 weeks she has €36. How much does she save each week?

10 Emma buys a packet of lollies at a cinema. She eats half of the packet, and her 3 friends share the rest. If her friends receive 6 lollies each, how many lollies were in the packet originally?

REVIEW SET 9B

1 **a** Is $x + 8 = 19 - 2$ true or false when $x = 9$?

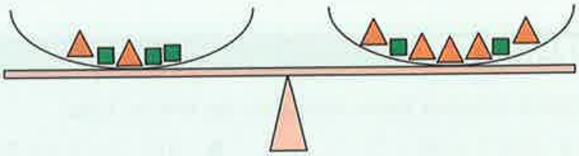
b Is $5x = 30 \div 3$ true or false when $x = 3$?

2 Find the equation which results from:

a adding 5 to both sides of $4x - 5 = 11$

b dividing both sides of $3(x - 2) = 15$ by 3.

3 This set of scales is balanced.
Find ■ if ▲ = 6.



4 Find x using an inverse operation:

a $3 + x = 7$ **b** $x - 4 = 11$ **c** $5x = -30$ **d** $\frac{x}{6} = -2$

5 When a number is divided by four, the result is 9. What is the number?

6 Copy and complete the following flowcharts:

a $\boxed{x} \xrightarrow{?} \boxed{4x} \xrightarrow{?} \boxed{4x - 7}$ **b** $\boxed{x} \xrightarrow{+\,9} \boxed{\quad\quad} \xrightarrow{\times\,2} \boxed{\quad\quad}$

7 Use a flowchart to show how to isolate x:

a $\frac{x-9}{4}$ **b** $2(x + 5)$

8 Solve for x:

a $4x + 5 = 29$ **b** $3x - 7 = -1$ **c** $\frac{x}{4} - 5 = -2$

d $3x + 8 = -25$ **e** $\frac{x-2}{5} = -4$ **f** $7(x + 3) = -49$

9 Monica asked each of her friends to buy a charity raffle ticket for £3. All but 4 of her friends bought a ticket, and she raised £18. How many friends did Monica ask?

10 Solve for x:

a $2x - 4 + 3x = 41$ **b** $x + (x + 40) + (x + 65) = 180$

Chapter 10

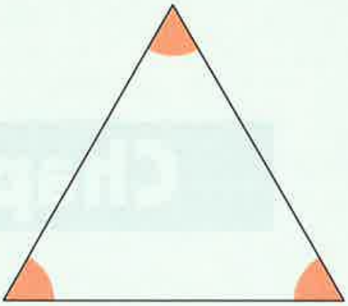
Polygons

Contents:

OPENING PROBLEM

Consider the triangle alongside.

- **a** Use a ruler to check that all of the sides are the same length.
- **b** Use a protractor to measure the size of each angle. What do you notice?
- **c** Find the sum of the angles measured in **b**.
- **d** Draw a triangle of your own, and find the sum of its angles. What do you notice?



Triangles are examples of **polygons**. We study polygons in the branch of mathematics called **plane geometry**. We are particularly interested in measuring the angles and side lengths of polygons.

A POLYGONS

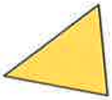
A shape that is drawn on a flat surface or plane is called a **plane figure**.

If the boundary of a shape has no beginning or end, it is said to be **closed**.

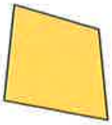
A **polygon** is a closed plane figure with straight line sides which do not cross.

Some simple examples of polygons are:

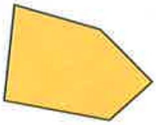
triangle
3 sides



quadrilateral
4 sides



pentagon
5 sides



Polygons are named according to the number of sides they have. For example, a 9-sided polygon can be called a 9-gon. However, many polygons are known by more familiar names, such as those in the table.

Number of sides	*Polygon name*
3	**Triangle**
4	**Quadrilateral**
5	**Pentagon**
6	**Hexagon**
7	**Heptagon**
8	**Octagon**
9	**Nonagon**
10	**Decagon**

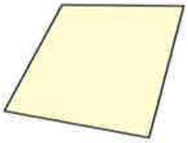
A **vertex** of a polygon is a point where two sides meet.

The plural of vertex is **vertices**.

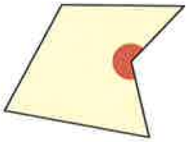
In any polygon, the number of sides equals the number of angles.

CONVEX POLYGONS

A **convex polygon** is a polygon which has no interior reflex angles.



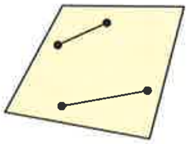
is a convex polygon.



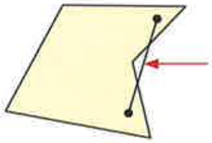
is a non-convex polygon.

Every pair of points inside a convex polygon can be joined by a straight line segment which remains inside the polygon.

For example:



All line segments between points remain inside the polygon, so the polygon is convex.



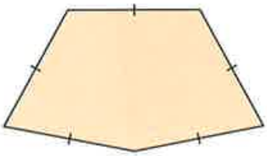
This line segment goes outside the polygon, so the polygon is not convex.

REGULAR POLYGONS

A **regular polygon** has all sides of equal length **and** all angles of equal measure.

The triangle in the **Opening Problem** is an example of a regular polygon.

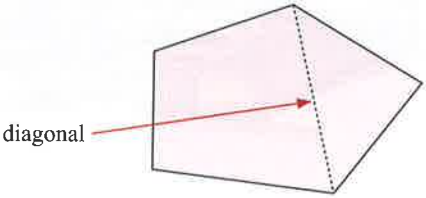
This polygon is not regular even though its sides are equal in length. Its angles are not all equal.



This polygon is not regular even though its angles are equal. Its sides are not all equal in length.

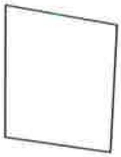
DIAGONALS OF A POLYGON

A **diagonal** of a polygon is a straight line segment which joins a pair of vertices across the polygon.



EXERCISE 10A

1 Which of these figures is a polygon?
Give a reason if the figure is not a polygon.

a


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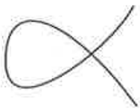
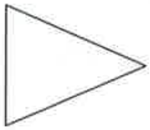
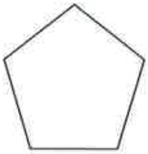
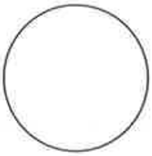
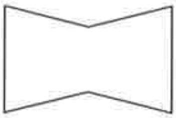


c


d


e


f


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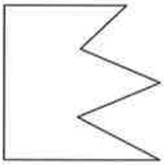

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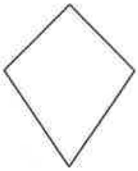

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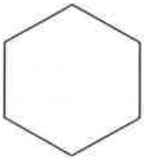

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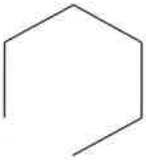


2 Write down the name given to a polygon with:

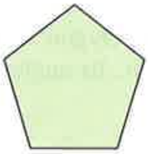
a three sides
b four sides
c six sides
d seven sides
e eight sides
f nine sides.

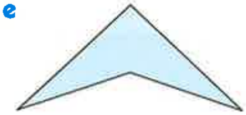
3 Name these polygons according to their number of sides and whether they are convex:

a


b


c


d


e


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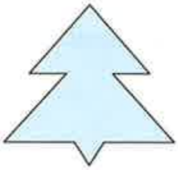

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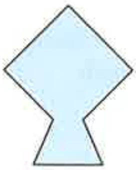

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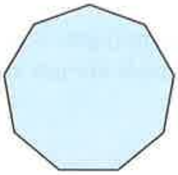
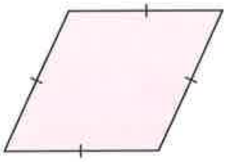


4 Sketch:

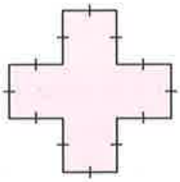
- **a** a convex 5-sided polygon
- **b** a non-convex 5-sided polygon
- **c** a convex 8-sided polygon
- **d** a non-convex 8-sided polygon.

5 Explain why these figures are not regular polygons:

a

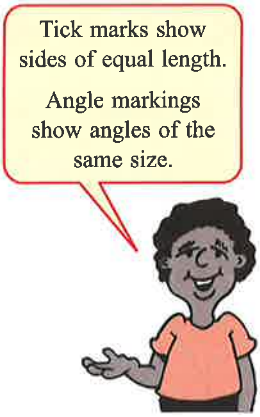


b



c

d

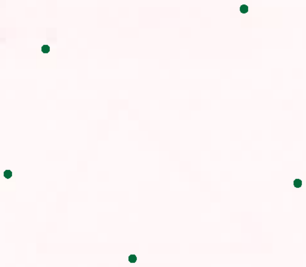


6 Sketch each of the following convex polygons, and draw all of their diagonals:

- **a** a quadrilateral
- **b** a pentagon
- **c** an octagon.

ACTIVITY CREATING POLYGONS

Suppose you are given a set of vertices. Your task is to connect the vertices to create a polygon. Is there only one way to do this?



What to do:

1 Click on the icon, and print the six sets of vertices.

SETS OF VERTICES

2 For each set of vertices, try to connect the vertices to create a polygon. You must use all of the vertices.

3 Compare your polygons with those of your classmates. Did everybody draw the same polygons?

4 Use your drawings to help answer these questions:

- **a** Is it always possible to draw a polygon with the given vertices?
- **b** Is it always possible to draw a *convex* polygon with the given vertices?
- **c** Is it ever possible to draw more than one polygon with the same set of vertices? What can you say about the convexity of the polygons if this is the case?

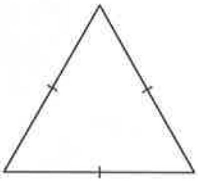
B TRIANGLES

A **triangle** is a polygon with three sides.

Triangles may be classified according to the measure of their sides or the measure of their angles.

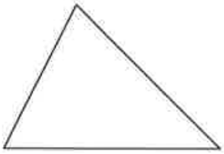
CLASSIFICATION BY SIDES

A triangle is:
- **equilateral** if its sides are all equal in length
- **isosceles** if at least two of its sides are equal in length
- **scalene** if none of its sides are equal in length.



equilateral

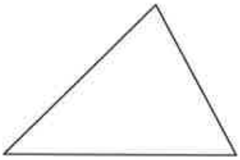
isosceles



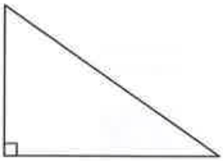
scalene

CLASSIFICATION BY ANGLES

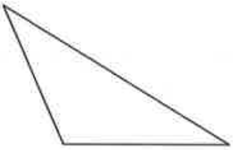
A triangle is:
- **acute angled** if *all* of its angles are acute
- **right angled** if one of its angles is a right angle (90°)
- **obtuse angled** if one of its angles is obtuse.



acute angled



right angled

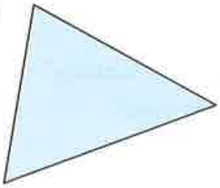


obtuse angled

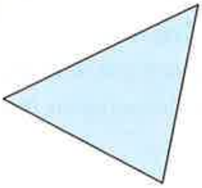
EXERCISE 10B

1 Measure the lengths of the sides of these triangles. Use your measurements to classify each triangle as equilateral, isosceles, or scalene.

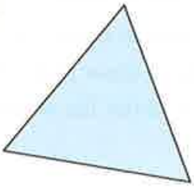
a



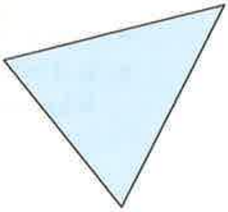
b



c

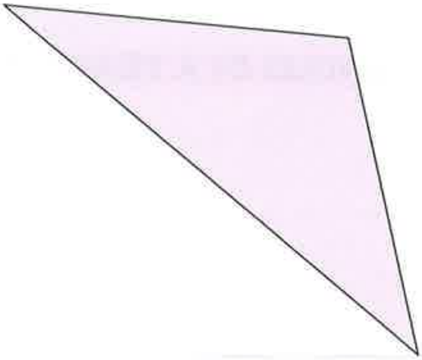


d

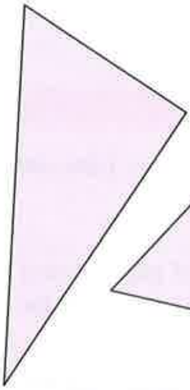


2 Measure the sizes of the angles of these triangles. Use your measurements to classify each triangle as acute, obtuse, or right angled.

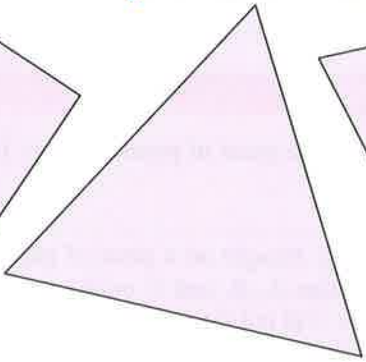
a



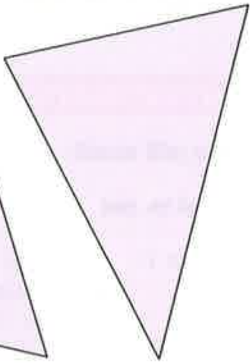
b



c

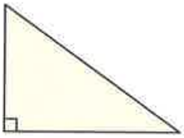


d

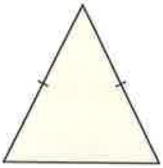


3 Use the indicated lengths of the sides *and* the sizes of the angles to classify each triangle. Include *two* descriptions for each triangle.

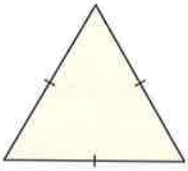
a



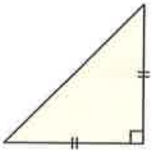
b



c



d

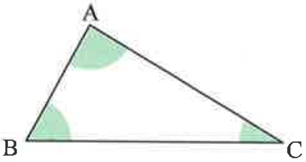


DISCUSSION

Is an equilateral triangle also isosceles?

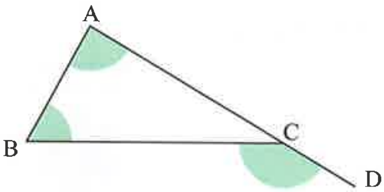
C ANGLES OF A TRIANGLE

When we talk about the angles of a triangle, we actually mean the **interior angles** or the angles *inside* the triangle.



The shaded angles are the interior angles of triangle ABC.

If we extend a side of the triangle we create an **exterior angle**.



Angle BCD is an exterior angle of triangle ABC.

DEMO

All triangles have *six* exterior angles.

There are rules or **theorems** which allow us to determine angle sizes in figures involving triangles.

ANGLE SUM OF A TRIANGLE

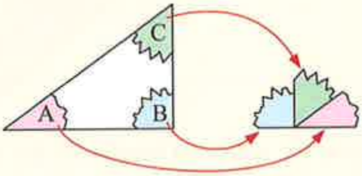
INVESTIGATION 1 — ANGLES OF A TRIANGLE

You will need: a large piece of paper, scissors, ruler, and pencil.

What to do:

Step 1: Draw any triangle on a piece of paper. Label the vertices A, B, and C on the inside of the triangle. Cut out the triangle.

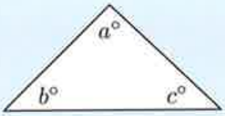
Step 2: Tear off each of the 3 angles. Place them adjacent to each other with vertices all meeting and not overlapping. What do you notice?



Step 3: Repeat this experiment with other triangles. What do you notice?

From **Investigation 1** and the **Opening Problem**, you should have discovered that:

The sum of the angles in a triangle is 180°.



$$a + b + c = 180$$

GEOMETRY PACKAGE

Proof:

Let triangle ABC have angles of a°, b°, and c° at the vertices A, B, and C.

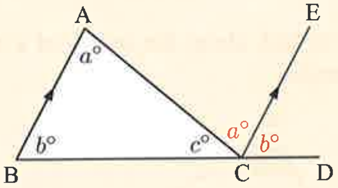
Extend [BC] to D, and draw [CE] parallel to [BA].

Now $A\widehat{C}E = B\widehat{A}C = a^\circ$ {equal alternate angles}

and $E\widehat{C}D = A\widehat{B}C = b^\circ$ {equal corresponding angles}

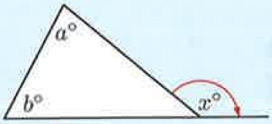
But $a + b + c = 180$ {angles on a line}

$\therefore$ $a + b + c = 180$



EXTERIOR ANGLE OF A TRIANGLE

An exterior angle of a triangle is equal in size to the sum of the interior opposite angles.



$$x = a + b$$

GEOMETRY PACKAGE

Proof:

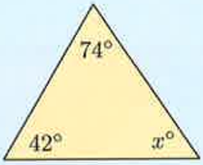
Following on from the angle sum of a triangle proof, we see that the exterior angle ACD $= a^\circ + b^\circ = a^\circ + b^\circ$, which is the sum of the interior opposite angles.

When we solve geometry problems, the diagrams are often not drawn to scale. We solve the problem using the given information on side lengths and angles, and known polygon properties.

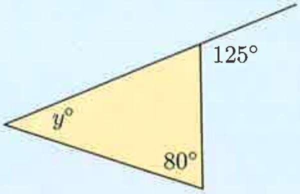
Example 1 Self Tutor

Find the unknowns in the following, giving brief reasons for your answers:

a



b



a $x + 42 + 74 = 180$ {angle sum of triangle}

$\therefore\ x + 116 = 180$

$\therefore\ x + 116 - 116 = 180 - 116$ {subtracting 116 from both sides}

$\therefore\ x = 64$

b $y + 80 = 125$ {exterior angle of triangle}

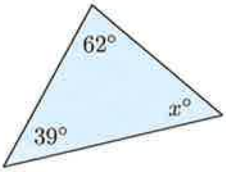
$\therefore\ y + 80 - 80 = 125 - 80$ {subtracting 80 from both sides}

$\therefore\ y = 45$

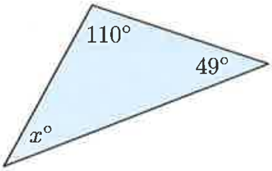
EXERCISE 10C

1 Find x:

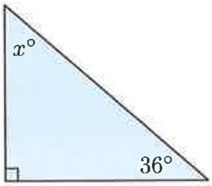
a



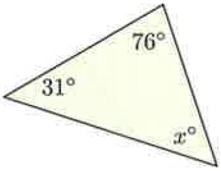
b



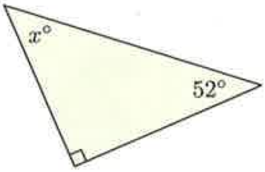
c



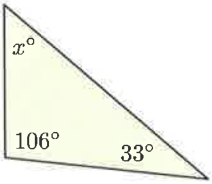
d



e

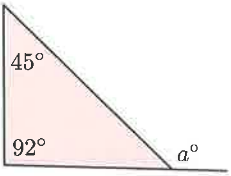


f



2 Find a:

a



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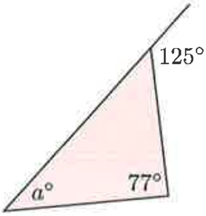

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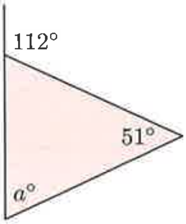

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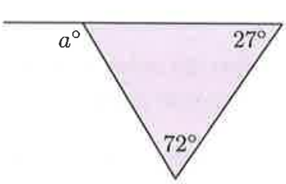
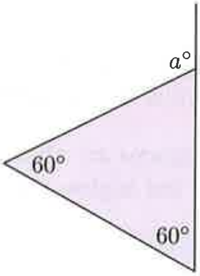


e



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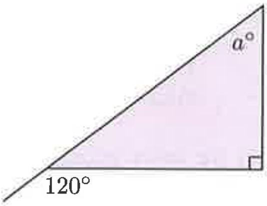
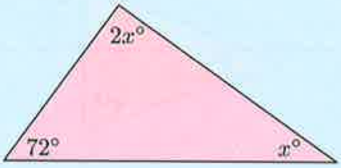


Example 2

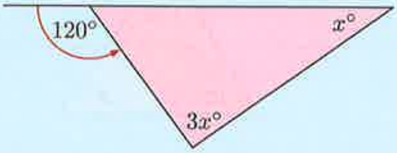
Self Tutor

Find x:

a



b



a

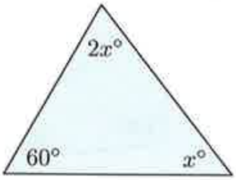
$$\begin{aligned} 2x + x + 72 &= 180 && \{\text{angle sum of triangle}\} \\ \therefore\ 3x + 72 &= 180 && \{\text{collecting like terms}\} \\ \therefore\ 3x + 72 - 72 &= 180 - 72 && \{\text{subtracting 72 from both sides}\} \\ \therefore\ 3x &= 108 \\ \therefore\ \frac{3x}{3} &= \frac{108}{3} && \{\text{dividing both sides by 3}\} \\ \therefore\ x &= 36 \end{aligned}$$

b

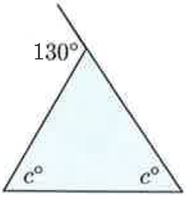
$$\begin{aligned} 3x + x &= 120 && \{\text{exterior angle of triangle}\} \\ \therefore\ 4x &= 120 && \{\text{collecting like terms}\} \\ \therefore\ \frac{4x}{4} &= \frac{120}{4} && \{\text{dividing both sides by 4}\} \\ \therefore\ x &= 30 \end{aligned}$$

3 Find the unknowns in these triangles:

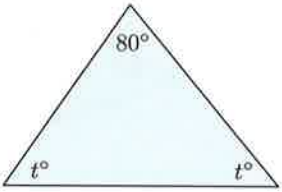
a



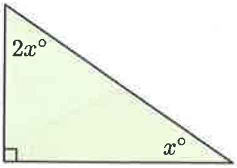
b



c



d



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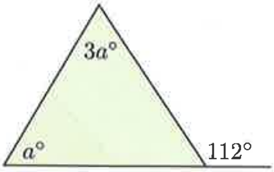

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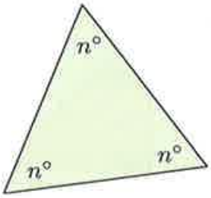
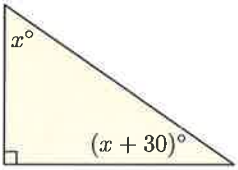


4 Explain why it is not possible to draw a triangle which has:

a two obtuse angles
b one obtuse angle and one right angle
c all angles less than 60°.

5 Find x:

a



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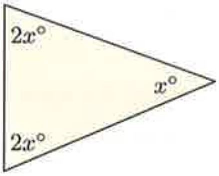
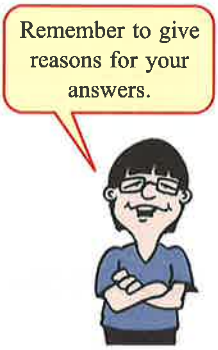
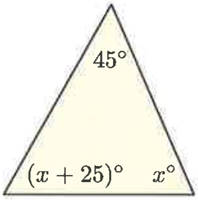
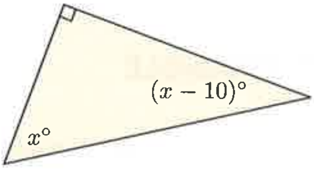




c



d



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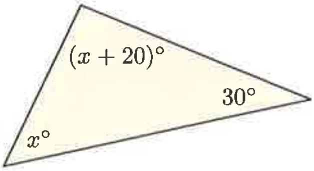

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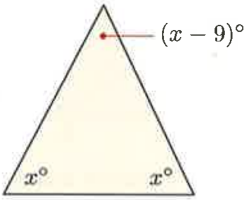
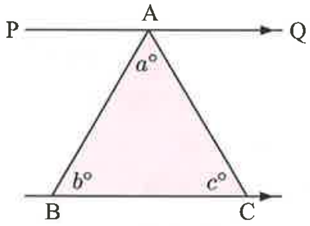


6 The figure shown can be used to prove the angles of a triangle theorem.

Copy and complete:

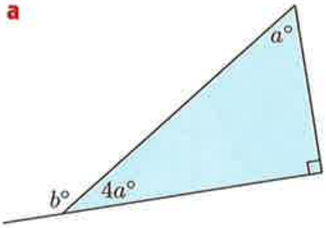
$\widehat{QAC}= ......$ {equal alternate angles}
$\widehat{PAB}= ......$ {equal alternate angles}

Now $\widehat{PAB}+\widehat{BAC}+\widehat{QAC}= ......$ {angles on a line}
So, $a+b+c= ......$

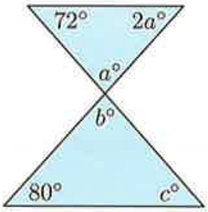


7 Find, in alphabetical order, the values of the unknowns:

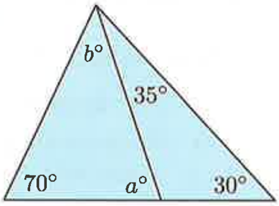
a



b



c

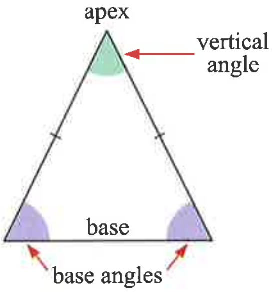


D ISOSCELES TRIANGLES

An **isosceles triangle** is a triangle which has at least two sides equal in length.

We label parts of an isosceles triangle as follows:

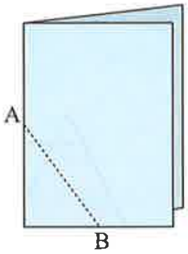
- the third side is called the **base**
- the vertex between the equal sides is called the **apex**
- the angle at the apex is called the **vertical angle**
- the angles opposite the equal sides are called the **base angles**.

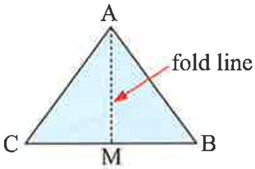


MAKING AN ISOSCELES TRIANGLE

We can make an isosceles triangle from a sheet of paper.

Fold the paper in half, then draw a straight line [AB] as shown. With the two sheets pressed tightly together, cut along [AB] through both sheets.





Keep the triangular piece of paper.

When you unfold it, you should obtain the isosceles triangle ABC shown.

DISCUSSION

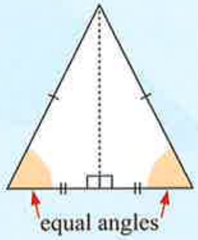
In the triangle ABC above, explain why:

- AB = AC
- $A\widehat{C}B = A\widehat{B}C$
- M is the midpoint of [BC]
- [AM] is at right angles to [BC].

From the demonstrations above we conclude that:

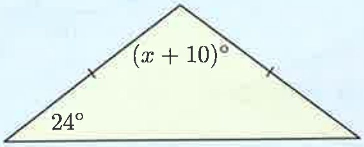
In any isosceles triangle:

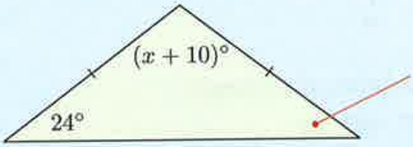
- the base angles are equal
- the line joining the apex to the midpoint of the base is perpendicular to the base.



Example 3 **Self Tutor**

Find x:





Since the triangle is isosceles, the base angles are equal in size.

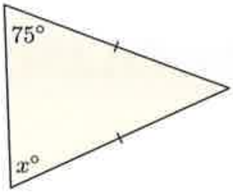
$\therefore$ this angle is also 24°.

$$\therefore\ (x+10)+24+24=180 \quad \{\text{angle sum of triangle}\}$$
$$\therefore\ x+58=180$$
$$\therefore\ x+58-58=180-58$$
$$\therefore\ x=122$$

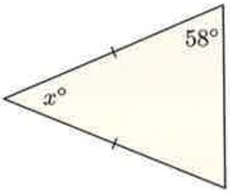
EXERCISE 10D

1 The following diagrams have not been drawn to scale, but the information given on them is correct. Use the information to find the unknowns:

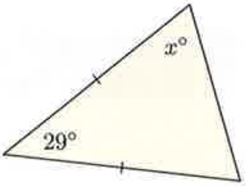
a



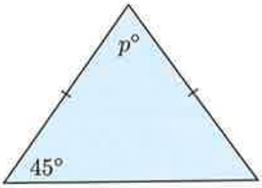
b



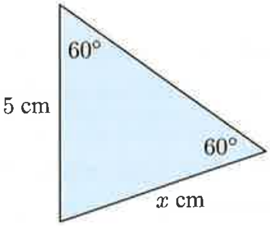
c



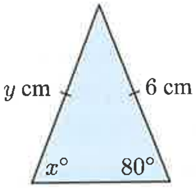
d



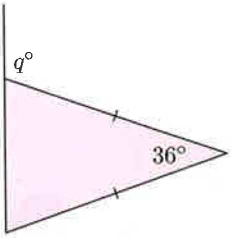
e



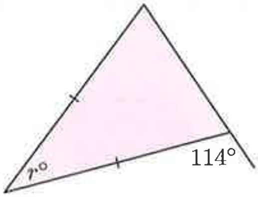
f



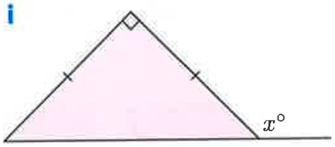
g



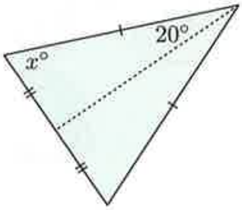
h



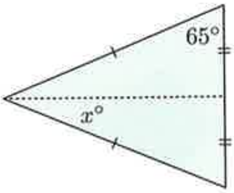
i



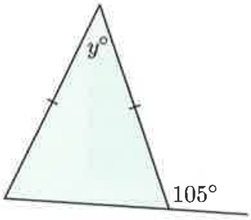
j



k

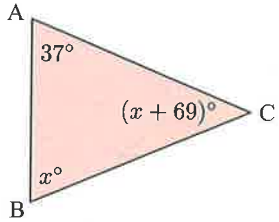


l

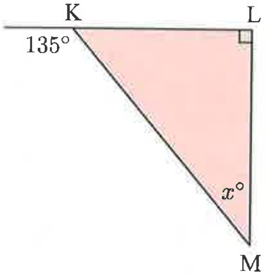


2 In each of the following triangles, find the value of x. Hence deduce something about the triangle.

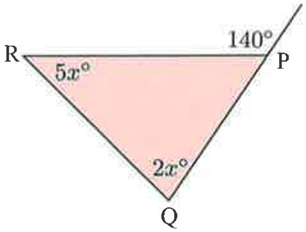
a



b



c



3 Match each set of angle sizes with the corresponding description of a triangle:

- **a** isosceles and acute angled
- **b** scalene and obtuse angled
- **c** scalene and right angled
- **d** isosceles and obtuse angled
- **e** scalene and acute angled
- **f** equilateral and acute angled
- **g** isosceles and right angled.

- **A** 25°, 90°, 65°
- **B** 50°, 60°, 70°
- **C** 90°, 45°, 45°
- **D** 125°, 35°, 20°
- **E** 40°, 100°, 40°
- **F** 40°, 70°, 70°
- **G** 60°, 60°, 60°

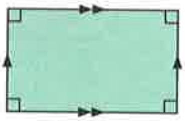
E QUADRILATERALS

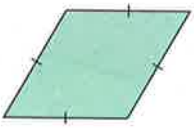
A **quadrilateral** is a polygon with four sides.

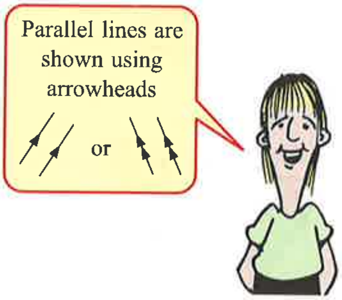
There are six special quadrilaterals:

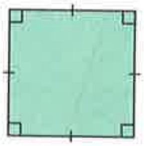
- A **parallelogram** is a quadrilateral which has opposite sides parallel.
- A **rectangle** is a parallelogram with four equal angles of 90°.
- A **rhombus** is a quadrilateral in which all sides are equal.
- A **square** is a rhombus with four equal angles of 90°.
- A **trapezium** is a quadrilateral which has exactly one pair of opposite sides parallel.
- A **kite** is a quadrilateral which has two pairs of adjacent sides equal.


parallelogram

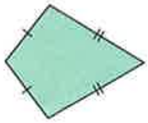

rectangle


rhombus




square


trapezium


kite

INVESTIGATION 2 PROPERTIES OF QUADRILATERALS

What to do:

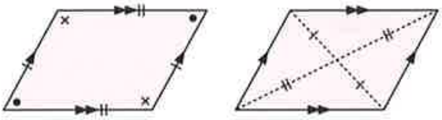
PRINTABLE WORKSHEET

1. Print the worksheets by clicking on the icon.
2. For each **parallelogram**:
   - **a** Measure the lengths of the opposite sides. What do you notice?
   - **b** Measure the sizes of the opposite angles. What do you notice?
   - **c** Draw the two diagonals. Measure the distances from each vertex to the point of intersection of the diagonals. What do you notice?
3. For each **rectangle**:
   - **a** Measure the lengths of the opposite sides. What do you notice?
   - **b** Measure the lengths of the diagonals. What do you notice?
   - **c** Copy and complete:
     A rectangle is a parallelogram with diagonals that are ....
4. For each **rhombus**:
   - **a** Check that opposite sides are parallel.
   - **b** Measure the sizes of the opposite angles. What do you notice?
   - **c** Draw the two diagonals. Measure the distances from each vertex to the point of intersection of the diagonals. What do you notice?
   - **d** At what angle do the diagonals intersect?
   - **e** Fold the rhombus along each diagonal. What do you notice about the angles formed?
5. For each **square**:
   - **a** Check that opposite sides are parallel.
   - **b** Fold the square along each diagonal. What do you notice about the angle where the diagonals intersect?
   - **c** What else do you notice about the diagonals?
6. For the **kite**:
   - **a** Measure the sizes of the opposite angles. What do you notice?
   - **b** Fold the kite along its diagonals. What do you notice?

From the **Investigation**, you should have discovered these properties of special quadrilaterals:

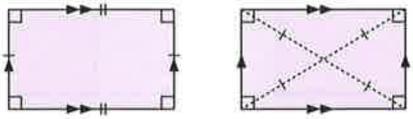
Parallelogram

- opposite sides are equal in length
- opposite angles are equal in size
- diagonals bisect each other (divide each other in half).

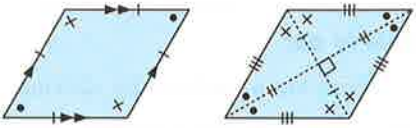


Rectangle

- opposite sides are equal in length
- diagonals are equal in length
- diagonals bisect each other.

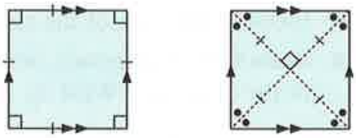


Rhombus



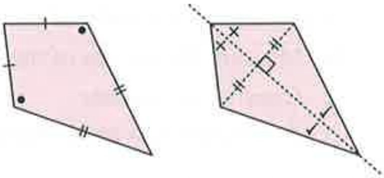
- opposite sides are parallel
- opposite angles are equal in size
- diagonals bisect each other at right angles
- diagonals bisect the angles at each vertex.

Square



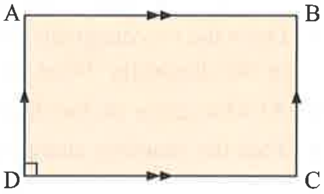
- opposite sides are parallel
- diagonals bisect each other at right angles
- diagonals bisect the angles at each vertex.

Kite



- *one* pair of opposite angles is equal in size
- diagonals cut each other at right angles
- *one* diagonal bisects one pair of angles at the vertices
- symmetrical about *one* diagonal.

PARALLEL AND PERPENDICULAR LINES



In the figure we notice that [AB] is **parallel** to [DC].
We write this as [AB] || [DC].

[AD] is at right angles or **perpendicular** to [DC].

We write this as [AD] ⊥ [DC].

> || reads *is parallel to.*
>
> ⊥ reads *is perpendicular to.*

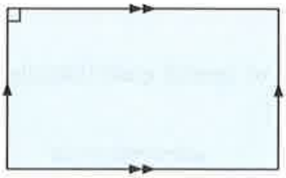
EXERCISE 10E

1 Sketch, and fully label:

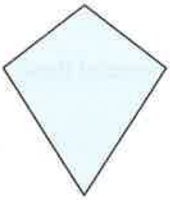
a a parallelogram **b** a rhombus **c** a kite.

2 Use a ruler to help classify the following:

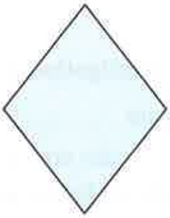
a



b

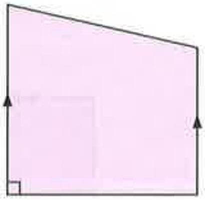


c

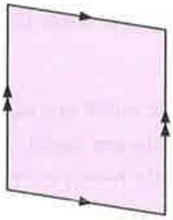


d

e



f

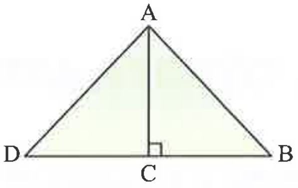


3 True or false?

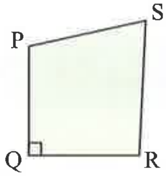
- **a** A parallelogram is a quadrilateral which has opposite sides parallel.
- **b** A rectangle is a parallelogram with four equal angles of 90°.
- **c** A rhombus is a quadrilateral in which all sides are equal.
- **d** A square is a rhombus with four equal angles of 90°.
- **e** A trapezium is a quadrilateral which has a pair of opposite sides parallel.
- **f** A kite is a quadrilateral which has two pairs of adjacent sides equal.
- **g** A kite has one pair of opposite angles equal.
- **h** The diagonals of a rhombus bisect each other at right angles and bisect the angles of the rhombus.

4 Using || and ⊥, write statements about the following figures:

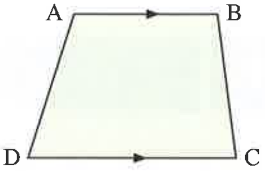
a



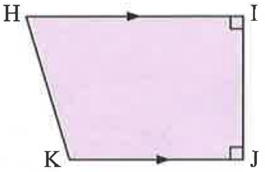
b



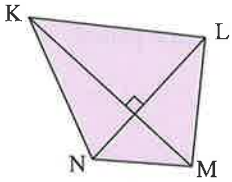
c



d



e



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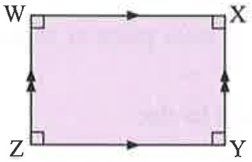
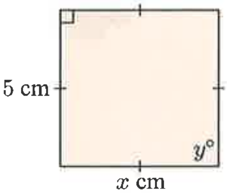


5 Sketch a figure to illustrate each of the following sets of information. Separate sketches are needed in each case. Make sure you label all of the given information.

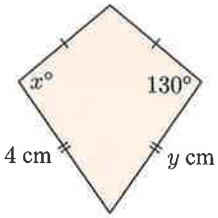
- **a** [AB] is 2 cm long. [AC] is 3 cm long. [AB] ⊥ [AC].
- **b** [KL] is 4 cm long. [MN] is 5 cm long. [KL] || [MN] and [KL] is 2 cm from [MN].
- **c** ABCD is a quadrilateral in which [AB] || [DC] and [AD] || [BC].
- **d** PQRS is a quadrilateral in which [PQ] is 3 cm long, [RS] is 4 cm long, and [PS] is 2 cm long. [PQ] || [SR], and [QP] ⊥ [PS].

6 Find the values of the variables in these figures, giving reasons for your answers:

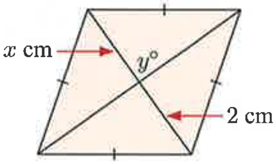
a



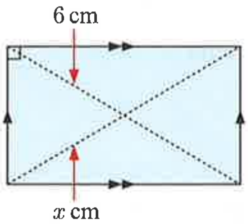
b



c



d



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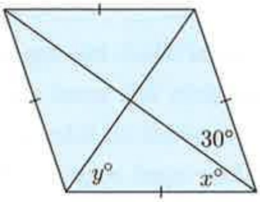

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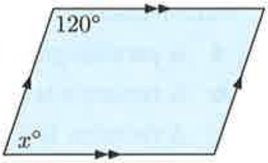
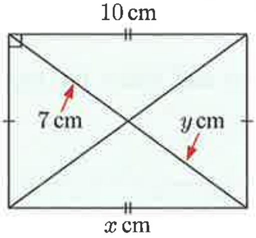
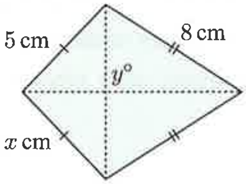


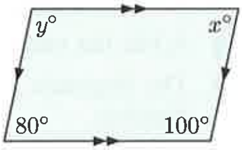
g



h



i



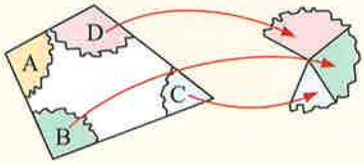
F ANGLES OF A QUADRILATERAL

INVESTIGATION 3 ANGLES OF A QUADRILATERAL

You will need:

a large piece of paper, scissors, ruler, and pencil.

What to do:



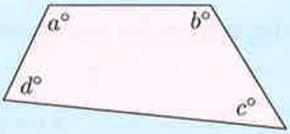
Step 1: Draw any quadrilateral on a piece of paper. Label the vertices A, B, C, and D on the inside of the quadrilateral. Cut out the quadrilateral.

Step 2: Tear off each of the 4 angles. Place them adjacent to each other with vertices all meeting and not overlapping. What do you notice?

Step 3: Repeat this experiment with a few other quadrilaterals. What do you notice?

From **Investigation 3** you should have discovered that:

The sum of the angles of a quadrilateral is 360°.



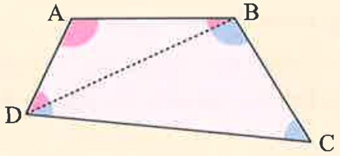
$$a + b + c + d = 360$$

GEOMETRY PACKAGE

Proof:

Suppose we divide quadrilateral ABCD into the two triangles ABD and BCD.

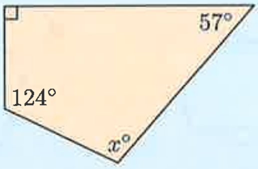
The sum of the interior angles of ABCD
$= \text{sum of angles of } \triangle ABD + \text{sum of angles of } \triangle BCD$
$= 180^\circ + 180^\circ$
$= 360^\circ$



Example 4

Self Tutor

Find x:



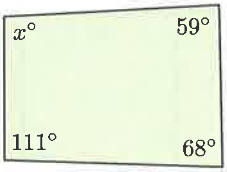
The sum of the angles of a quadrilateral is 360°.

$$x + 57 + 90 + 124 = 360$$
$$\therefore \quad x + 271 = 360$$
$$\therefore \quad x + 271 - 271 = 360 - 271$$
$$\therefore \quad x = 89$$

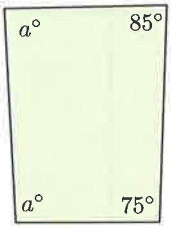
EXERCISE 10F

1 Find the values of the variables:

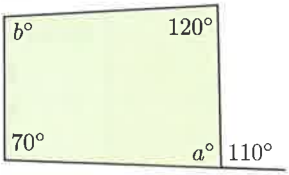
a



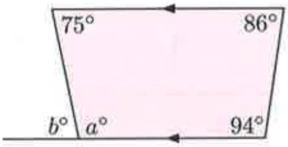
b



c

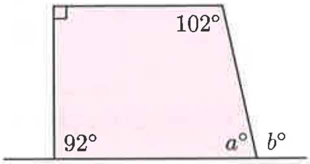


d



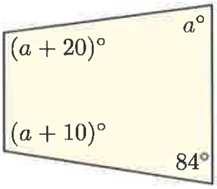
e

f

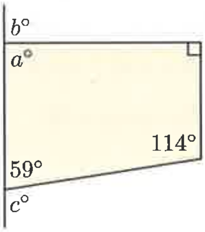


2 Find the values of the variables:

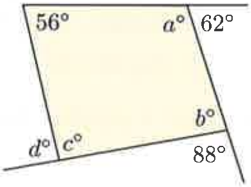
a



b



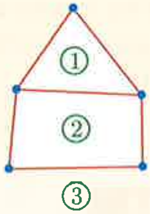
c



INVESTIGATION 4 EULER'S RULE

In this Investigation we will find a connection between the number of vertices, edges, and regions of any plane figure.

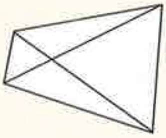
For example, this figure has 5 vertices, 6 edges, and 3 regions.
Notice that we include the region outside the figure.



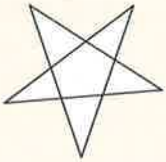
What to do:

1 Consider the following figures:

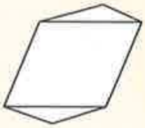
a



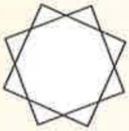
b



c



d



Copy and complete the following table. **e** to **h** are for four diagrams of figures like those above, but of your choice.

Figure	*Vertices* (V)	*Regions* (R)	*Edges* (E)	$V+R-2$
Given example	5	3	6	6
a				
b				
c				
d				
e				
f				
g				
h				

Comment on your results.

You should have found that, for any figure, $E = V + R - 2$. This is known as **Euler's rule**.

2 Use Euler's rule to find the number of:

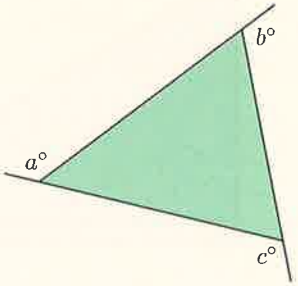
- **a** edges for a figure with 5 vertices and 4 regions
- **b** vertices for a figure with 6 edges and 5 regions
- **c** regions for a figure with 10 edges and 8 vertices.

3 Draw a possible figure for each of the cases in **2**.

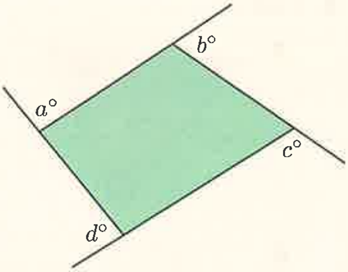
INVESTIGATION 5 EXTERIOR ANGLES OF POLYGONS

What to do:

1 Draw any triangle and measure one exterior angle from each vertex. These are a, b, and c in the figure alongside. Find the sum of these angles.
Repeat this procedure with two other triangles of your choice.
Discuss your results.



2 Draw any quadrilateral and measure one exterior angle from each vertex. Find the sum of the exterior angles.
Repeat this procedure with two other quadrilaterals.
Discuss your results.



3 Predict a value for the sum of the exterior angles of any polygon. Check your value by drawing any pentagon and hexagon, measuring each of the exterior angles, and calculating their sum.

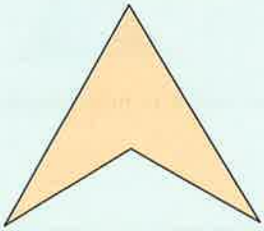
KEY WORDS USED IN THIS CHAPTER

- acute
- convex polygon
- Euler's rule
- isosceles
- parallelogram
- quadrilateral
- rhombus
- square
- vertex
- apex
- diagonal
- exterior angle
- kite
- plane figure
- rectangle
- right angled
- trapezium
- vertical angle
- base angles
- equilateral
- interior angle
- obtuse
- polygon
- regular polygon
- scalene
- triangle
- vertices

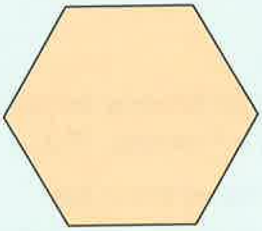
REVIEW SET 10A

1 Name these polygons according to their number of sides and whether they are convex:

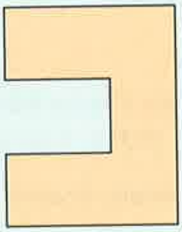
a



b

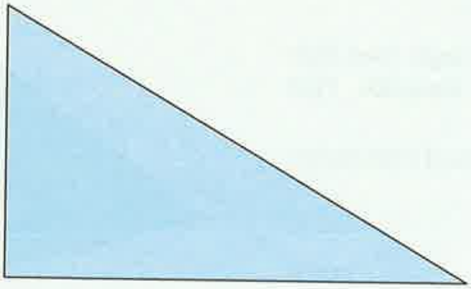


c

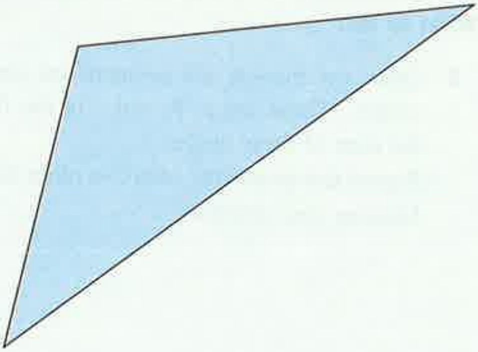


2 Use a protractor to classify the following triangles as acute, obtuse, or right angled.

a

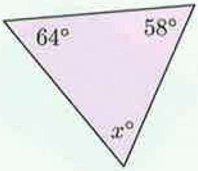


b

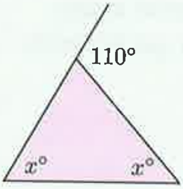


3 Find x:

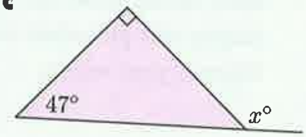
a



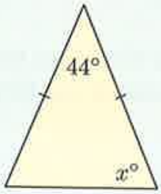
b



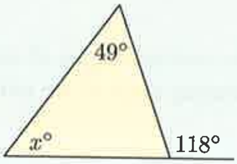
c



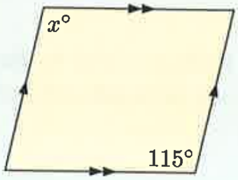
d



e

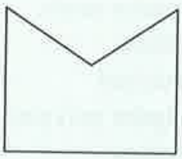


f

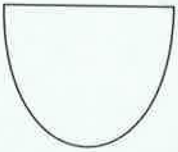


4 Which of these figures is a polygon?

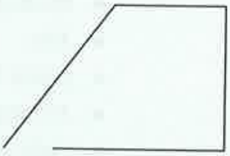
a



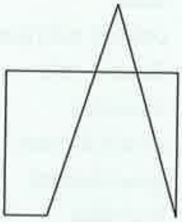
b



c

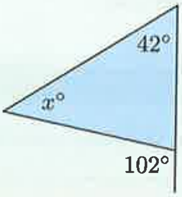


d

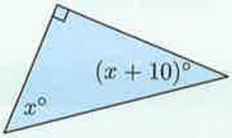


5 Find x:

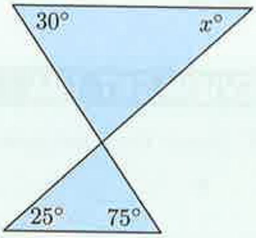
a



b



c



6 Sketch a figure that illustrates the following information. Make sure it is fully labelled.
[PQ] is 3 cm long. [PR] is 4 cm long. [PQ] ⊥ [PR].

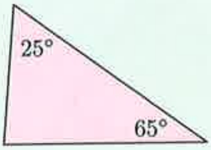
7 Determine whether the following are true or false:

a All squares are kites.

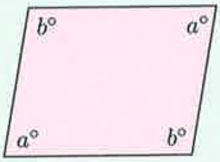
b All parallelograms are rectangles.

8 Using the information given, name each of the following figures. Give reasons for your answers.

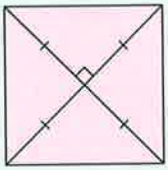
a



b

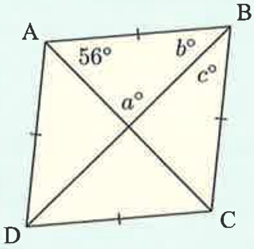


c



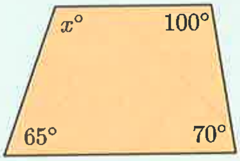
9 ABCD is a rhombus. Find the value of:

a a **b** b **c** c

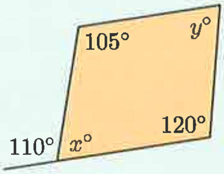


10 Find the values of the variables:

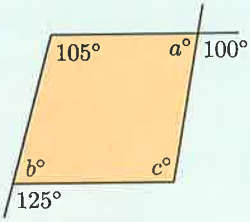
a



b



c



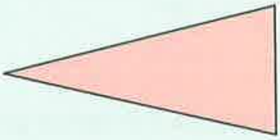
REVIEW SET 10B

1 Sketch a hexagon which is:

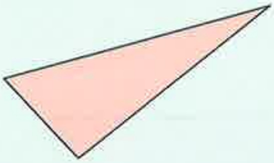
a regular **b** convex but not regular **c** non-convex.

2 Use a ruler to classify the following triangles as equilateral, isosceles, or scalene:

a

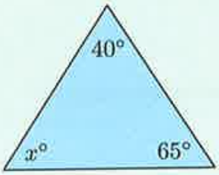


b

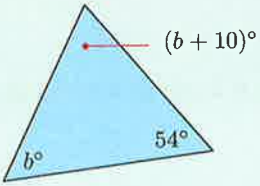


3 Find the values of the variables in the following:

a

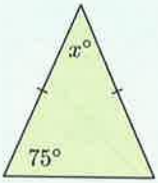


b

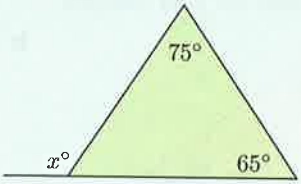


4 Find x:

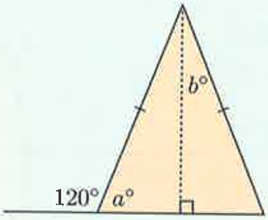
a



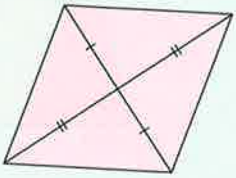
b



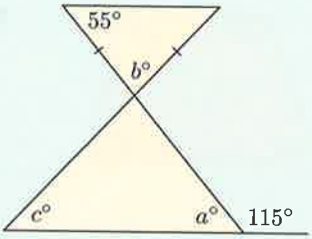
5 Find the values of a and b.



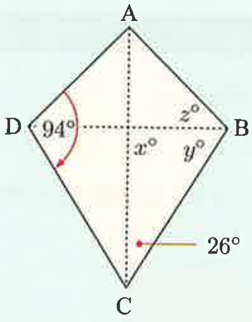
6 Name the following figure using the information given.
Give reasons for your answer.



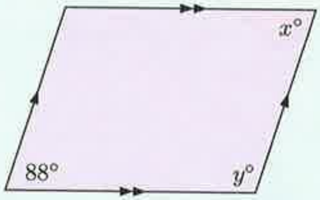
7 Find the values of a, b, and c.



8 ABCD is a kite.
Find the values of x, y, and z.



9



a Name this figure using the information given.

b Find the values of x and y.

10 Explain why it is not possible to draw a quadrilateral which has all acute angles.

Chapter 11

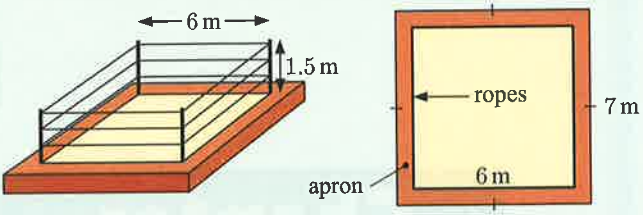
Measurement: Length and area

Contents:

OPENING PROBLEM

A boxing ring has the dimensions shown. There are 3 ropes on each side of the ring, and each rope is 6 m long. The ropes are connected to the corner posts which are 1.5 m high.

The ring is 7 m long on all sides. The part of the ring outside the ropes is called the *apron*.



Things to think about:

- **a** What is the total length of the ropes?
- **b** What is the area of the ring inside the ropes?
- **c** What is the area of the apron?

Measurements of length, area, volume, capacity, and time enable us to answer questions such as:

- How far is it to work?
- How big is the swimming pool?
- How long does the bus take to get to school?
- How many litres of petrol did you buy?

We also use measurements to compare quantities.

The **metric system** of units was developed in France in 1789. It is an easy system to work with because it uses powers of 10 for all conversions. Common prefixes are used when naming related units.

Greek prefixes are used to make the base units **larger**.

For example,

kilo	means	1000
mega	means	1 000 000

Latin prefixes are used to make the base units **smaller**.

For example,

centi	means	$\frac{1}{100}$
milli	means	$\frac{1}{1000}$

The metric system is more correctly called **Le Système International d'Unités** or **SI** for short.

A LENGTH

The **metre** (m) is the base unit for length in the metric system.

A metre is about the average length of an adult's stride.

HISTORICAL NOTE

Originally the **metre** was defined as one ten-millionth of the distance from the north pole to the equator along the line of longitude through Paris, France. After difficult and exhaustive surveys, a piece of platinum alloy was prepared to this length and called the **standard metre**.

The standard metre was kept at the International Bureau of Weights and Measures at Sèvres, near Paris. However, this meant it was not easily accessible to scientists around the world.

From 1960 to 1983, the metre was defined as 1 650 763.73 wavelengths of orange-red light from the isotope Krypton 86, measured in a vacuum.

Finally, in 1983 the metre was redefined as the distance light travels in a vacuum in $\frac{1}{299\,792\,458}$ of a second.

From the metre, other units of length were devised to measure smaller and larger distances:

1 **kilometre** (km) = 1000 metres (2.5 laps of a running track)

1 **centimetre** (cm) = $\frac{1}{100}$ metre (about the width of a fingernail)

1 **millimetre** (mm) = $\frac{1}{1000}$ metre (about the width of a coin)

or $\frac{1}{10}$ centimetre

CONVERSION OF LENGTH UNITS

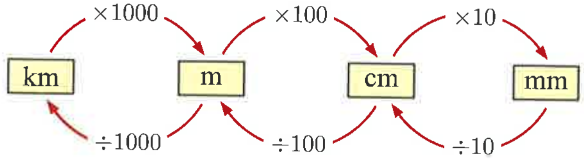
When we convert from a **larger** unit to a **smaller** unit there will be more of the smaller units, so we must **multiply**.

For example, 2 m = (2 × 100) cm = 200 cm.

When we convert from a **smaller** unit to a **larger** unit there will be less of the larger units, so we must **divide**.

For example, 2000 m = (2000 ÷ 1000) km = 2 km.

In the SI system of units we must multiply or divide by powers of 10.



1 km = 1000 m

1 m = 100 cm

1 cm = 10 mm

Example 1 **Self Tutor**

Express in centimetres:

a 3.2 m **b** 423 mm **c** 6 km

a 3.2 m
$= (3.2 \times 100)$ cm
$= 320$ cm

b 423 mm
$= (423 \div 10)$ cm
$= 42.3$ cm

c 6 km
$= (6 \times 1000)$ m
$= 6000$ m
$= (6000 \times 100)$ cm
$= 600\,000$ cm

EXERCISE 11A

1 Give the unit you would use to measure:

a the distance of a plane flight **b** the width of a staple
c the height of a tree **d** the length of a pen
e the length of an oval **f** the width of a blade of grass.

2 Choose the correct answer:

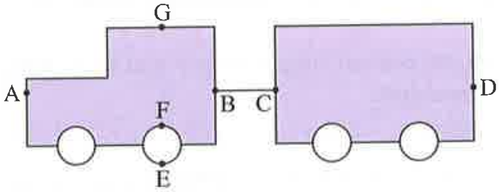
a The length of a car would be:
A 50 cm **B** 5 m **C** 5 mm **D** 500 mm

b The length of a mosquito would be:
A 11 cm **B** 1.1 m **C** 11 mm **D** 11 km

c The distance from London to Berlin would be:
A 9000 m **B** 9 km **C** 900 km **D** 900 m

3 The diagram shows a model train. Use your ruler to measure the following lengths in millimetres:

a the length of the engine [AB]
b the length of the carriage [CD]
c the distance between the engine and the carriage [BC]
d the total length [AD]
e the height of a wheel [EF]
f the train's height [EG].



4 Express in centimetres:
a 4 m **b** 20 mm **c** 2.9 m **d** 3 km

5 Express in metres:
a 800 cm **b** 7 km **c** 120 km **d** 32 000 mm

6 Express in millimetres:
a 9 cm **b** 3 m **c** 120 cm **d** 450 m

7 Express in kilometres:
a 15 000 m **b** 750 000 cm **c** 600 m **d** 2 500 000 mm

8 Convert to the units shown:

- **a** 6 cm = ... mm
- **b** 700 cm = ... m
- **c** 3000 m = ... km
- **d** 80 mm = ... cm
- **e** 11 m = ... cm
- **f** 4 km = ... m
- **g** 3.2 cm = ... mm
- **h** 240 cm = ... m
- **i** 3800 m = ... km
- **j** 17 mm = ... cm
- **k** 7.8 m = ... cm
- **l** 0.6 km = ... m

Example 2 **Self Tutor**

Find the sum of 3 km + 350 m + 220 cm in metres.

3 km + 350 m + 220 cm
= 3000 m + 350 m + 2.2 m {converting to metres}
= 3352.2 m

9 Find the sum of:

- **a** 4 m + 50 cm + 95 mm in centimetres
- **b** 5 m + 12 cm + 7 mm in centimetres
- **c** 3 km + 430 m + 220 cm in metres
- **d** 8 km + 920 m + 650 cm in metres.

Example 3 **Self Tutor**

Calculate the number of 50 m laps Ian swims in his 4.2 km pool session.

We first convert the 4.2 km to m.

$$4.2 \text{ km} = (4.2 \times 1000) \text{ m}$$
$$= 4200 \text{ m}$$

$$\therefore \text{ the number of laps Ian swims} = \frac{4200 \text{ m}}{50 \text{ m}}$$
$$= 84$$

10 Calculate the number of 8 m pipes required to lay a 320 km pipeline.

11 Tyler stacks 28 dominoes, one on top of the other. Each domino is 7 mm high. Calculate the height of the stack, in centimetres.

12 Find the number of 7.5 m lengths of string which can be cut from a 1.5 km reel.

13 Marie's average step length is 90 cm. In one day she took 12 000 steps. How many kilometres did she walk?

B PERIMETER

In English the word *perimeter* refers to the boundary of a region.

For example, we say that:

- the boundary line of a hockey field is the playing *perimeter*
- the boundary of a prison is protected by a *perimeter* fence.

However, in mathematics the word *perimeter* refers to the distance around a figure.

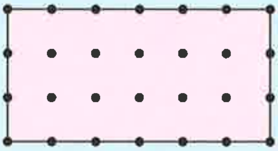
The **perimeter** of a closed figure is a measurement of the distance around the boundary of the figure.

The perimeter of a polygon can be found by adding the lengths of its sides.

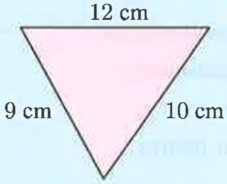
Example 4 **Self Tutor**

Find the perimeter of these polygons:

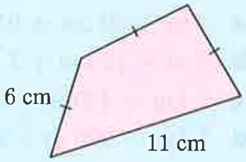
a



b



c



a Perimeter
$= 6 + 3 + 6 + 3$ units
$= 18$ units

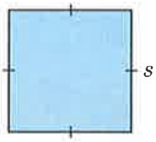
b Perimeter
$= 9 + 10 + 12$ cm
$= 31$ cm

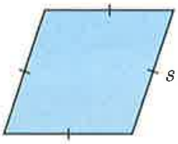
c Perimeter
$= 11 + (3 \times 6)$ cm
$= 11 + 18$ cm
$= 29$ cm

We can derive **formulae** for the perimeters of common quadrilaterals:

A **formula** is a rule which connects two or more variables. **Formulae** is the plural of formula.

SQUARES AND RHOMBUSES





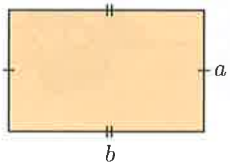
All 4 sides are equal in length.

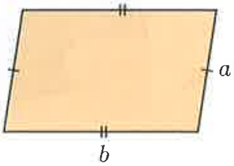
perimeter = 4 × side length

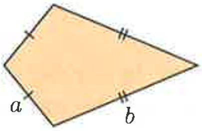
We write this as the formula:

$$P = 4s$$

RECTANGLES, PARALLELOGRAMS, AND KITES







These polygons have 2 pairs of sides of equal length.

perimeter $= a + b + a + b$

We write this as the formula:

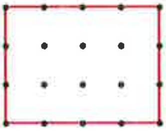
$$P = 2(a + b)$$

EXERCISE 11B

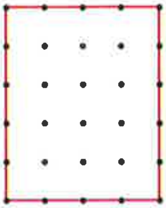
1 Find the perimeter of each figure:

a

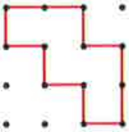
b



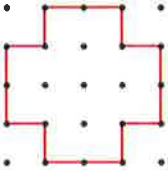
c



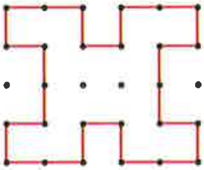
d



e

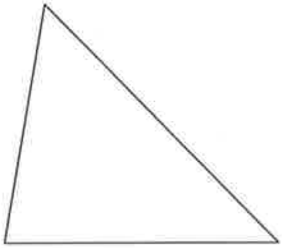


f

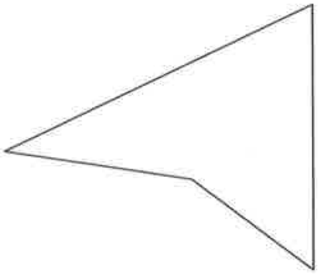


2 Use your ruler to measure the side lengths of each figure, and hence find its perimeter.

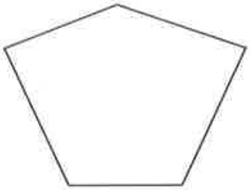
a



b

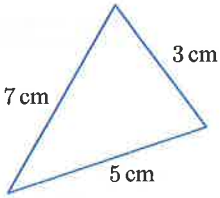


c



3 Find the perimeter of each figure:

a



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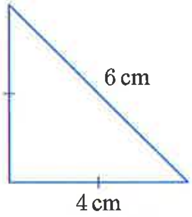

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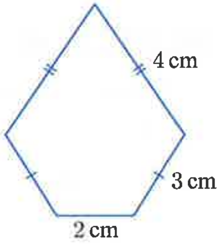

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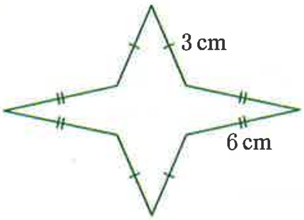

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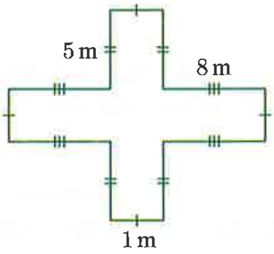

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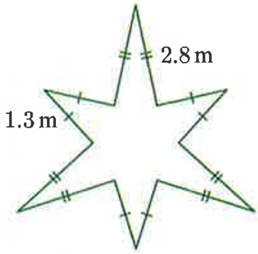
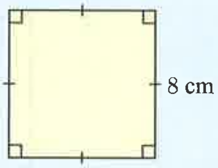


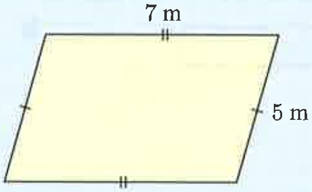
Example 5 Self Tutor

Find the perimeter of:

a



b



a $P = 4s$

$\therefore\ P = 4 \times 8$ cm

$\therefore\ P = 32$ cm

b $P = 2(a + b)$

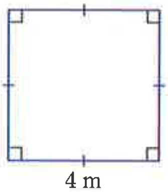
$\therefore\ P = 2(7 + 5)$ m

$\therefore\ P = 2 \times 12$ m

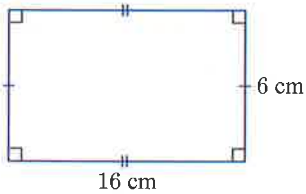
$\therefore\ P = 24$ m

4 Use the formulae to find the perimeter of each figure:

a



b



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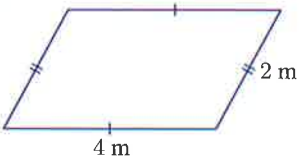

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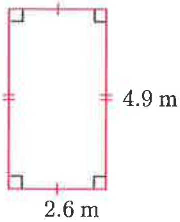

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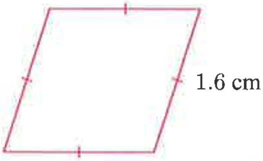

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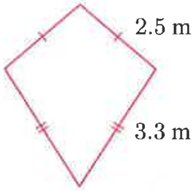
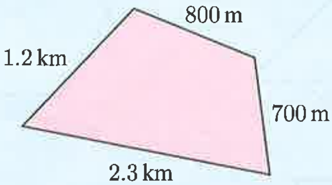


Example 6 Self Tutor

Calculate the perimeter of:

a



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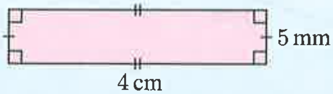


a $P = 2.3$ km $+ 1.2$ km $+ 800$ m $+ 700$ m {adding all sides}

$\therefore\ P = 2300$ m $+ 1200$ m $+ 800$ m $+ 700$ m {converting to m}

$\therefore\ P = 5000$ m

b $P = 2(a + b)$ {perimeter of a rectangle}

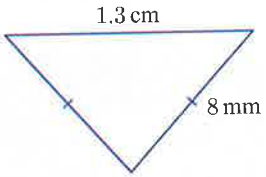
$\therefore\ P = 2(40 + 5)$ mm {converting to mm}

$\therefore\ P = 2 \times 45$ mm

$\therefore\ P = 90$ mm

5 Calculate the perimeter of each figure:

a



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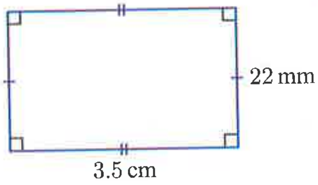

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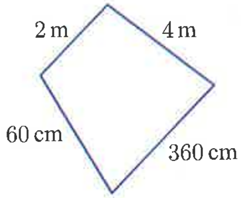

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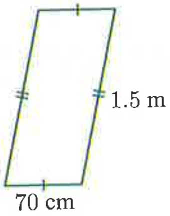

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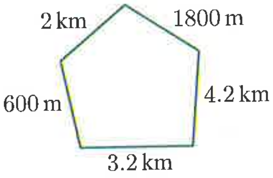

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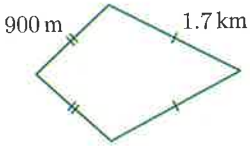
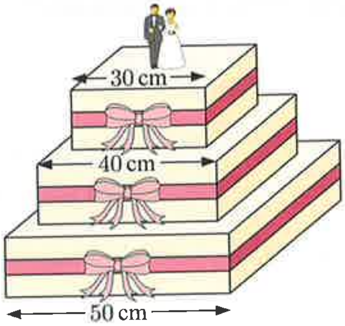


6 A piece of glass 150 cm long by 90 cm wide is placed within an 8 cm wide metal frame to make a table top. Find the perimeter of:

a the glass **b** the table top.

7 A square field has sides of length 850 metres. Find the cost of fencing the field with 3 strands of wire if the wire costs $1.35 per metre.

8 A wedding cake has three square layers. The lengths of these layers are 30 cm, 40 cm, and 50 cm. A ribbon is placed around each layer, and is tied with a bow. Allowing 20 cm for each bow, how much ribbon is needed in total?



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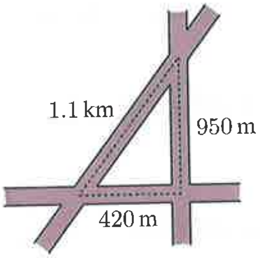


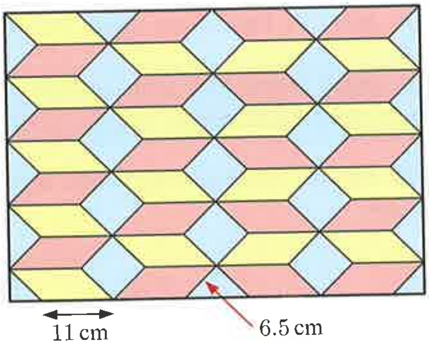
A cyclist trains by cycling around the triangular block shown. If she completes 14 laps in training, how far has she cycled? Give your answer in kilometres.

10 A stained-glass window is created from a combination of squares, parallelograms, and triangles.

a Find the perimeter of each:

i square **ii** parallelogram.

b Find the total length of glue strips required to join the pieces of glass together.



ACTIVITY

COURT LINES

What to do:

1 Research the dimensions of the courts that these sports are played on:

- **a** badminton
- **b** basketball
- **c** netball
- **d** tennis
- **e** volleyball.

Draw a diagram of each court, including all dimensions.

2 Find, in metres, the outer perimeter of each court in **1**.

3 Find, in metres, the total length of the lines of each court in **1**.

4 Write the courts in order of least to most paint required for lines.

5 The new school gymnasium will have 1 basketball court, 4 badminton courts, and 2 volleyball courts. Find the total length of lines to be painted.

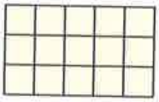
C AREA

All around your school there are many flat surfaces such as paths, floors, desk tops, ceilings, walls, and courts for playing sport. All of these surfaces have boundaries which define their shape.

The **area** of a region is the amount of **surface** within its boundaries.

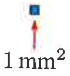
The area of a closed figure is the number of square units it encloses.

For example, the rectangle alongside has an area of 15 square units.



In the metric system, the units we use for the measurement of area are related to the units we use for length.

1 **square millimetre** (mm^2) is the area enclosed by a square of side length 1 mm.



The area of a computer chip might be measured in mm^2.

1 **square centimetre** (cm^2) is the area enclosed by a square of side length 1 cm.

The area of a book cover might be measured in cm^2.

1 **square metre** (m^2) is the area enclosed by a square of side length 1 m.

The area of a brick paving would be measured in m^2.

1 **hectare** (ha) is the area enclosed by a square of side length 100 m.

Larger areas are often measured in hectares.

Tennis stadium at Crandon Park, Miami, Florida, USA

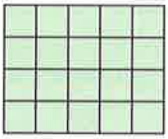
1 **square kilometre** (km^2) is the area enclosed by a square of side length 1 km.

The area of a country or continent would be measured in km^2.

EXERCISE 11C.1

1 Determine the shaded area in the following, giving your answers in square units:

a



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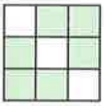

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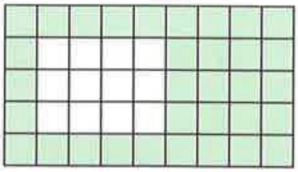

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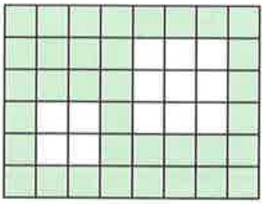


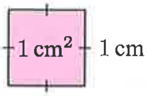
2 State the units of measurement that would be most appropriate for measuring:

- **a** the size of the floor of a room
- **b** the area of a sheet of paper
- **c** the area of Tanzania
- **d** the size of a dot on a die
- **e** the area of a farm.

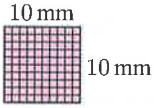
CONVERSION OF AREA UNITS

We can use the length unit conversions to help us convert from one area unit to another.

We have already seen that a square with sides of length 1 cm has area 1 cm^2:



We could also measure the sides of this square in millimetres:



Each of the small squares has area 1 mm^2.

There are $10 \times 10 = 100$ square millimetres in the square, so $1 \text{ cm}^2 = 100 \text{ mm}^2$.

Likewise,

$$\begin{aligned} 1\text{ m}^2 &= 1\text{ m} \times 1\text{ m} \\ &= 100\text{ cm} \times 100\text{ cm} \\ &= 10\,000\text{ cm}^2 \end{aligned}$$

$$\begin{aligned} 1\text{ ha} &= 100\text{ m} \times 100\text{ m} \\ &= 10\,000\text{ m}^2 \end{aligned}$$

$$\begin{aligned} 1\text{ km}^2 &= 1\text{ km} \times 1\text{ km} \\ &= 1000\text{ m} \times 1000\text{ m} \\ &= 1\,000\,000\text{ m}^2 \\ &= 100 \times 10\,000\text{ m}^2 \\ &= 100\text{ ha} \end{aligned}$$

$$\begin{array}{ll} 1\text{ cm}^2 = 100\text{ mm}^2 & 1\text{ ha} = 10\,000\text{ m}^2 \\ 1\text{ m}^2 = 10\,000\text{ cm}^2 & 1\text{ km}^2 = 100\text{ ha} \end{array}$$

AREA UNIT CONVERSIONS

$$\begin{tikzcd}
\boxed{\text{km}^2} \arrow[r, bend left, "\times 100"] & \boxed{\text{ha}} \arrow[r, bend left, "\times 10\,000"] \arrow[l, bend left, "\div 100"] & \boxed{\text{m}^2} \arrow[r, bend left, "\times 10\,000"] \arrow[l, bend left, "\div 10\,000"] & \boxed{\text{cm}^2} \arrow[r, bend left, "\times 100"] \arrow[l, bend left, "\div 10\,000"] & \boxed{\text{mm}^2} \arrow[l, bend left, "\div 100"]
\end{tikzcd}$$

Example 7 **Self Tutor**

Convert:

a 2.2 ha to m^2 **b** 540 mm^2 to cm^2

a
$$\begin{aligned} &2.2\text{ ha} \\ &= (2.2 \times 10\,000)\text{ m}^2 \\ &= 22\,000\text{ m}^2 \end{aligned}$$

b
$$\begin{aligned} &540\text{ mm}^2 \\ &= (540 \div 100)\text{ cm}^2 \\ &= 5.4\text{ cm}^2 \end{aligned}$$

To convert from larger to smaller units we multiply.

To convert from smaller to larger units we divide.

EXERCISE 11C.2

1 Convert:

a 5 cm^2 to mm^2 **b** 2500 mm^2 to cm^2 **c** 7 ha to m^2

d 3.6 m^2 to cm^2 **e** 0.4 km^2 to ha **f** 83 cm^2 to mm^2

g 80 ha to m^2 **h** 15 600 cm^2 to m^2 **i** 1200 ha to km^2

j 900 mm^2 to cm^2 **k** 76 000 m^2 to ha **l** 2.8 cm^2 to mm^2

m 0.25 km^2 to ha **n** 12.48 m^2 to cm^2 **o** 0.0092 m^2 to mm^2

2 A photograph has area 150 cm^2. Express this area in mm^2.

3 Three farmers Alessio, Bruno, and Carlos own blocks of land with the following areas:
Alessio 2.15 km^2, Bruno 320 ha, Carlos 640 000 m^2.
Which farmer owns the:

a largest block **b** smallest block?

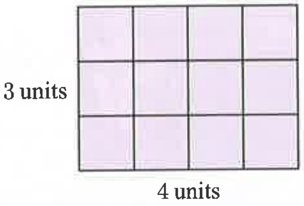
4 A bag of fertiliser recommends that 0.06 kg of fertiliser are applied to every m^2 of lawn. The gardener of a school has 5.7 ha of lawn to manage. He wants to buy fertiliser in 25 kg bags, each costing €19.85.

- **a** Convert the area to be fertilised into m^2.
- **b** How much fertiliser will be required?
- **c** How much will the gardener spend on fertiliser?

D AREAS OF POLYGONS

Dividing shapes into unit squares and then counting these unit squares is not a very convenient way of calculating areas. Instead, we use formulae to find the areas of polygons such as rectangles, triangles, parallelograms, and trapezia.

RECTANGLES



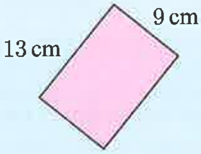
Consider a rectangle 4 units long and 3 units wide. The area of this rectangle is 12 units^2, and we can find this by multiplying $4 \times 3 = 12$.

Area of rectangle = length × width

Example 8 **Self Tutor**

Find the area of each rectangle:

a



b

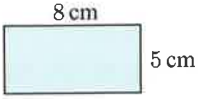
a Area
$= \text{length} \times \text{width}$
$= 13 \text{ cm} \times 9 \text{ cm}$
$= 117 \text{ cm}^2$

b Area
$= \text{length} \times \text{width}$
$= 1.5 \text{ m} \times 0.8 \text{ m}$
$= 1.2 \text{ m}^2$

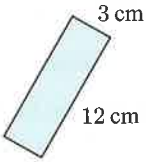
EXERCISE 11D.1

1 Find the area of each rectangle:

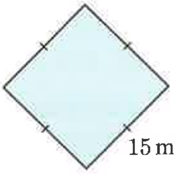
a



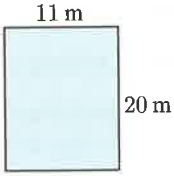
b



c



d



2 **a** Estimate the area of an A4 sheet of paper in square centimetres.

b Measure the dimensions of the sheet. Use your measurements to calculate the area of the sheet to the nearest square centimetre. How close was your estimate?

Example 9 **Self Tutor**

A hallway is 8 m long and 90 cm wide.

a Find the area of the hallway in m^2.

b If carpet costs $40 per square metre, how much will it cost to carpet the hallway?

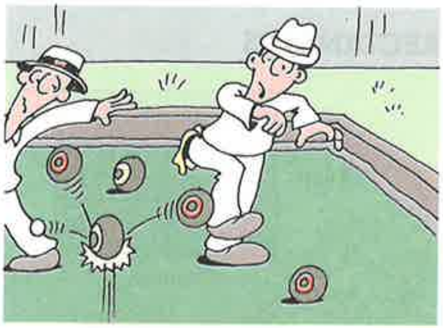
a Area $=$ length $\times$ width
$= 8 \text{ m} \times 0.9 \text{ m}$ {converting to metres}
$= 7.2 \text{ m}^2$

b Cost $= \$40 \times 7.2$
$= \$288$

3 A lawn bowls club has a green with dimensions 40 m by 60 m.

a Find the area of the green.

b How long will it take to mow the whole green if 30 square metres can be mowed each minute?



4 The top face of brick pavers are 15 cm by 25 cm.

a How many pavers would you need to pave a 3 m by 9 m driveway?

b If each paver costs £5.50, find their total cost.

5 A wheat field is 7.2 km by 2000 m. Find the:

a area of the field in hectares

b value of the wheat if the farmer earns $1200 per ha.

Example 10 **Self Tutor**

A rectangle is 13 m long. Its area is 65 m^2. Find the width of the rectangle.

Let the width of the rectangle be x m.

Now area $=$ length $\times$ width

$\therefore\ 65 = 13 \times x$

$\therefore\ \frac{65}{13} = x$ {dividing both sides by 13}

$\therefore\ x = 5$

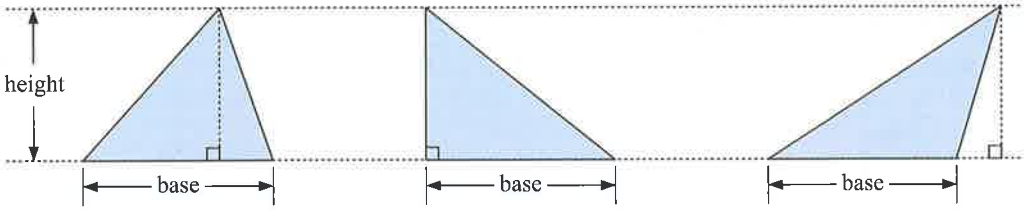
The width of the rectangle is 5 m.

6 A rectangle is 8 cm long. It has area 48 cm^2. Find the width of the rectangle.

7 A rectangle has area 84 m^2. It is 7 m wide. Find the length of the rectangle.

8 According to local rules, advertising signs on a street must be no larger than 12 m^2. Neville wants his sign to be 2.5 m high. What is the maximum length his sign can be?

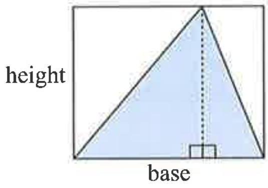
TRIANGLES

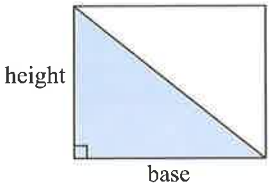


$$\textbf{Area of triangle} = \tfrac{1}{2}(\textbf{base} \times \textbf{height})$$

DEMO

The first two cases are demonstrated easily by drawing a rectangle with the same base and height as the triangle.

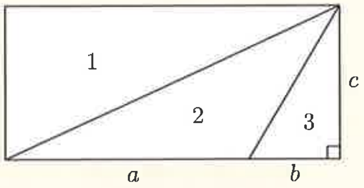




$$\text{Area} = \tfrac{1}{2}(\text{area of rectangle})$$
$$= \tfrac{1}{2}(\text{base} \times \text{height})$$

The third case is demonstrated in the **Investigation** below.

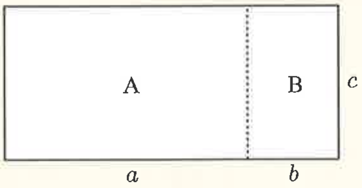
INVESTIGATION — AREA OF A TRIANGLE



Alongside are two rectangles with length $(a+b)$ and height c.

The first rectangle has been divided into three triangles.

The second rectangle has been divided into two smaller rectangles A and B.



What to do:

1 In terms of a, b, and c, write down formulae for:

a area A **b** area B **c** area 3

2 Notice that area 2 + area 3 = area 1

$$= \tfrac{1}{2} \text{ of the complete rectangle}$$
$$= \tfrac{1}{2} \text{ of area A} + \tfrac{1}{2} \text{ of area B}$$

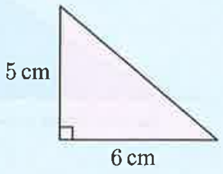
Use your formulae in **1** to copy and complete:

$$\text{area 2} + \text{area 3} = \tfrac{1}{2} \ldots\ldots + \tfrac{1}{2} \ldots\ldots$$
$$\therefore \ \text{area 2} + \ldots\ldots = \ldots\ldots + \ldots\ldots$$
$$\therefore \ \text{area 2} = \ldots\ldots$$
$$= \tfrac{1}{2}(\text{base} \times \text{height})$$

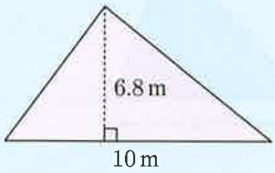
Example 11 **Self Tutor**

Find the area of each triangle:

a



b



a Area $= \frac{1}{2}(\text{base} \times \text{height})$

$\therefore \; A = \frac{1}{2}(6 \times 5) \text{ cm}^2$

$\therefore \; A = 15 \text{ cm}^2$

b Area $= \frac{1}{2}(\text{base} \times \text{height})$

$\therefore \; A = \frac{1}{2}(10 \times 6.8) \text{ m}^2$

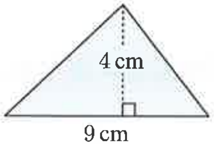
$\therefore \; A = \frac{1}{2} \times 68 \text{ m}^2$

$\therefore \; A = 34 \text{ m}^2$

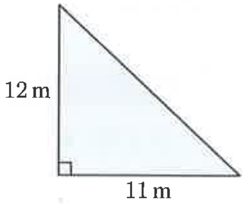
EXERCISE 11D.2

1 Find the area of each triangle:

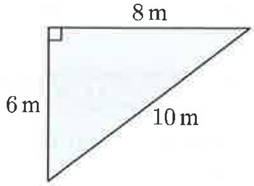
a



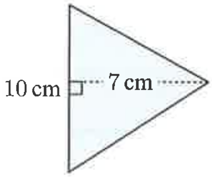
b



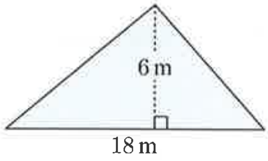
c



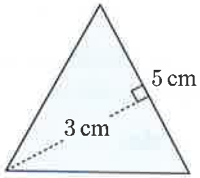
d



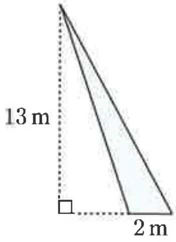
e



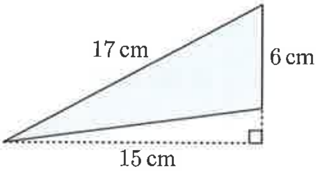
f



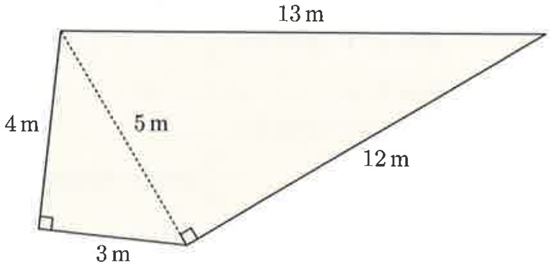
g



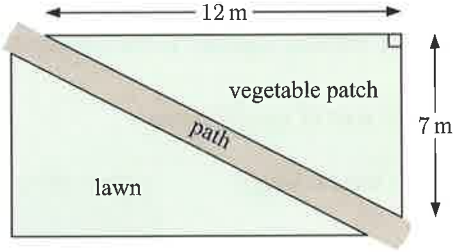
h



2 Find the area of this quadrilateral by considering it as the sum of the areas of two triangles:

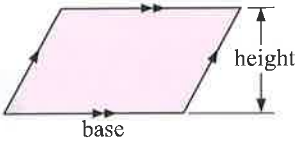


3 A path cuts across a backyard, as shown. Find the area of the vegetable patch.



4 A triangle has area 15 cm^2, and its base has length 5 cm. Find the height of the triangle.

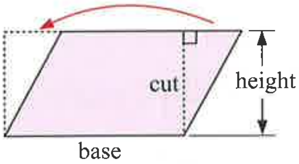
PARALLELOGRAMS

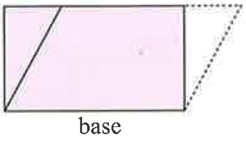


Area of parallelogram = base × height

We can demonstrate this formula by cutting out a triangle from one end of the parallelogram and shifting it to the other end. The resulting shape is a rectangle with the same base and height as the parallelogram.

DEMO



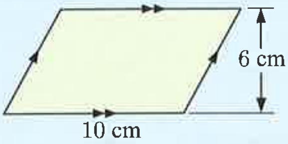


Perform this demonstration for yourself using paper and scissors.

Example 12

Self Tutor

Find the area of:

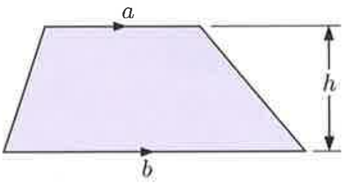


Area = base × height

$\therefore\ A = 10 \text{ cm} \times 6 \text{ cm}$

$\therefore\ A = 60 \text{ cm}^2$

TRAPEZIA

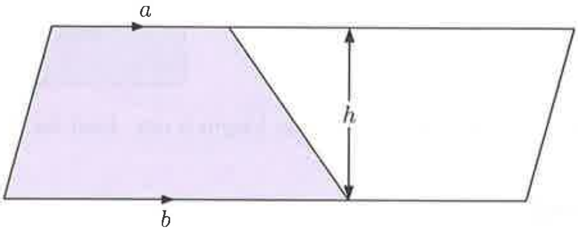


Area = average length of parallel sides × distance between parallel sides

or **Area** $= \left(\dfrac{a+b}{2}\right) \times h$

We can demonstrate this result using a second identical trapezium. We place the trapezia together to form a parallelogram.

$$\begin{aligned}\text{Area} &= \frac{1}{2} \text{ of area of parallelogram}\\ &= \frac{1}{2} \times \text{base} \times \text{height}\\ &= \frac{1}{2} \times (a+b) \times h\end{aligned}$$



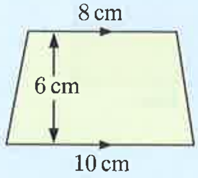
Perform this demonstration for yourself using paper and scissors.

DEMO

Example 13

Self Tutor

Find the area of the trapezium:



$$\text{Area} = \left(\frac{a+b}{2}\right) \times h$$

$$\therefore \quad A = \left(\frac{8+10}{2}\right) \times 6 \text{ cm}^2$$

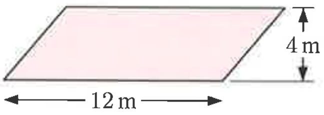
$$\therefore \quad A = 9 \times 6 \text{ cm}^2$$

$$\therefore \quad A = 54 \text{ cm}^2$$

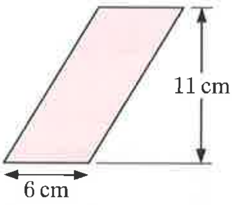
EXERCISE 11D.3

1 Find the area of each parallelogram:

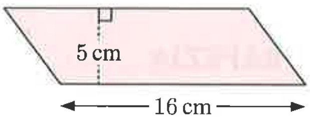
a



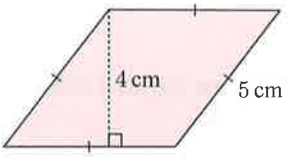
b



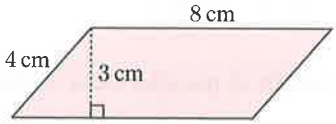
c



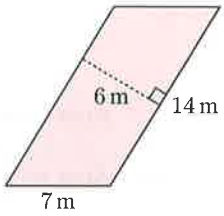
d



e

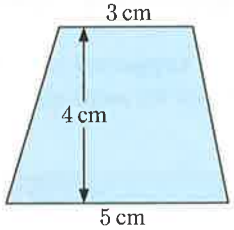


f



2 Find the area of each trapezium:

a



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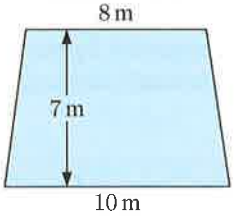
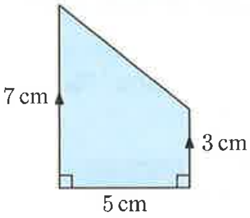


c



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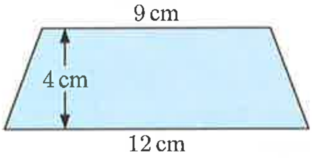
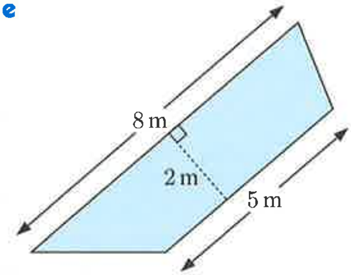
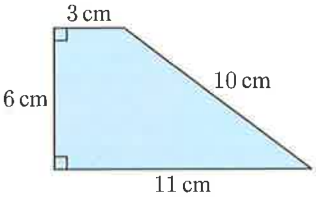


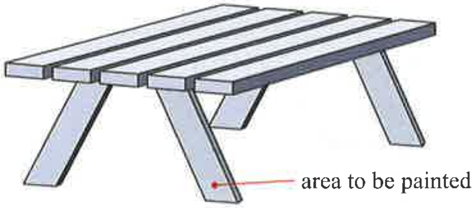
e



f

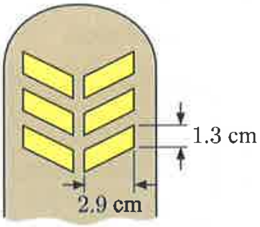


3



The surface of the park bench leg indicated must be repainted due to graffiti. The leg is 10 cm wide at the base, and the top of the leg is 60 cm above the ground. Find the area that needs to be repainted.

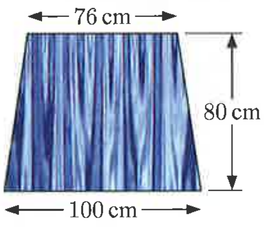
4 Find the total area of the stripes on this army soldier's uniform.



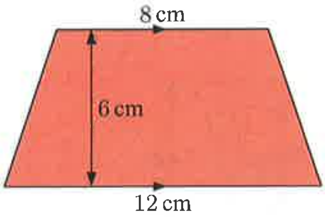
5 Joan is using a trapezium-shaped piece of material to make a skirt.

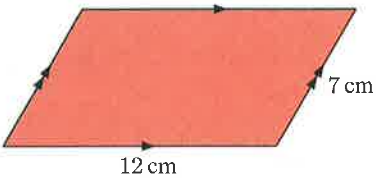
a Find the area of the material.

b The material is worth £16 per square metre. Find the total value of the material.



6





These figures have the same area. Find the height of the parallelogram.

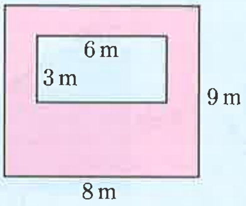
E AREAS OF COMPOSITE FIGURES

The figures in this Section are called **composite figures**. They are made up or **composed** of two or more standard figures. Their areas can be calculated using addition and subtraction of the areas of the standard figures.

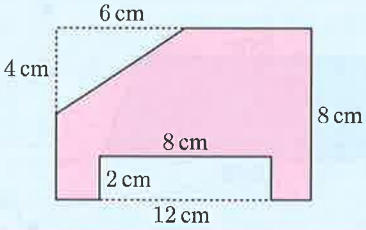
Example 14 **Self Tutor**

Find the pink shaded area:

a



b



a Area

= area large rectangle − area small rectangle

$= (9 \times 8 - 6 \times 3)\ \text{m}^2$

$= (72 - 18)\ \text{m}^2$

$= 54\ \text{m}^2$

b Area

= area large rectangle − area triangle − area small rectangle

$= (12 \times 8 - \frac{1}{2} \times 6 \times 4 - 8 \times 2)\ \text{cm}^2$

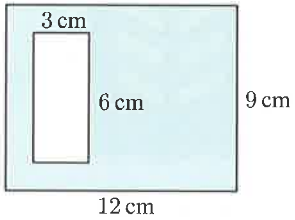
$= (96 - 12 - 16)\ \text{cm}^2$

$= 68\ \text{cm}^2$

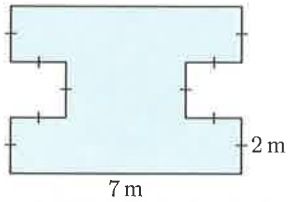
EXERCISE 11E

1 Find the shaded area:

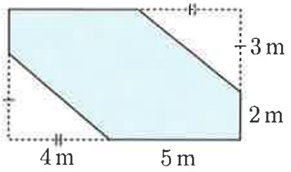
a



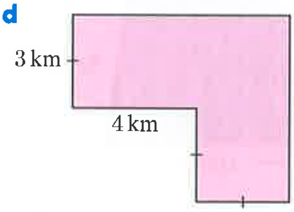
b



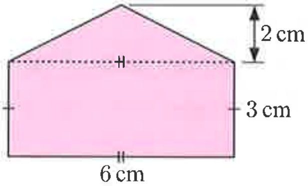
c



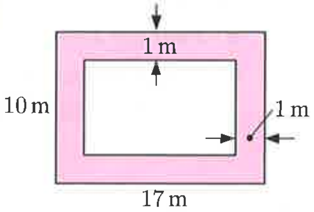
d



e

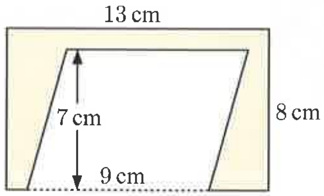


f

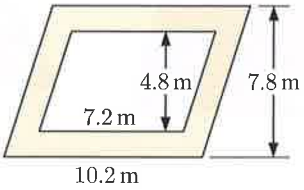


2 Find the shaded area:

a



b

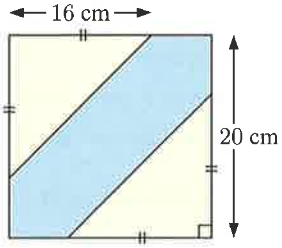


3 A rectangular swimming pool 8 m long and 5 m wide has a 0.8 m wide path around it.

- **a** Draw a diagram to illustrate the situation.
- **b** Find the area of the path.
- **c** Find the cost of covering the path with slate at \$32 per m^2.

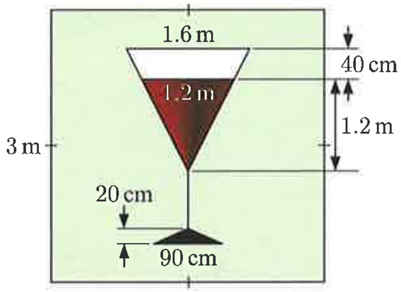
4 My school house has the logo shown. It is a square with two isosceles triangles in opposite corners.

- **a** Find the area of the logo.
- **b** Find the area of each triangle.
- **c** Hence find the area of the stripe across the middle.

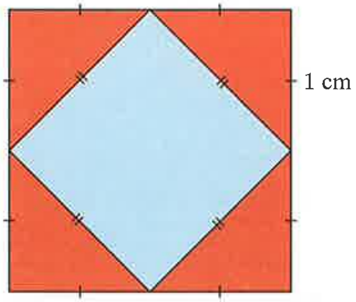


5 The sign alongside is displayed outside a winery. Find the area of the sign which is:

a white **b** red **c** black **d** green.



6
- **a** Find the area of the whole square.
- **b** Find the area of each red triangle.
- **c** Hence find the area of the blue square.
- **d** Find the side length of the blue square.



Global context

click here

Shikaku puzzles

Statement of inquiry:	Solving mathematical puzzles can help us to better understand mathematical concepts.
Global context:	Scientific and technical innovation
Key concept:	Logic
Related concepts:	Pattern, Measurement
Objective:	Investigating patterns
Approaches to learning:	Thinking, Social

KEY WORDS USED IN THIS CHAPTER

- area
- hectare
- metre
- centimetre
- kilometre
- millimetre
- composite figure
- length
- perimeter

REVIEW SET 11A

1 Which unit of area would be most appropriate for measuring:

a the area of a gymnasium floor **b** the area of India

c the size of a postcard?

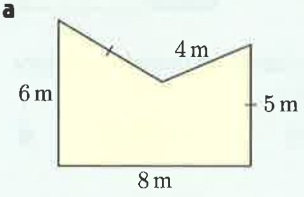
2 Convert:

a 12.9 cm to mm **b** 3.95 km to m **c** 2.43 m to cm

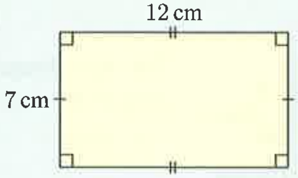
d 1459 cm^2 to m^2 **e** 9.4 ha to m^2 **f** 12.8 cm^2 to mm^2

3 Find the perimeter of:

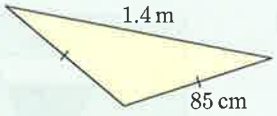
a



b

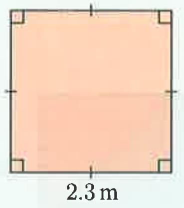


c

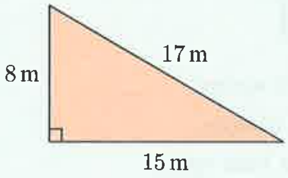


4 Find the area of:

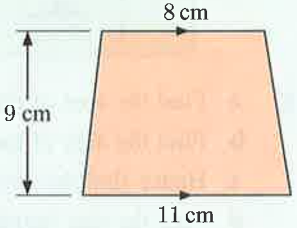
a



b



c



5 Find the sum of 2 km + 510 m + 190 cm in metres.

6 A rectangular recreation reserve is 800 m wide and 2.3 km long.

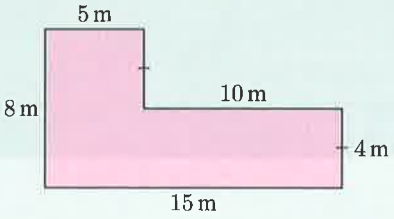
a Find the area of the reserve in hectares.

b If 20 trees are planted in each hectare, how many trees are planted in total?

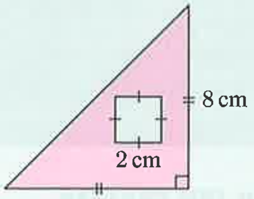
7 A length of wire 360 m long is cut into equal lengths which are then bent into equilateral triangles with sides 15 cm. How many triangles can be made?

8 Find the pink shaded area:

a

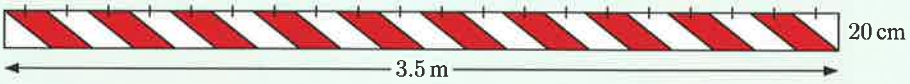


b



9 Answer the **Opening Problem** on page **228**.

10 The boom gate of a railway crossing contains a pattern of parallelograms, with a triangle at each end.



a Find the area of each:

i parallelogram **ii** triangle.

b Check your answers by adding the areas of the sections and comparing this with the total area of the rectangular boom gate. Show all working.

REVIEW SET 11B

1 What unit of length would be most appropriate for measuring:

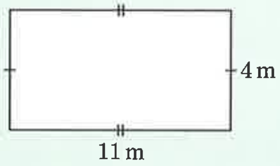
a the width of a street **b** the length of an eraser?

2 Convert:

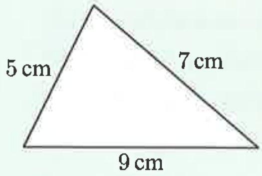
a 49 mm to cm **b** 299 mm^2 to cm^2 **c** 6.84 km^2 to ha

3 Find the perimeter of:

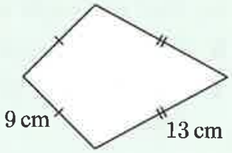
a



b

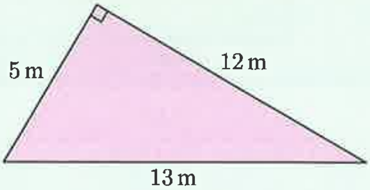


c

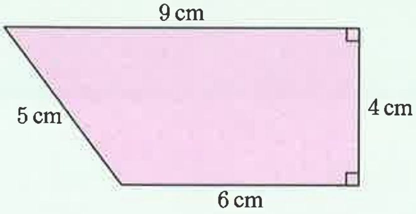


4 Find the area of:

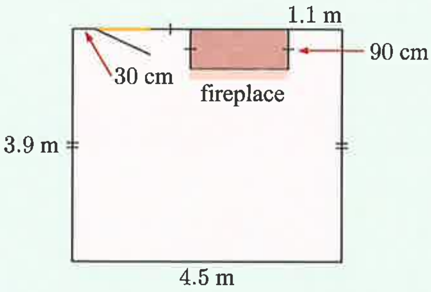
a



b



5 Adam wants to place skirting board along the bottom of the walls in his room.
How many metres of board does Adam need?

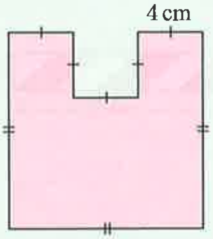


6 A soccer pitch has dimensions 100 m by 75 m. Find the cost of fertilising the pitch if 1 kg of fertiliser covers 10 square metres and fertiliser costs $15 for a 20 kg bag.

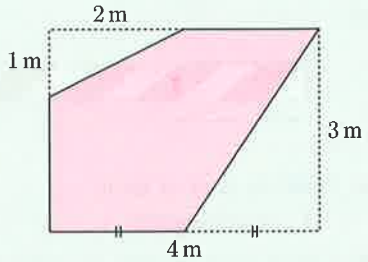
7 A rug measuring 2.5 m by 3.5 m was placed in a room 6.4 m long and 8.2 m wide. What area of floor is not covered by the rug?

8 Find the pink shaded area:

a



b



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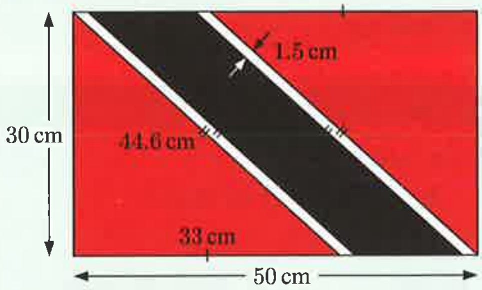


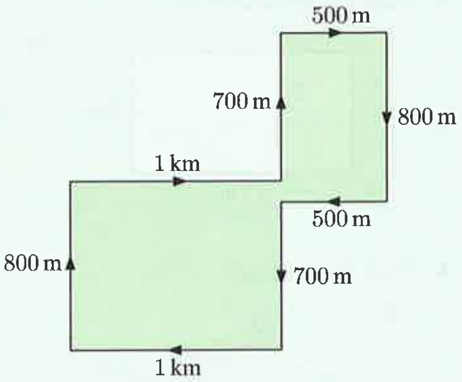
A flag of Trinidad & Tobago has the dimensions shown.

The white stripes have width 1.5 cm.

- **a** Find the area of the flag.
- **b** Find the area of:
  - **i** a red triangle
  - **ii** a white stripe.
- **c** Hence find the area of the black stripe.

10 A street circuit for a car race is shown alongside.

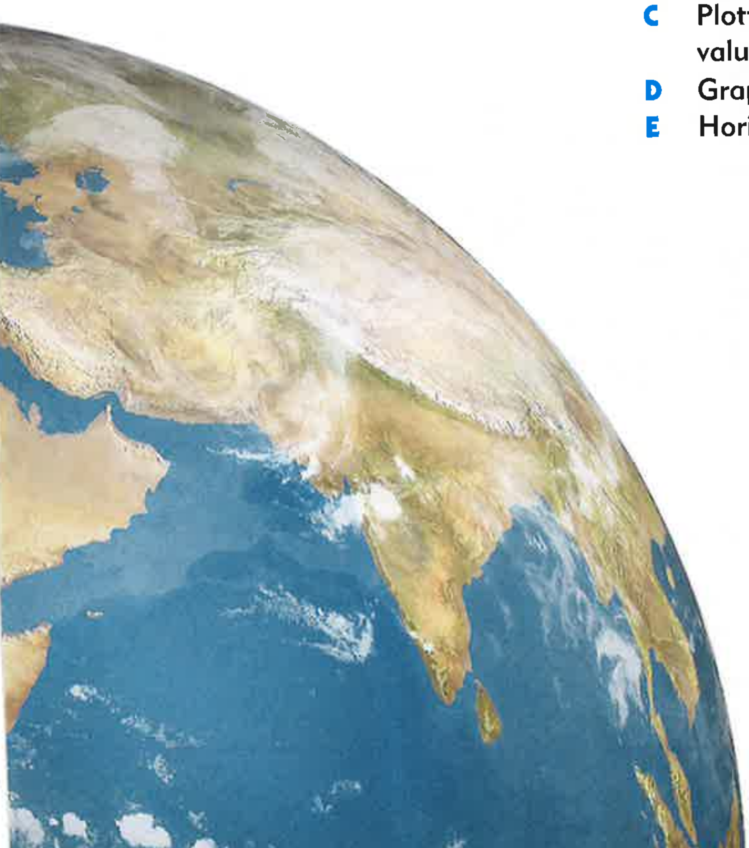
- **a** Find the length of one lap around the circuit.
- **b** How far will cars travel during a 50 lap race?
- **c** Find the area of the region inside the race track.



Chapter 12

Coordinate geometry

Contents:



OPENING PROBLEM

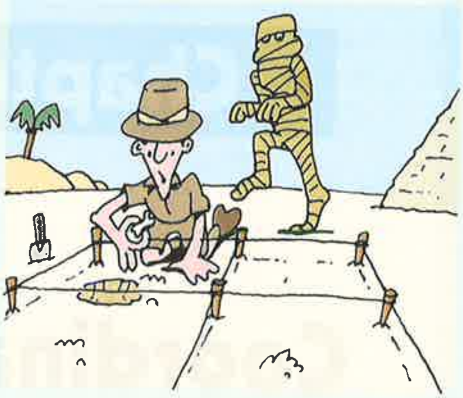
Scientists and archeologists often use grids when searching for fossils and ancient artefacts. They do this so they can accurately record the location where each object is found.

Things to think about:

a Professor Johnson has used pegs and ropes to form a grid over his excavation site. What else does he need so he can record the positions of his discoveries?

b How can Professor Johnson improve his accuracy in identifying positions? Discuss your ideas with your class.

c Professor Johnson wants to record the position of the object in his grid *and* the depth at which it was found. Suggest a way in which he could do this.



You have probably seen **map references** before in a street directory or an atlas.

Horizontal and vertical lines divide the map into regions. We can describe the location of a feature using a letter and number combination.

For example, the combination A5 refers to the region shaded.

We can see that Royce Hall is found in region A5. We say that A5 is the **map reference** for Royce Hall. It does not describe the *exact* location of Royce Hall, but it tells us where to look.

To describe a location more accurately, we can use **coordinates** on a **number grid**.

© OpenStreetMap contributors

HISTORICAL NOTE

Frenchman **René Descartes** found a method for describing the position of a point in a plane. His work led to a new branch of mathematics called **coordinate geometry**.

One of Descartes' principles was "never to accept anything as true which I do not clearly and distinctly see to be so". This is a good piece of advice for your own study of mathematics.

A NUMBER GRIDS

A **number grid** can be used to locate the exact position of any point on a plane.

The number grid contains horizontal and vertical **axes** of reference. We label both axes with numbers, and the numbers are placed on grid lines, not in the regions between them.

The horizontal axis is called the **x-axis**.

The vertical axis is called the **y-axis**.

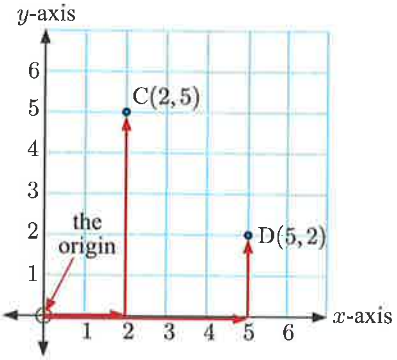
The point of intersection is called the **origin**, **O**.

To get from the origin to point C, we first move 2 units in the x-direction and then 5 units in the y-direction. We say that C has **coordinates** (2, 5). The **x-coordinate** is 2 and the **y-coordinate** is 5.

To get from the origin to point D, we first move 5 units in the x-direction and then 2 units in the y-direction. We say that D has coordinates (5, 2).

These coordinates are called **ordered pairs** because we move first in the x-direction and then in the y-direction.

Notice that C(2, 5) and D(5, 2) are at different positions in the number plane.

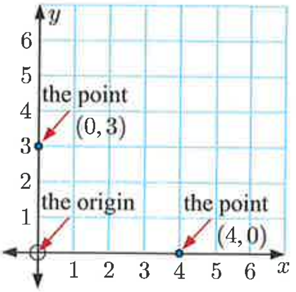


POINTS ON THE AXES

Consider a point with x-coordinate 0. It lies on the y-axis, because there is no movement to the right, only up.

Now consider a point with y-coordinate 0. It lies on the x-axis, because there is no movement up, only to the right.

The **origin O** has coordinates (0, 0). It is marked with a small circle at the intersection of the axes.

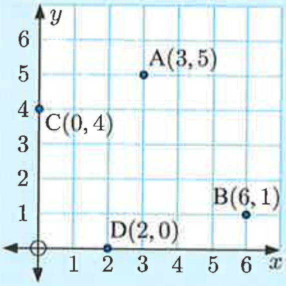


Example 1

Self Tutor

On the same set of axes, plot and label the points with coordinates:

A(3, 5), B(6, 1), C(0, 4), D(2, 0).



EXERCISE 12A

1 On graph paper, draw a set of axes. Plot and label the following points:

a A(2, 2)
b B(4, 8)
c C(3, 1)
d D(7, 0)
e E(0, 5)
f F(5, 4)
g G(9, 1)
h H(6, 0)
i I(0, 1)
j J(7, 7)
k K(0, 0)
l L(8, 3)

2 Copy and complete:

a The ......-coordinate of a point on the x-axis is 0.

b The ......-coordinate of a point on the y-axis is 0.

3 Write down:

a the x-coordinate of:

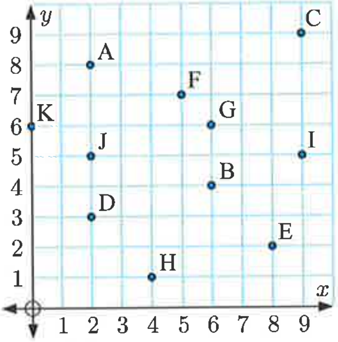
i B **ii** A **iii** C **iv** G

b the y-coordinate of:

i E **ii** H **iii** J **iv** K

c the coordinates of each point

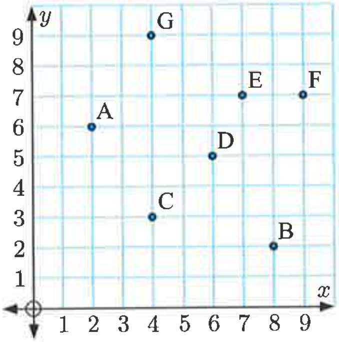
d the coordinates of the origin, O.



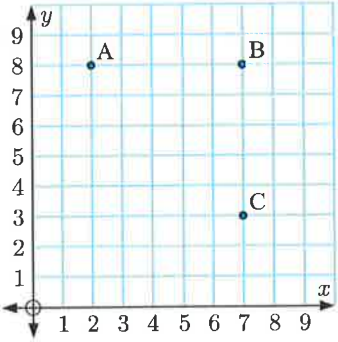
4 **a** Name two points with the same x-coordinate. What do you notice about these points?

b Name two points with the same y-coordinate. What do you notice about these points?

c Name the point whose x-coordinate is equal to its y-coordinate.



5 ABCD is a square. A, B, and C are marked on the grid. Write down the coordinates of D.



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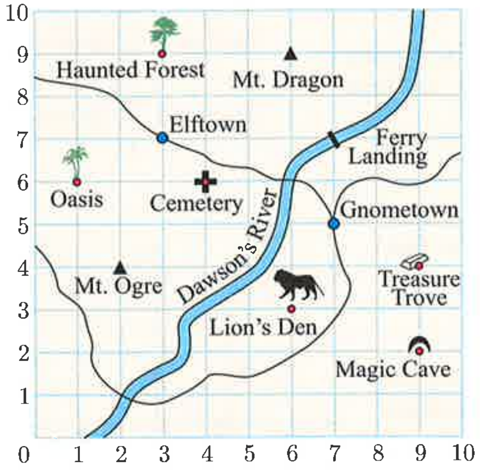


Use the map to find:

a the grid coordinates for:
- **i** Gnometown
- **ii** Magic Cave
- **iii** Ferry Landing
- **iv** where the roads cross Dawson's River

b the places located at:
- **i** (9, 4) **ii** (6, 3)
- **iii** (2, 4) **iv** (1, 6)

7 **a** On a set of axes, plot and label the points A(3, 1), B(6, 2), C(9, 3), and D(12, 4).

b Join these points. What do you notice?

c If the pattern continues, what will the next point be?

8 **a** On a set of axes, plot and label the points A(0, 10), B(1, 8), C(2, 6), and D(3, 4).

b If the pattern continues, what will the coordinates of the next two points be?

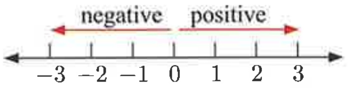
ACTIVITY HOPPING AROUND A NUMBER PLANE

For this Activity, click on the icon to obtain instructions and a printable grid.

ACTIVITY

B POSITIVE AND NEGATIVE COORDINATES

In **Chapter 3** we saw how the number line was extended in two directions to include positive and negative numbers.



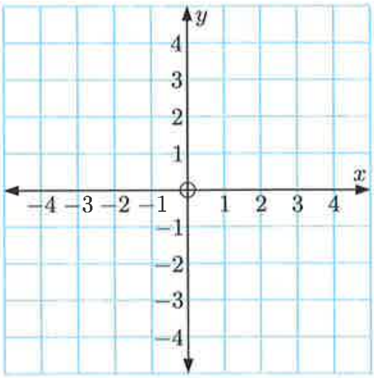
To extend the number plane studied in the last Section, we extend both the x-axis and the y-axis in two directions. This allows us to consider positive *and* negative coordinates.

In the centre of the number plane is the origin O.

The x-axis is positive to the right of O, and negative to the left of O.

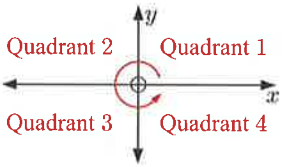
The y-axis is positive above O, and negative below O.

This number plane is called the **Cartesian plane**.



The axes divide the plane into four **quadrants**.

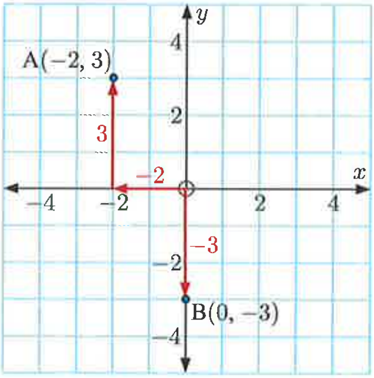
The quadrants are numbered in an anticlockwise direction, starting with the upper right hand quadrant in which x and y are both positive.



We can now describe and plot points in any of the four quadrants or on either axis.

For example:

- To plot the point A$(-2, 3)$, we move 2 units to the *left* of the origin, then 3 units up. A is in the second quadrant.
- To plot the point B$(0, -3)$, we do not move left or right, but we move 3 units down. B is on the y-axis.

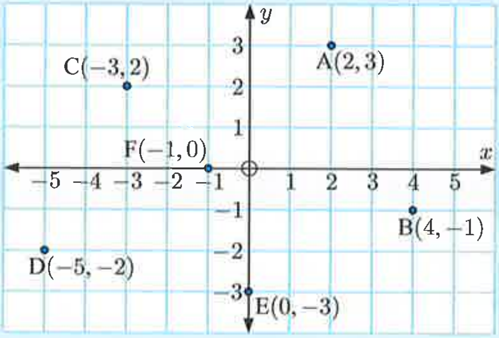


Example 2

Self Tutor

Plot the following points on the Cartesian plane:

A$(2, 3)$, B$(4, -1)$, C$(-3, 2)$, D$(-5, -2)$, E$(0, -3)$, F$(-1, 0)$.



EXERCISE 12B

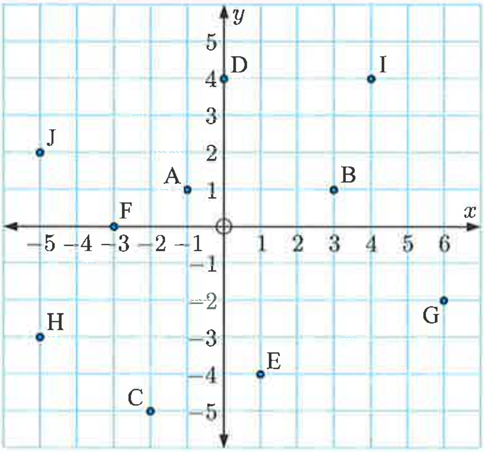
1 Draw a set of axes, then plot and label the following points:

a A$(3, 4)$
b B$(6, 2)$
c C$(-3, 0)$
d D$(-5, -5)$
e E$(0, -1)$
f F$(4, 0)$
g G$(3, -4)$
h H$(0, 6)$
i I$(-5, -2)$
j J$(-4, -1)$
k K$(3, -3)$
l L$(-5, 4)$

2 On a set of axes, plot the points with coordinates given below. Join the points with straight line segments in the order given:

$(-3, 4)$, $(-1, 5)$, $(1, 5)$, $(3, 4)$, $(1, 3)$, $(-1, 3)$, $(-3, 4)$, $(-3, -2)$, $(-1, -3)$, $(1, -3)$, $(3, -2)$, $(3, 4)$, $(3, 3)$, $(4, 3)$, $(5, 2)$, $(5, 0)$, $(4, -1)$, $(3, -1)$, $(3, 0)$, $(4, 0)$, $(4, 2)$, $(3, 2)$

3 Consider the points on the set of axes shown.



a Write down the x-coordinate of:

i D **ii** B **iii** J **iv** G

b Write down the y-coordinate of:

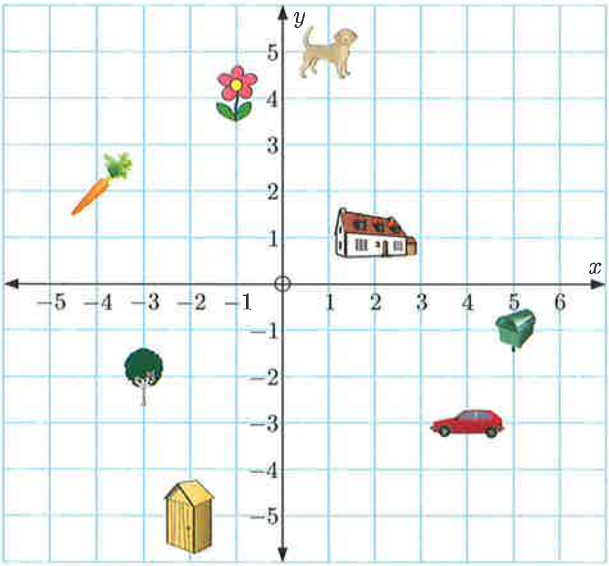
i A **ii** C **iii** F **iv** I

c Write down the coordinates of all points.

d Which of the points lie:

- **i** in the first quadrant
- **ii** in the second quadrant
- **iii** in the third quadrant
- **iv** in the fourth quadrant
- **v** on the x-axis
- **vi** on the y-axis?

4 Consider the map alongside.



a Write down the coordinates of the:

- **i** house
- **ii** tree
- **iii** flower garden
- **iv** car
- **v** dog
- **vi** carrot patch
- **vii** letterbox
- **viii** toolshed

b Which of the things lie in the:

- **i** first quadrant
- **ii** second quadrant
- **iii** third quadrant
- **iv** fourth quadrant?

5 In which quadrant would you find a point where:

a both x and y are positive **b** both x and y are negative

c x is negative and y is positive **d** x is positive and y is negative?

6 Determine the quadrant in which the following points lie:

a A(3, 5) **b** B(2, −2) **c** C(−1, −3) **d** D(−4, 2)

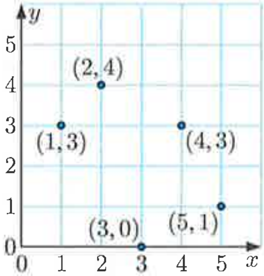
e E(5, −3) **f** F(4, −4) **g** G(−2, −1) **h** H(−3, 5)

C PLOTTING POINTS FROM A TABLE OF VALUES

Tony plays lacrosse for his local club. The numbers of goals he has scored in the five games so far this season are shown below in a **table of values**:

Game number (x)	1	2	3	4	5
Goals scored (y)	3	4	0	3	1

We can display these values graphically by plotting the x and y-values on a number plane. The points $(1, 3)$, $(2, 4)$, $(3, 0)$, $(4, 3)$, and $(5, 1)$ are shown below.



EXERCISE 12C

1 Mike kept a record of the cars sold at his car yard each day last week:

Day number (x)	1	2	3	4	5	6	7
Cars sold (y)	3	1	2	0	4	7	5

Plot these points on a number plane.

2 While on ski camp, Ned recorded the minimum temperature reached each night. The results are given in the table below:

Night number (x)	1	2	3	4	5
Temperature (y °C)	−2	0	−1	3	1

Plot these points on a number plane.

3 For each of the following tables of values, plot the points on a number plane:

a

x	1	2	3	4
y	2	4	1	2

b

x	0	1	2	3	4
y	3	−2	0	5	−1

c

x	−3	−1	0	2	3
y	4	2	−1	4	−2

d

x	−2	−1	0	1	2
y	3	0	2	−4	−3

4 **a** Plot the points for this table of values on a number plane.

x	−3	−1	0	2	3
y	0	2	3	5	6

b What do you notice about these points?

c Can you see a relationship between the x and y-coordinates of these points?

D GRAPHING STRAIGHT LINES

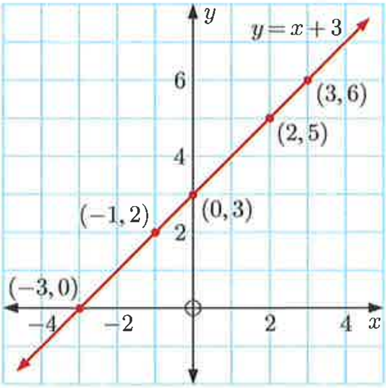
A **straight line** consists of an infinite number of points in a particular direction. We cannot list *all* of the points on a line in a table of values, but if we know *some* points on the line then we can plot them and hence draw the line through them.

THE EQUATION OF A LINE

The **equation of a line** is a rule which connects the x and y-coordinates of **all** points on the line.

In question **4** of the previous Exercise, you should have noticed that the plotted points lie in a straight line. For each of the points, the y-coordinate is 3 more than the x-coordinate.

The rule connecting the x and y-coordinates of each point on the line is $y = x + 3$. We say that $y = x + 3$ is the **equation** of the line.



Example 3 **Self Tutor**

For each point on a line, the y-coordinate is 2 less than the x-coordinate. State the equation of the line.

The equation of the line is $y = x - 2$.

Suppose we know the equation of a line. If we are given the x-coordinate of any point on the line, we can use the equation to find the y-coordinate.

Example 4 **Self Tutor**

The point P lies on the line with equation $y = x - 5$.
The x-coordinate of P is 3. Find the y-coordinate of P.

Substituting $x = 3$ into $y = x - 5$ gives $y = 3 - 5$

$\therefore\ y = -2$

So, the y-coordinate of P is -2.

EXERCISE 12D.1

1 State the equation of a line if, for each point on the line:

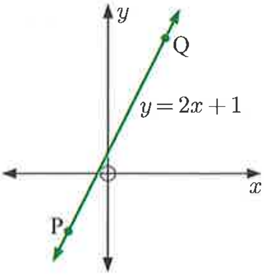
- **a** the y-coordinate is 5 more than the x-coordinate
- **b** the y-coordinate is 7 less than the x-coordinate
- **c** the y-coordinate is 3 times the x-coordinate
- **d** the y-coordinate is half the x-coordinate.

2 For each of the lines in **1**, state one point which lies on the line.

3 The point P lies on the line with equation $y = x + 6$. The x-coordinate of P is 4. Find the y-coordinate of P.

4 The point Q lies on the line with equation $y = 5x$. The x-coordinate of Q is -3. Find the y-coordinate of Q.

5 The graph of the line with equation $y = 2x + 1$ is shown alongside. P has x-coordinate -2, and Q has x-coordinate 3. Find the coordinates of P and Q.



GRAPHING STRAIGHT LINES

If we are given the equation of a line, we can graph the line using these steps:

Step 1: For each of the x-coordinates -2, -1, 0, 1, and 2, find the corresponding y-coordinate.

Hence complete a table of values like this:

x	-2	-1	0	1	2
y					

Step 2: Plot the points on a number plane.

Step 3: Draw a straight line through the points.

Step 4: Place arrows at the ends of the line to indicate that the line extends forever in both directions.

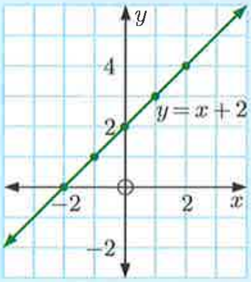
Example 5 **Self Tutor**

Draw the graph of the line with equation $y = x + 2$.

When $x = -2$, $y = -2 + 2 = 0$.
When $x = -1$, $y = -1 + 2 = 1$.
When $x = 0$, $y = 0 + 2 = 2$.
When $x = 1$, $y = 1 + 2 = 3$.
When $x = 2$, $y = 2 + 2 = 4$.

The table of values is:

x	-2	-1	0	1	2
y	0	1	2	3	4



EXERCISE 12D.2

1 Use a table of values to draw the graph of the line with equation:

a $y = x$ **b** $y = x + 4$ **c** $y = x - 2$

2 Draw the graph of the line with equation:

a $y = x + 1$ **b** $y = x - 4$ **c** $y = 2x$

d $y = 1 - x$ **e** $y = 2x + 1$ **f** $y = 2x - 3$

g $y = -3x$ **h** $y = \frac{1}{2}x$ **i** $y = 3 - 2x$

3 **a** Draw the graph of the line with equation $y = \frac{1}{2}x + 2$.

b Find the coordinates of the point where the graph cuts the: **i** y-axis **ii** x-axis.

DISCUSSION

Examine the graphs you have drawn, and the corresponding equations. What part of the equation do you think controls:

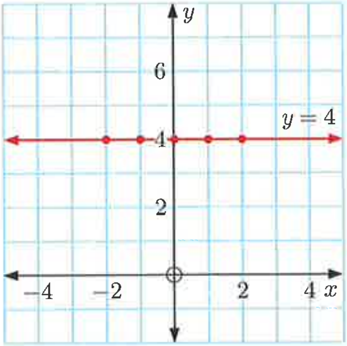
- the steepness of the line
- whether the graph slopes upwards or downwards
- where the graph cuts the y-axis?

E HORIZONTAL AND VERTICAL LINES

Consider the line with equation $y = 4$.

At first it may be unclear how we should complete our table of values, because x is not mentioned in the equation. However, the equation means that no matter what the value of x is, the value of y is always 4.

x	-2	-1	0	1	2
y	4	4	4	4	4

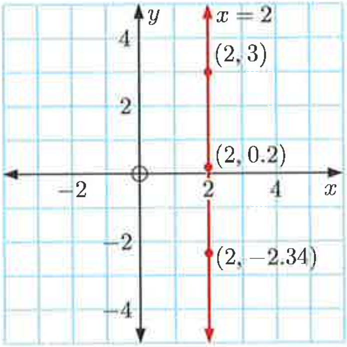


We can plot these points on a number plane. The result is a **horizontal line**. It includes all points with y-coordinate 4.

All **horizontal lines** have equations of the form $y = k$.

Similarly, the line with equation $x = 2$ consists of all points with x-coordinate 2. For example, $(2, 3)$, $(2, 0.2)$, and $(2, -2.34)$ all lie on this line. The line is **vertical**.

All **vertical lines** have equations of the form $x = k$.



Example 6

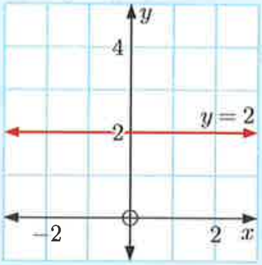
Self Tutor

Draw the graph of:

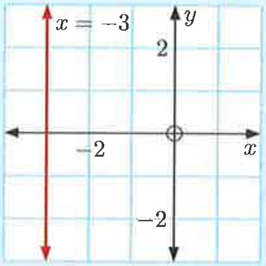
a $y = 2$

b $x = -3$

a The line $y = 2$ consists of all points with y-coordinate 2. It is a horizontal line.



b The line $x = -3$ consists of all points with x-coordinate -3. It is a vertical line.



EXERCISE 12E

1 Draw the graph of:

a $y = 1$

b $x = 3$

c $y = -2$

d $x = -4$

e $x = 1.5$

f $y = -\frac{1}{2}$

g $x = 0$

h $y = 0$

2 **a** On the same set of axes, draw the graphs of $x = 4$ and $y = -3$.

b Write down the coordinates of the point where the lines meet.

KEY WORDS USED IN THIS CHAPTER

- axes
- Cartesian plane
- coordinates
- equation
- number grid
- number plane
- ordered pair
- origin
- quadrant
- table of values
- x-axis
- x-coordinate
- y-axis
- y-coordinate

REVIEW SET 12A

1 Write down the coordinates of point:

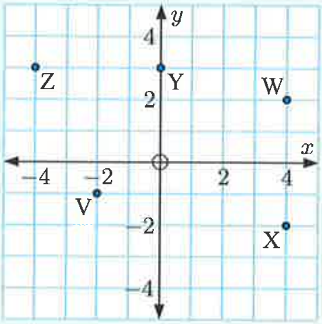
a V

b W

c X

d Y

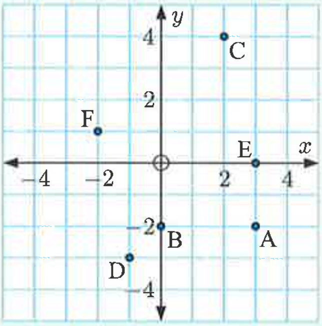
e Z



2 Plot and label the following points: F(4, −2), G(−5, −3), H(−1, 3), I(0, −4).

3 Write down:

- **a** the x-coordinates of A and D
- **b** the y-coordinates of B and C
- **c** the coordinates of A, B, E, and F.



4 Determine the quadrant in which the following points lie:

a $(-2, 7)$ **b** $(-3, -6)$ **c** $(0, -2)$ **d** $(5, 1)$

5 The height of a plant is recorded on a weekly basis:

Plot these points on a number plane.

Week number (x)	1	2	3	4	5
Height (y cm)	6	9	11	13	14

6 **a** On a set of axes, plot and label the points A$(-3, 2)$, B$(-2, 3)$, and C$(-1, 4)$.

b If the pattern continues, what will the next point be?

7 For each table of values, plot the points on a number plane:

a

x	1	2	3	4	5
y	4	2	5	0	2

b

x	−2	−1	0	1	2
y	−7	−4	−1	2	5

8 State the equation of a line if, for each point on the line:

- **a** the y-coordinate is 3 less than the x-coordinate
- **b** the y-coordinate is twice the x-coordinate.

9 Using a table of values, graph the line with equation $y = 2x - 1$.

10 **a** Copy and complete the table of values for the line with equation $y = -5$:

x	−2	−1	0	1	2
y					

b Use your table of values to draw the graph of $y = -5$.

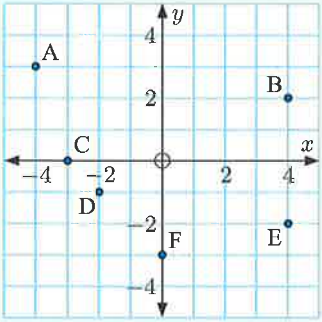
REVIEW SET 12B

1 Match each ordered pair with the correct point on the number plane:

a $(4, -2)$ **b** $(0, -3)$

c $(-4, 3)$ **d** $(4, 2)$

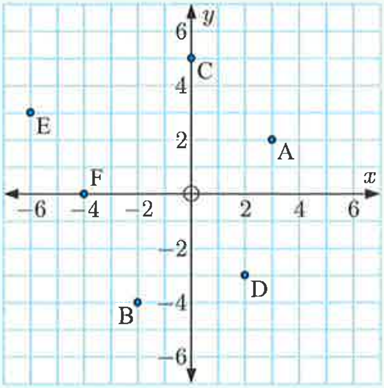
e $(-3, 0)$ **f** $(-2, -1)$



2 In which quadrant would I find a point with negative x and y-coordinates?

3 Write down:

- **a** the x-coordinate of D
- **b** the y-coordinate of E
- **c** the coordinates of A and B.



4 On the same set of axes, plot and label the points A(−3, 2), B(1, 5), C(−4, −2), and D(0, −1).

5 Are (3, 4) and (4, 3) the same point on the number plane? Use an illustration in your answer.

6 The y-coordinate of each point on a line is two less than its x-coordinate.

- **a** Write down the equation of this line.
- **b** Find the coordinates of the point where this line crosses the y-axis.

7 Tina measured the rainfall at her house each day for 5 days. Her results are shown below:

Day number (x)	1	2	3	4	5
Rainfall (y mm)	5	2	9	0	4

Plot these points on a set of axes.

8 The point P lies on the line with equation $y = 3x - 2$. The x-coordinate of P is −2. Find the y-coordinate of P.

9 Draw the graph of the line with equation:

- **a** $y = -\frac{1}{2}x$
- **b** $y = 2x - 4$

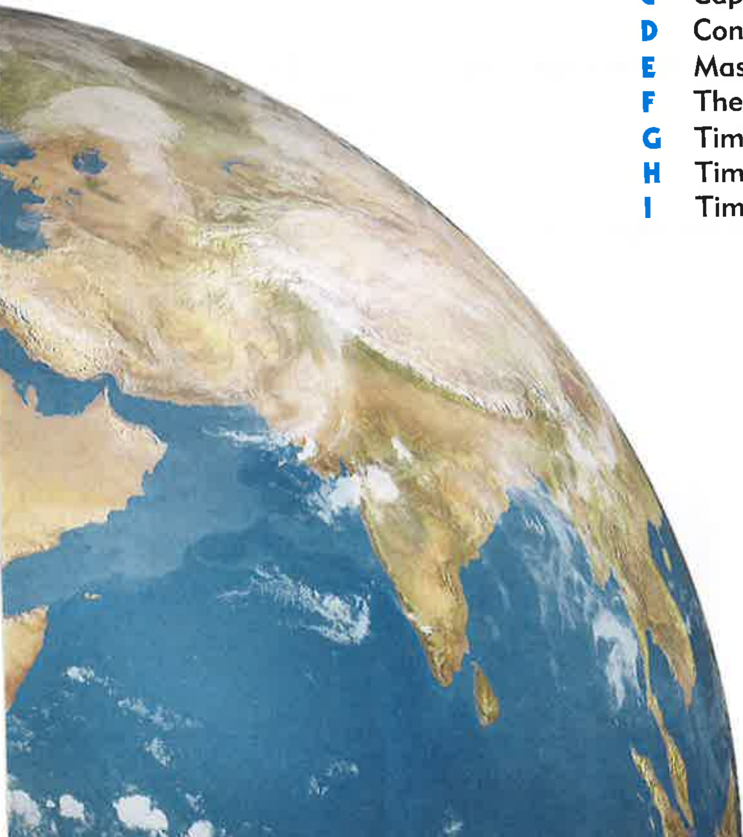
10 Draw the graph of:

- **a** $x = 1$
- **b** $y = 3$
- **c** $x = -2.5$
- **d** $y = \frac{1}{4}$

Chapter 13

Further measurement

Contents:



OPENING PROBLEM

Chun's roof is leaking. 10 mL of water is dripping onto her floor every minute. At 2:30 pm she places a 20 cm by 10 cm by 5 cm container under the leak to catch the drops.

Things to think about:

- **a** How much water can the container hold?
- **b** How long will it take for the container to fill?
- **c** At what time will the container overflow?

In this chapter we complete our study of measurement by looking at **volume**, **capacity**, **mass**, and **time**.

A VOLUME

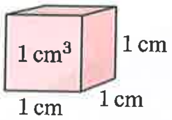
The **volume** of a three-dimensional object is the amount of space it occupies. This space is measured in **cubic units**.

The units we use for measuring volume are related to the units we use for length.

1 **cubic millimetre** (mm^3) is the volume of a cube with side length 1 mm.

The volume of a small marble might be measured in mm^3.

1 **cubic centimetre** (cm^3) is the volume of a cube with side length 1 cm.



The volume of a petrol tank might be measured in cm^3.

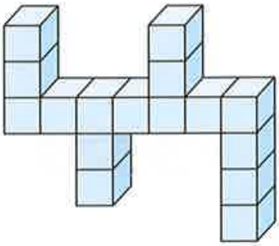
1 **cubic metre** (m^3) is the volume of a cube with side length 1 m.

The volume of rock mined from a quarry might be measured in m^3.

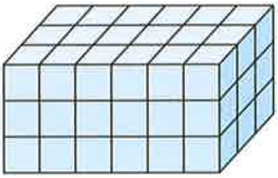
EXERCISE 13A.1

1 Find the number of cubic units in each of the following solids:

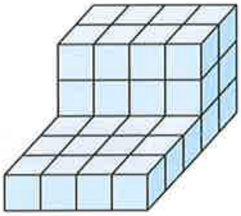
a



b



c



2 Which of the solids in **1** has the:

a greatest volume

b least volume?

3 Give the units of volume that would be most suitable for measuring the space occupied by:

a a textbook

b an apple

c a truck

d a paper clip

e a mobile phone

f a football

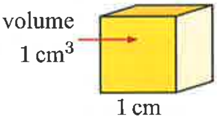
g an elephant

h an eraser

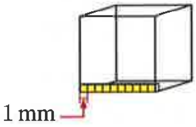
i a single plant seed.

CONVERSION OF VOLUME UNITS

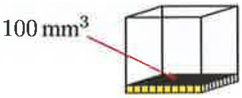
Consider a cube with side length 1 cm. This cube has a volume of 1 cm^3.



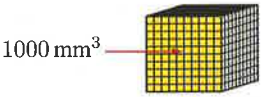
Since 1 cm = 10 mm, we can fit 10 cubic millimetres in a row along one side of the cube.



We can fit 10 of these rows on the bottom surface of the cube, using $10 \times 10 = 100$ cubic millimetres in total.



We can fit 10 of these layers in the cube, using $10 \times 100 = 1000$ cubic millimetres in total.



This demonstration shows us that one cubic centimetre occupies the same amount of space as 1000 cubic millimetres.

$1 \text{ cm}^3 = 10 \text{ mm} \times 10 \text{ mm} \times 10 \text{ mm} = 1000 \text{ mm}^3$

Similarly, $1 \text{ m}^3 = 100 \text{ cm} \times 100 \text{ cm} \times 100 \text{ cm} = 1\,000\,000 \text{ cm}^3$.

We can convert between the units of volume using this conversion diagram:

$$\begin{tikzcd} \text{m}^3 \arrow[r, bend left, "\times 1\,000\,000"] & \text{cm}^3 \arrow[l, bend left, "\div 1\,000\,000"] \arrow[r, bend left, "\times 1000"] & \text{mm}^3 \arrow[l, bend left, "\div 1000"] \end{tikzcd}$$

Example 1 **Self Tutor**

Convert:

a 4.56 cm^3 to mm^3 **b** 324 000 cm^3 to m^3

a
$$\begin{aligned} & 4.56\ \text{cm}^3 \\ &= (4.56 \times 1000)\ \text{mm}^3 \\ &= 4560\ \text{mm}^3 \end{aligned}$$

b
$$\begin{aligned} & 324\,000\ \text{cm}^3 \\ &= (324\,000 \div 1\,000\,000)\ \text{m}^3 \\ &= 0.324\ \text{m}^3 \end{aligned}$$

EXERCISE 13A.2

1 Convert:

a 48 cm^3 to mm^3
b 29 000 cm^3 to m^3
c 1.2 m^3 to cm^3
d 12 485 mm^3 to cm^3
e 0.000 45 m^3 to cm^3
f 14 500 cm^3 to m^3
g 0.295 cm^3 to mm^3
h 1.43 mm^3 to cm^3
i 0.0056 m^3 to mm^3

2 A slab of granite has a volume of 25 000 cm^3. 60 of these slabs are packed into a truck. Find the total volume of the slabs, in cubic metres.

3 To make the concrete for a path, Vic mixed 50 000 cm^3 of sand, 25 000 cm^3 of cement powder, and 0.16 m^3 of gravel. Find the total volume of these components.

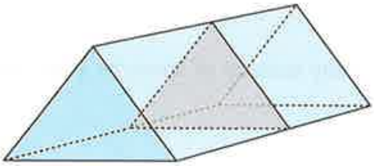
4 A slab of freeze dried coffee with volume 2000 cm^3 is broken into tiny pieces, each with volume 10 mm^3. Find the total number of pieces.

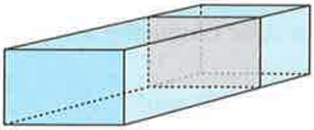
B VOLUME FORMULAE

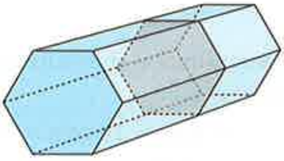
We can use formulae to calculate the volumes of common solids. In this Section we consider the volumes of a group of solids called **prisms**.

A **prism** is a solid with a uniform cross-section that is a polygon.

If we take any slice of a prism parallel to its end, the exposed surface will be exactly the same shape and size as the end. This is what we mean by a **uniform cross-section**.

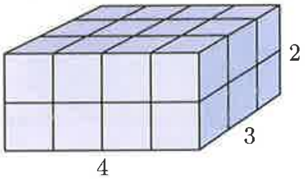

triangular prism


rectangular prism


hexagonal prism

RECTANGULAR PRISMS

A simple example of a prism is this rectangular prism.



Check that you agree with the following facts:

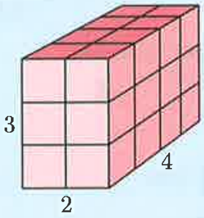
- There are $4 \times 3 = 12$ cubes in the top layer of blocks.
- There are 2 layers.
- There are $12 \times 2 = 24$ cubes in total.
- The volume of this rectangular prism is 24 units^3.
- The volume can be found by the multiplication length × width × height.

Volume of rectangular prism = length × width × height = lwh

Example 2

Self Tutor

Find the number of cubic units in:



Volume $= lwh$

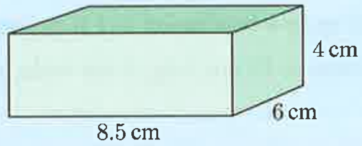
$\therefore \quad V = 2 \times 4 \times 3$

$\therefore \quad V = 24$ units^3

Example 3

Self Tutor

Find the volume of the rectangular prism:



Volume = length × width × height

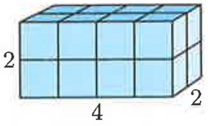
$\therefore \quad V = 8.5 \text{ cm} \times 6 \text{ cm} \times 4 \text{ cm}$

$\therefore \quad V = 204 \text{ cm}^3$

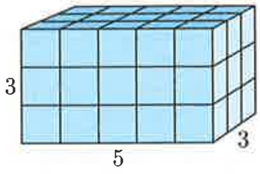
EXERCISE 13B.1

1 Find the number of cubic units in each of the following solids:

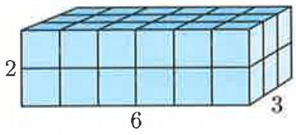
a



b

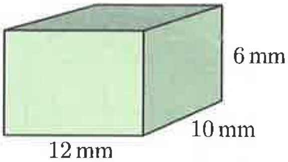


c

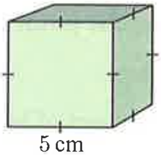


2 Find the volume of each prism:

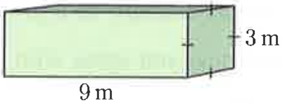
a



b



c



3 Find the volume of:

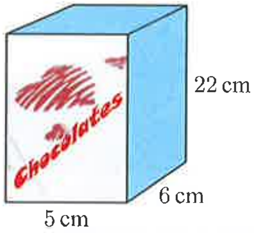
a the match box

b the skip bin

4 Which of these chocolate boxes has the larger volume?

A

B



5 Find the volume of a rectangular tank 5 m by 1.2 m by 2 m.

6 Find the volume of air in a box which is 1.5 m long, 1.2 m wide, and 80 cm high.

7 A rectangular container is 10 cm long, 6 cm wide, and has volume 240 cm^3. Find the height of the container.

ACTIVITY

FLY-BY-NIGHT AIRLINES

Airline companies frequently place restrictions on the size of packages which passengers can carry with them onto an aircraft.

Fly-By-Night Airlines have the following package policy:

- *All packages must be rectangular.*
- *The sum of the length, width, and height of any package must not exceed* 90 *cm.*

Your task is to determine the rectangular package of largest volume which is allowed to be taken on the plane.

What to do:

1 Copy and complete the following table for packages where the sum of the length, width, and height is always 90 cm. Add your own choices of dimensions for the second half of the table.

Length	*Width*	*Height*	*Volume*
10	20	60	12 000
10	30		
10	40		
20	20		
20	25		
20	30		
20	35		

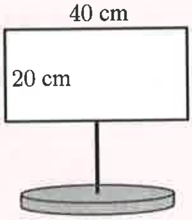
Length	*Width*	*Height*	*Volume*

2 What do you suspect are the dimensions of the package of greatest volume?

3 Fly-By-Night decides to introduce a further restriction to ensure all packages will fit in the overhead lockers:

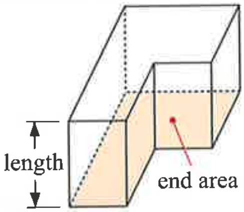
- *All packages must pass through a* 40 *cm by* 20 *cm rectangle.*

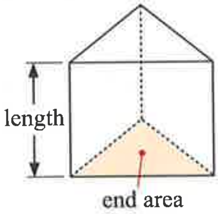
Find the package of greatest volume given this new restriction.

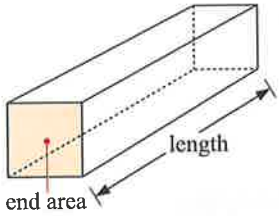


OTHER PRISMS

All prisms contain two identical end faces, connected by straight edges. They are **solids of uniform cross-section**.







To find the volume of a prism, we multiply the area of the end by the length of the prism.

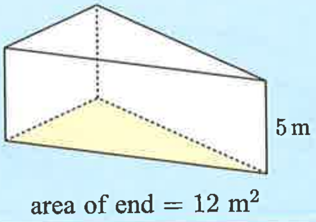
Volume = area of end × length

The formula for the volume of a rectangular prism is a special case of this formula.

Example 4

Self Tutor

Find the volume of this solid.



Volume = area of end × length

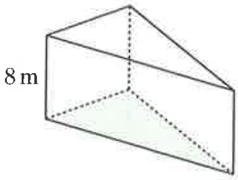
$\therefore \ V = (12 \times 5)\ \text{m}^3$

$\therefore \ V = 60\ \text{m}^3$

EXERCISE 13B.2

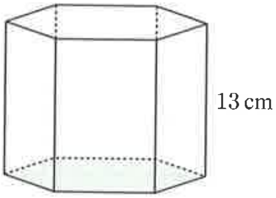
1 Find the volume of each solid:

a



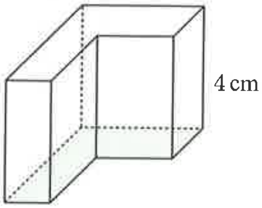
area of end $= 14\text{ m}^2$

b



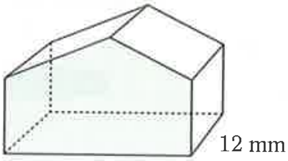
area of end $= 60\text{ cm}^2$

c



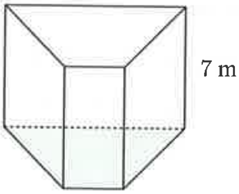
area of end $= 11\text{ cm}^2$

d



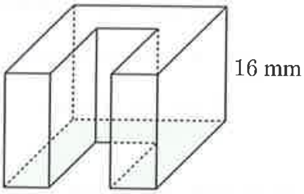
area of end $= 16\text{ mm}^2$

e



area of end $= 17\text{ m}^2$

f



area of end $= 4\text{ cm}^2$

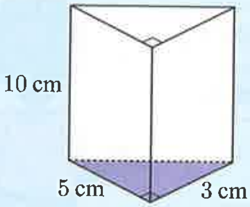
2 An empty classroom has floor area 56 m^2 and ceiling height 3 m. Find the volume of air in the classroom.

3 A solid of uniform cross-section has end area 38.5 cm^2 and volume 308 cm^3. How long is the solid?

Example 5

Self Tutor

Find the volume of this prism:



Volume

$= \text{area of end} \times \text{length}$

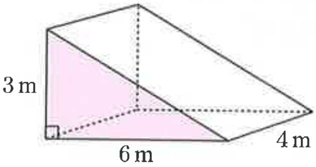
$= \left[\frac{1}{2}(\text{base} \times \text{height})\right] \times \text{length}$ {area of triangle formula}

$= \frac{1}{2}(3 \times 5) \times 10\text{ cm}^3$

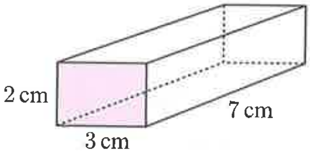
$= 75\text{ cm}^3$

4 Find the volume of each solid:

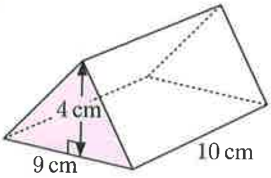
a



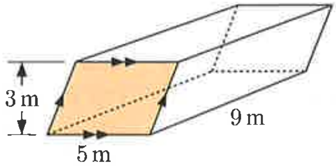
b



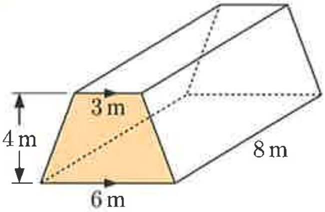
c



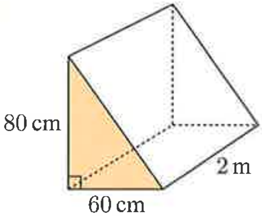
d



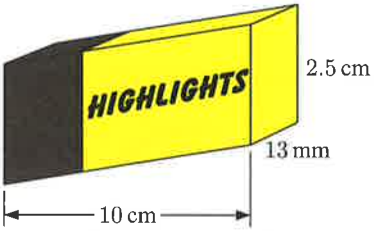
e



f



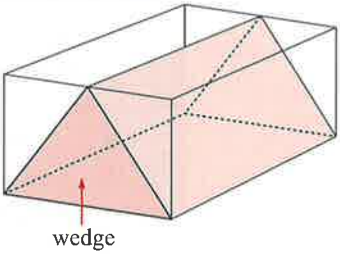
5 A highlighter has the dimensions shown. Find the volume of the highlighter.



6 Mulch costs \$58 per cubic metre. How much would it cost to lay mulch in a garden 13 m long and 4.5 m wide to a depth of 8 cm?

7 A wooden block is 60 cm long, 10 cm wide, and 8 cm high. A wedge in the shape of a triangular prism is cut from the block as shown.

- **a** Find the volume of the original block.
- **b** Find the volume of the wedge.
- **c** What fraction of the block was used to make the wedge?



C CAPACITY

The **capacity** of a container is a measure of the volume it can hold. We can think of it as the space within the container.

We use the term **capacity** when we talk about fluids or gases.

For example, the capacity of a cup is the amount of liquid it can hold.

The **litre** (L) is the basic unit for the measurement of capacity. A standard carton of milk has a capacity of 1 litre.

Other units of capacity include the **millilitre** (mL), **kilolitre** (kL), and **megalitre** (ML).

1 L = 1000 mL
1 kL = 1000 L
1 ML = 1000 kL

This table shows some capacities of familiar objects:

Item	*Capacity*
Medicine glass	25 mL
Bucket	8 L
Hot water system	170 L
Cup	250 mL
Petrol tank	65 L
50 m swimming pool	1500 kL
Reservoir	1000 ML

We can convert between the units of capacity using the following conversion diagram:

$$\begin{tikzcd}
\boxed{\text{ML}} \arrow[r, bend left, "\times 1000"] & \boxed{\text{kL}} \arrow[l, bend left, "\div 1000"] \arrow[r, bend left, "\times 1000"] & \boxed{\text{L}} \arrow[l, bend left, "\div 1000"] \arrow[r, bend left, "\times 1000"] & \boxed{\text{mL}} \arrow[l, bend left, "\div 1000"]
\end{tikzcd}$$

Example 6 **Self Tutor**

Convert:

a 4500 mL to L **b** 350 kL to L

a 4500 mL
$= (4500 \div 1000)$ L
$= 4.5$ L

b 350 kL
$= (350 \times 1000)$ L
$= 350\,000$ L

EXERCISE 13C

1 A test tube would most likely have a capacity of:

A 5 L **B** 0.5 mL **C** 50 mL **D** 5 kL **E** 50 L

2 A large drink bottle would most likely have a capacity of:

A 80 mL **B** 8 L **C** 800 L **D** 800 mL **E** 8 kL

3 A lake would most likely have a capacity of:

A 8.5 ML **B** 85 L **C** 8500 mL **D** 85 kL **E** 85 mL

4 A kitchen sink would most likely have a capacity of:

A 28 mL **B** 2.8 L **C** 28 L **D** 28 kL **E** 2.8 ML

5 Convert:

a 8000 mL to L **b** 2 ML to kL **c** 786 L to kL
d 40 mL to L **e** 3.95 kL to L **f** 1 ML to mL

6 How many 375 mL bottles of soda can be filled from a 24 L container?

7 George's shower releases 16 litres of water per minute. If George takes a 10 minute shower every day, how many kilolitres of water will he use showering in the month of January?

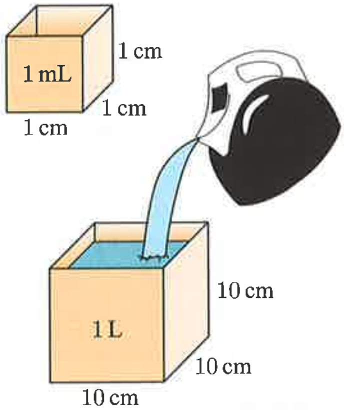
D CONNECTING VOLUME AND CAPACITY

The units for **capacity** and the units for **volume** are closely related.

1 mL of fluid will fill a cube 1 cm × 1 cm × 1 cm.

The cube has volume 1 cm^3, so we say 1 cm^3 *is equivalent to* 1 mL. We write:

$$1\ \text{cm}^3 \equiv 1\ \text{mL}$$



1 L of fluid will fill a cube 10 cm × 10 cm × 10 cm.

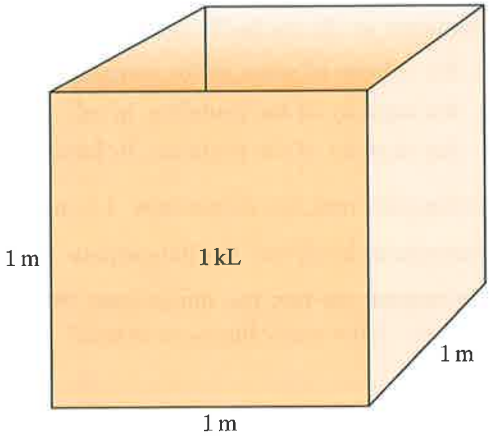
The cube has volume 1000 cm^3, so we write:

$$1000\ \text{cm}^3 \equiv 1\ \text{L}$$

1 kL of fluid will fill a cube 1 m × 1 m × 1 m.

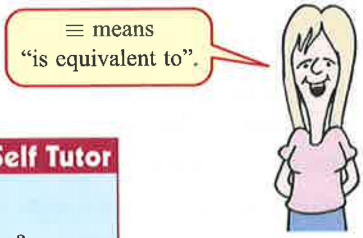
The cube has volume 1 m^3, so we write:

$$1\ \text{m}^3 \equiv 1\ \text{kL}$$



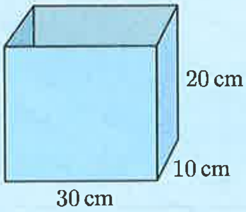
We can summarise the connection between volume units and capacity units as follows:

$$\begin{aligned} 1\ \text{cm}^3 &\equiv 1\ \text{mL} \\ 1000\ \text{cm}^3 &\equiv 1\ \text{L} \\ 1\ \text{m}^3 &\equiv 1\ \text{kL} \end{aligned}$$



Example 7 **Self Tutor**

Calculate the capacity of the container:



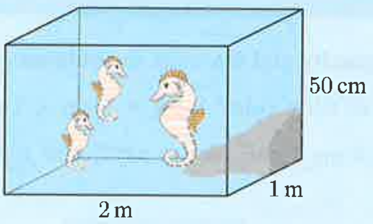
Volume
$= 30 \times 10 \times 20\ \text{cm}^3$
$= 6000\ \text{cm}^3$

$\therefore$ capacity
$= 6000$ mL
$= (6000 \div 1000)$ L
$= 6$ L

Example 8 **Self Tutor**

Find the capacity of a fishtank 2 m by 1 m by 50 cm.

Give your answer in litres.



$$\begin{aligned} \text{Volume} &= \text{length} \times \text{width} \times \text{height} \\ &= 200\text{ cm} \times 100\text{ cm} \times 50\text{ cm} \quad \{\text{units must all be the same}\} \\ &= 1\,000\,000\text{ cm}^3 \\ \therefore\ \text{capacity} &= 1\,000\,000\text{ mL} \\ &= (1\,000\,000 \div 1000)\text{ L} \\ &= 1000\text{ L} \end{aligned}$$

EXERCISE 13D

1 A container is 25 cm by 20 cm by 15 cm. Find:

- **a** the volume of space in the container, in cm^3
- **b** the capacity of the container, in mL
- **c** the capacity of the container, in litres.

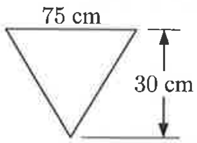
2 A rectangular tank has dimensions 1.5 m by 2 m by 4 m. Find its capacity in kL.

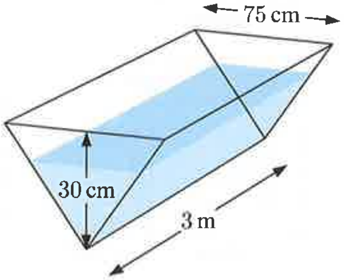
3 A rectangular lunch box has dimensions 20 cm by 12 cm by 10 cm. Find its capacity in L.

4 A rectangular ice box has dimensions 80 cm by 30 cm by 30 cm. How many litres can it hold?

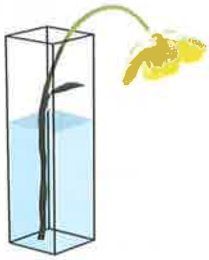
5 A 3 m long water trough has the triangular cross-section shown. Find:

- **a** the area of the triangle in cm^2
- **b** the volume of space in the trough, in cm^3
- **c** the capacity of the trough in:
  - **i** litres
  - **ii** kilolitres.





6



A vase has a square base with sides 8 cm long. The vase is 30 cm high. It is filled with 1.6 litres of water. How far from the top will the water reach?

E MASS

The **mass** of an object is the amount of matter it contains.

The **kilogram** (kg) is the base unit of mass in the SI System. Other units of mass which are commonly used are the **milligram** (mg), **gram** (g), and **tonne** (t).

An ant weighs approximately 5 milligrams.

A paper clip weighs approximately 1 gram.

A pineapple weighs approximately 1 kilogram.

A small car weighs approximately 1 tonne.

CONVERSION OF MASS UNITS

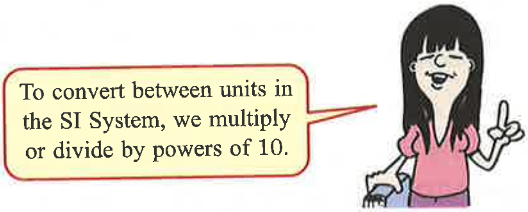
As we have done with other units of measurement, we can convert from one unit of mass to another.

$1 \text{ g} = 1000 \text{ mg}$

$1 \text{ kg} = 1000 \text{ g}$

$1 \text{ t} = 1000 \text{ kg}$

$$\begin{tikzcd}
\boxed{\text{t}} \arrow[r, bend left, "\times 1000"] & \boxed{\text{kg}} \arrow[l, bend left, "\div 1000"] \arrow[r, bend left, "\times 1000"] & \boxed{\text{g}} \arrow[l, bend left, "\div 1000"] \arrow[r, bend left, "\times 1000"] & \boxed{\text{mg}} \arrow[l, bend left, "\div 1000"]
\end{tikzcd}$$



Example 9 **Self Tutor**

Convert to grams:

a 3.2 kg **b** 735 mg **c** 4.5 tonnes

a 3.2 kg
$= (3.2 \times 1000)$ g
$= 3200$ g

b 735 mg
$= (735 \div 1000)$ g
$= 0.735$ g

c 4.5 tonnes
$= (4.5 \times 1000 \times 1000)$ g
$= 4\,500\,000$ g

Example 10 **Self Tutor**

Convert to kilograms:

a 8.6 t **b** 5860 g **c** 39 000 mg

a 8.6 t
$= (8.6 \times 1000)$ kg
$= 8600$ kg

b 5860 g
$= (5860 \div 1000)$ kg
$= 5.86$ kg

c 39 000 mg
$= (39\,000 \div 1000)$ g
$= 39$ g
$= (39 \div 1000)$ kg
$= 0.039$ kg

EXERCISE 13E

1 State the units you would use to measure the mass of:

a an apple **b** a bicycle **c** a pen
d a flea **e** a bus **f** a pig

2 Convert to grams:

a 3 kg **b** 6790 mg **c** 29 mg **d** 0.54 kg
e 1.2 t **f** 5249 mg **g** 10.37 kg **h** 0.25 t

3 Convert to kilograms:

a 6 t **b** 4000 g **c** 350 g **d** 400 000 mg
e 2.4 t **f** 285 g **g** 1436 mg **h** 50 mg

Example 11 **Self Tutor**

On average, an egg has mass 55 g. Estimate the total mass of eggs in 50 cartons, each containing 12 eggs.

Total mass ≈ 55 g $\times$ 12 eggs $\times$ 50 cartons
$\approx 33\,000$ g
$\approx (33\,000 \div 1000)$ kg
≈ 33 kg {kg is more appropriate than g}

4 Find the total mass of a packet of 12 muesli bars, each of mass 42 g.

5 If each lollipop has a mass of 4.5 g, how many lollipops are there in a 9 kg box?

6 How many bricks of mass 1.75 kg will I receive in my 7 tonne shipment of bricks?

7 Find the total mass, in kilograms, of 5000 candles, each with mass 240 g.

8 If the mass of 400 suitcases is 7.6 tonnes, find the average mass of each suitcase.

9 Paper is graded according to its weight. It is measured in *grams per square metre* or *gsm*.

A4 photocopy paper is usually 80 gsm, and a sheet of A4 paper measures 29.7 cm by 21 cm.

- **a** Find the mass of 1 m^2 of 80 gsm photocopy paper.
- **b** Find the area of 16 A4 sheets of paper, in m^2.
- **c** Find the mass of one sheet of A4 paper.

F THE RELATIONSHIP BETWEEN UNITS

The units for volume, capacity, and mass in the SI System are related as follows:

1000 cm^3 or 1 L of pure water at 4°C has mass 1 kg.
1 cm^3 or 1 mL of pure water at 4°C has mass 1 g.

Example 12 **Self Tutor**

a Find the mass of water which will fill a bucket with capacity 4 L.

b If the empty bucket has mass 250 g, what is the total mass of the bucket of water?

a 1 L of water has mass 1 kg.
$\therefore$ 4 L of water has mass 4 kg.
$\therefore$ 4 kg of water will fill the bucket.

b Mass of the water-filled bucket
= 4 kg + 250 g
= 4 kg + 0.25 kg
= 4.25 kg or 4250 g

EXERCISE 13F

1 Find the mass of 6 mL of pure water at 4°C.

2 Find the mass of 4000 cm^3 of pure water at 4°C.

3 A watering can has mass 450 g. Find the total mass of the watering can when it is filled with 3 L of water.

4 A rectangular tray is filled with water to allow laboratory equipment to soak. The tray is 40 cm long, 20 cm wide, and 12 cm high.

- **a** Find the capacity of the tray.
- **b** What mass of water is required to completely fill the tray?
- **c** When empty, the tray has mass 1.2 kg. If the tray is filled with water to a level 3 cm from the top of the tray, find the total mass of the tray and water.

G TIME

The measurement of **time** is a very important part of our lives. We encounter it in bus timetables, television guides, and school schedules.

An understanding of time allows us to organise our day and schedule events.

UNITS OF TIME

The units of time we use today are based on the rotation of the Earth and its movement around the Sun.

The time taken for the Earth to complete one rotation about its axis is called a **day**. The day is divided into hours, minutes, and seconds.

The time taken for the Earth to complete an orbit of the Sun is called a **year**.

The base unit of time in the SI System is the **second**, abbreviated **s**.

1 minute = 60 seconds

1 hour = 60 minutes = 3600 seconds

1 day = 24 hours

1 week = 7 days

1 year = $365\frac{1}{4}$ days

Example 13 **Self Tutor**

Convert:

a 4 hours and 35 minutes to minutes **b** 90 000 seconds to hours.

a 4 hours and 35 minutes $= (4 \times 60)$ min $+$ 35 min {60 min in 1 hour}
$= 275$ min

b 90 000 seconds $= (90\,000 \div 60)$ min {60 s in 1 min}
$= 1500$ min
$= (1500 \div 60)$ hours {60 min in 1 hour}
$= 25$ hours

EXERCISE 13G

1 Convert to minutes:

a 17 hours **b** 1380 seconds **c** 3 days

d 4 hours and 28 minutes **e** 5 days 11 hours and 33 minutes

2 Convert to days:

a 9 years **b** 8640 minutes **c** 1152 hours **d** 259 200 seconds

3 Convert to seconds:

a 6 hours **b** 13 minutes

c 12 hours and 16 minutes **d** 8 weeks

4 Jeremy's geography class lasted 80 minutes. Write this in hours and minutes.

5 The film went for 2 hours and 14 minutes. Write this in minutes.

6 Marcia has spent 45 minutes exercising each day for the last 5 years. Calculate the total time she has spent exercising in the last 5 years. Give your answer to the nearest day.

7 A cake decorator decorated 12 cakes in 25 hours and 24 minutes. On average, how long did it take to decorate each cake? Give your answer in hours and minutes.

H TIME CALCULATIONS

The **duration** of an event is the time the event lasts for. We often talk about the duration of a movie, a talk, or a flight.

The duration of an event is found by calculating the difference between its starting and finishing times.

Example 14 **Self Tutor**

What is the time difference between 8:45 am and 2:30 pm?

8:45 am to 9:00 am = 15 min
9:00 am to 2:00 pm = 5 h
2:00 pm to 2:30 pm = 30 min
$\therefore$ the time difference is 5 h 45 min

EXERCISE 13H

1 Find the time difference between:
- **a** 4:10 am and 8:35 am
- **b** 10:33 am and 5:49 pm
- **c** 3:20 pm and 6:08 pm
- **d** 9:52 am and 1:38 pm
- **e** 9:27 pm and 6:30 am the next day
- **f** 7:45 am and 10:05 am the next day.

2 A theatre show started at 7:30 pm and finished at 9:12 pm. How long was the show?

3 Paula started a marathon at 10:50 am and finished at 1:13 pm. How long did she take to complete the marathon?

4 Tai went to sleep at 10:15 pm and woke up at 7:10 am the next morning.
- **a** For how long did Tai sleep?
- **b** The school bus leaves at 8:02 am. How long does Tai have to get ready?

5 Study the bus timetable for the route from Darcy Downs to the City.

	Darcy Downs	*Rollings Rise*	*Standburn*	*Emmington*	*Marysville*	*City*
am	7:04	7:09	7:21	7:28	7:32	7:41
am express	8:15			8:31		8:48
pm	4:09	4:13	4:20	4:26	4:29	4:33

Determine how long it takes for the bus to get from Darcy Downs to the City:

a in the morning **b** using the morning express **c** in the afternoon.

6 A hairdresser works from 8 am to 6 pm. Her schedule for today is shown.

a Who has the longest appointment?

b Who has the shortest appointment?

c Find the longest continuous time period with no appointments.

d Another customer requests a 90 minute appointment. Will the hairdresser be able to see her today?

e The hairdresser charges £50 per hour. How much money will she earn today?

8:30 - 9:30	Tracey
10:15 - 10:30	Terry
10:30 - 11:00	Adrian
11:15 - 1:00	Deborah
1:30 - 2:15	Carol
3:30 - 4:00	Ihaka
4:00 - 4:30	Alex
4:30 - 6:00	Michelle

Example 15 **Self Tutor**

What is the time:

a $4\frac{1}{2}$ hours after 11:20 am **b** $2\frac{1}{4}$ hours before 5:10 pm?

a $4\frac{1}{2}$ hours after 11:20 am $= 11{:}20$ am $+ 4$ h $+ 30$ min
$= 3{:}20$ pm $+$ 30 min
$= 3{:}50$ pm

b $2\frac{1}{4}$ hours before 5:10 pm $= 5{:}10$ pm $- 2$ h $- 15$ min
$= 3{:}10$ pm $-$ 15 min
$= 2{:}55$ pm

7 Calculate the time:

a 6 hours after 2:12 pm **b** 3 hours before 7:09 pm

c $3\frac{1}{2}$ hours after 9:00 am **d** $8\frac{1}{2}$ hours after 8:40 am

e $2\frac{1}{4}$ hours before 10:20 am **f** 11 hours after 5:30 pm.

8 Lauren drives from her house to her sister's house which is $2\frac{1}{4}$ hours away. If Lauren leaves her house at 11:30 am, at what time will she arrive at her sister's house?

9 Russ is making dinner for a party, and wants it to be ready for 7 pm. The dinner will take $1\frac{1}{2}$ hours to prepare, and 45 minutes to cook. At what time should Russ start making the dinner?

10 The daily schedule of performances at a marine park is shown opposite.

	Duration	*Times*
Diving Dolphins	25 min	9:50, 1:40, 4:00
Whale Mania	30 min	10:15, 12:45, 3:15
Seal of Approval	35 min	11:30, 2:00
Otter Odyssey	20 min	10:00, 3:45
3D Underwater World	40 min	10:10, 12:30, 3:50
Marine Park Parade	25 min	4:00

a Which performance is shortest?

b Which performance finishes latest?

c At what time will the 1:40 dolphin performance end?

d Which performances will be in progress at 10:30?

e Justine arrives at the park $1\frac{3}{4}$ hours before the first seal performance starts. At what time does she arrive?

f Jim wants to see the 10:00 otter performance, then the next available whale performance. How much time will he have to wait between the performances?

g Harriet wants to see all six performances in one day.

- **i** Find the total amount of time she will spend watching performances.
- **ii** Determine the time at which she will need to see each performance.

I TIME ZONES

At any given time, different parts of the world are experiencing different phases of day and night.

For example, when it is the middle of the day in New York, Bangkok is in complete darkness.

This means the time of day varies depending on where you are.

Until around the year 1500, every city and town would calculate their own time by measuring the position of the Sun. This meant that cities that were only a short distance from each other would use slightly different times.

To solve this problem, the world was divided into **time zones**.

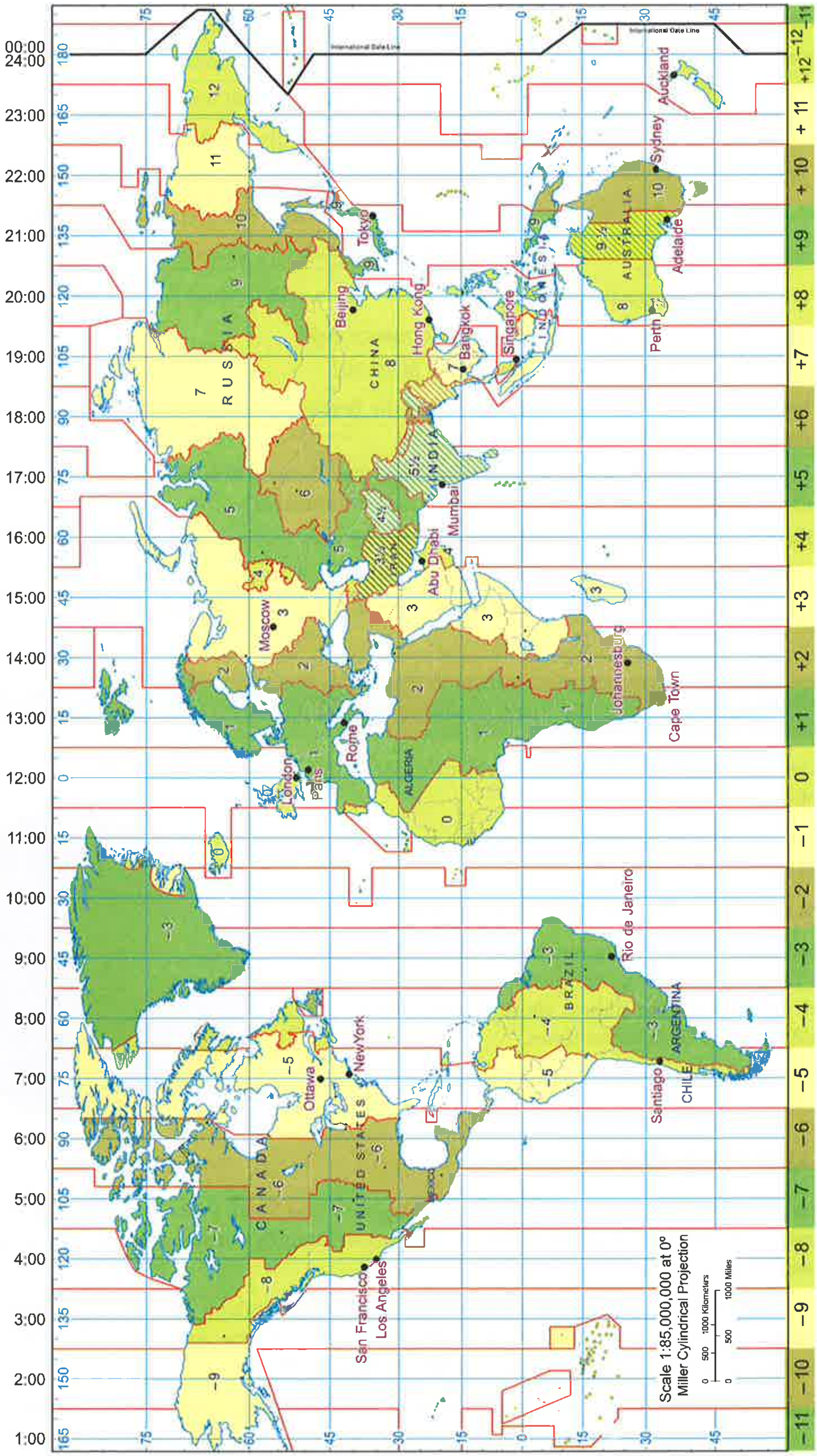
STANDARD TIME ZONES

The map on page **284** shows lines that run between the North and South Poles. They are not straight lines like the lines of longitude, but rather follow the borders of countries or regions, and natural boundaries such as rivers and mountains.

The first line of longitude, 0°, passes through Greenwich near London. This first or **Prime Meridian** is the starting point for 12 time zones west of Greenwich, and 12 time zones east of Greenwich. Time along the Prime Meridian is called **Greenwich Mean Time (GMT)**.

Places which lie in the same time zone share the same **standard time**. Standard Time Zones are usually measured in 1 hour units, but there are also a few $\frac{1}{2}$ hour units around the world.

STANDARD TIME ZONES OF THE WORLD



https://www.cia.gov/library/publications/the-world-factbook/reference_maps/pdf/time_zones.pdf

Places to the **east** of the Prime Meridian are **ahead** of GMT.

Places to the **west** of the Prime Meridian are **behind** GMT.

The map on page **284** shows the main time zones of the world. The numbers in the zones show how many hours have to be added or subtracted from Greenwich Mean Time to find the standard time in that zone.

Example 16 **Self Tutor**

If it is 12 noon in Greenwich, what is the standard time in:

a Mumbai **b** Los Angeles?

a Mumbai is in a zone marked $+5\frac{1}{2}$

$\therefore$ the standard time in Mumbai is $5\frac{1}{2}$ hours ahead of GMT.

$\therefore$ the standard time in Mumbai is 5:30 pm.

b Los Angeles is in a zone marked -8

$\therefore$ the standard time in Los Angeles is 8 hours behind GMT.

$\therefore$ the standard time in Los Angeles is 4 am.

EXERCISE 13I

1 If it is 12 noon in Greenwich, what is the standard time in:

a Ottawa **b** Cape Town **c** Hong Kong **d** Perth?

2 If it is 11 pm on Tuesday in Greenwich, what is the standard time in:

a Auckland **b** Abu Dhabi **c** Tokyo **d** Santiago?

3 If it is 5 pm on Friday in Sydney, what is the standard time in:

a Los Angeles **b** Paris **c** Moscow **d** Beijing?

4 Clint lives in Ottawa. At 11 am Ottawa time, he rang his brother Kirk, who lives in Rome. At what time did Kirk receive the call in Rome?

5 The 2014 World Cup football final started at 4 pm in Rio de Janeiro. What time did the game start for people watching the game in:

a Santiago **b** Paris **c** Tokyo?

Example 17 **Self Tutor**

A flight from Perth to Sydney leaves at 7 am Perth time, and takes 4 hours. What is the local time when the plane arrives in Sydney?

The flight leaves at 7 am Perth time, and takes 4 hours.

$\therefore$ the plane arrives in Sydney at 11 am Perth time.

Now Perth is in a zone marked $+8$, and Sydney is in a zone marked $+10$.

$\therefore$ the standard time in Sydney is 2 hours ahead of Perth.

$\therefore$ the plane arrives in Sydney at 1 pm local time.

6 Drew takes a 2:00 pm flight from Los Angeles to New York. The flight takes $5\frac{3}{4}$ hours. What is the time in New York when he arrives?

7 A flight from Mumbai to Johannesburg leaves at 11:50 pm. The flight takes 11 hours and 40 minutes. What is the local time in Johannesburg when the plane arrives?

8 Alison takes a flight from Sydney to Singapore. She leaves at 10 am Sydney time, and arrives at 4 pm Singapore time. How long was the flight?

KEY WORDS USED IN THIS CHAPTER

- capacity
- cubic millimetre
- hour
- mass
- millilitre
- second
- time zone
- volume
- cubic centimetre
- day
- kilogram
- megalitre
- minute
- standard time
- tonne
- week
- cubic metre
- gram
- kilolitre
- milligram
- prism
- time
- uniform cross-section
- year

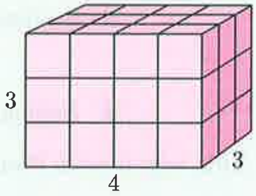
REVIEW SET 13A

1 Convert:

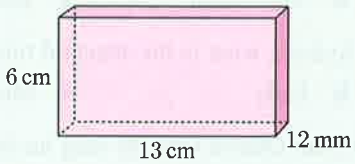
a 5860 mL to L
b 4 days and 9 hours to minutes
c 46 L to kL
d 2.5 t to kg
e 2.36 m^3 to cm^3
f 7020 cm^3 to L

2 Find the volume of:

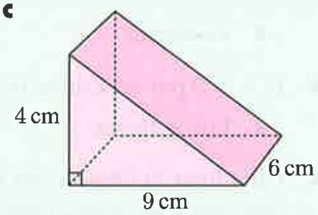
a



b



c



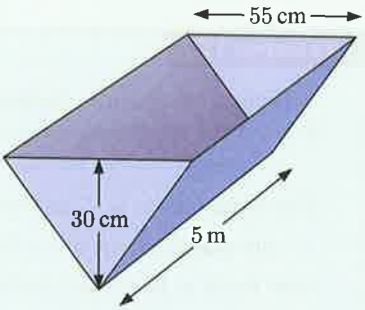
3 A truck driver charges €19.40 per cubic metre for delivering dirt. How much should the driver charge for filling a rectangular excavation 13 m long by 8 m wide by 2.2 m deep?

4 Find the total mass of 6000 textbooks, each with mass 750 g.

5 A water trough 5 m long has the triangular cross-section shown.

a Find the capacity of the trough in kilolitres.

b Find the mass of water required to fill the trough.

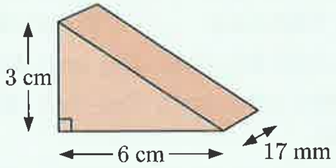


6 A ferry takes 12 minutes to cross a river. How many river crossings does the ferry make if it travels continuously for 6 hours?

7 Calculate the time:

a 3 hours after 10:20 am
b $1\frac{1}{2}$ hours before 5:16 pm
c $2\frac{1}{4}$ hours after 2:22 pm
d $4\frac{1}{2}$ hours before 12:45 pm

8 Find the volume of wood in this door wedge.



9 Answer the **Opening Problem** on page **266**.

10 If it is 3 pm in Beijing, what is the standard time in:

a Abu Dhabi
b Ottowa?

REVIEW SET 13B

1 Convert:

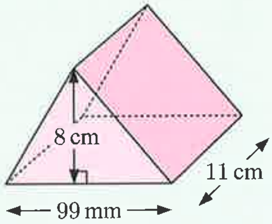
a 7 h 43 min to min
b 32 700 mm^3 to cm^3
c 5700 mg to g
d 3.9 L to mL
e 1200 hours to days
f 1.2 kL to cm^3

2 A cubic metre of wood is cut into dominoes with dimensions 5 cm × 2 cm × 1 cm. How many dominoes are made?

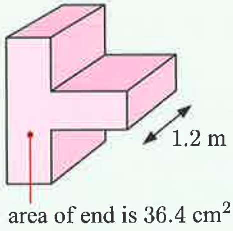
3 How long is it between 1:49 pm and 6:53 am the next day?

4 Find the volume of each prism:

a



b



5 There are seven 2 L bottles and six 375 mL cans of soft drink in a fridge. How many litres of soft drink are in the fridge?

6 A rectangular tank with base measuring 3.2 m by 1.5 m, contains water to a height of 50 cm. If the level of the water is increased by 10 cm, how much more water has been added to the tank?

7 Jess leaves the house at 7:20 am. She walks for 12 minutes to the bus stop, then waits 6 minutes for the bus. The ride to school takes 35 minutes. Jess takes 8 minutes to walk to her classroom. At what time does Jess arrive at class?

8 A large water bottle holds 7 litres of water.

a Find the mass of the water.
b If the bottle has a mass of 950 g, find the total mass of the water-filled bottle.

9 Caleb takes a 1:00 am flight from Hong Kong to Rome. The flight takes $12\frac{1}{2}$ hours. What is the time in Rome when he arrives?

10 A furniture store has the opening hours shown.

a For how long is the store open on:

i Wednesday **ii** Saturday?

b On which day is the store open the longest?

c For how many hours does the store open each week?

OPENING HOURS

Mon - Wed	9:00 am - 7:00 pm
Thursday	9:00 am - 9:00 pm
Friday	8:30 am - 6:00 pm
Saturday	10:00 am - 5:30 pm
Sunday	11:00 am - 5:00 pm

Chapter 14

Ratio

Contents:

OPENING PROBLEM

To make a chocolate milkshake, Joel usually combines 20 mL of chocolate topping with 200 mL of milk. However, when he looks in the fridge he finds there is only 100 mL of milk left.

Things to think about:

a If Joel still adds 20 mL of chocolate topping to the milk, will it taste the same as usual?

b How much chocolate topping should Joel add so that it tastes the way he likes it?

A RATIO

We often hear statements about:

- a team's win-loss ratio
- the teacher-student ratio in a school
- mixing ingredients in a particular ratio.

A **ratio** is an ordered comparison of quantities of the **same kind**.

Carol bought some industrial strength disinfectant for use in her hospital ward. It is important to mix the disinfectant and water in the correct ratio so that the disinfectant will kill germs without wasting chemicals unnecessarily.

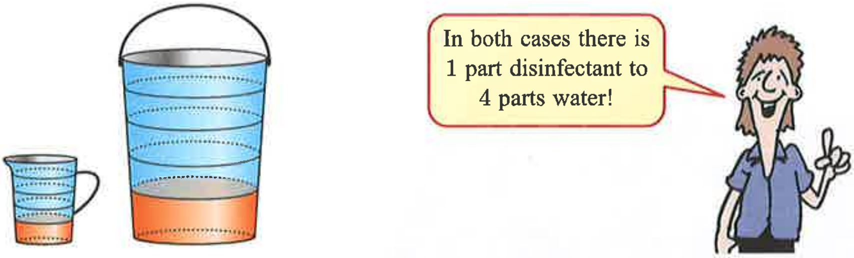
The bottle instructs her to "mix one part disinfectant to four parts water".

Disinfectant and water are both liquids, so this statement can be written as a **ratio**.

We say the ratio of disinfectant to water is 1 : 4 or "1 is to 4".

Notice that the ratio is written *without units* such as mL or L. However, when the disinfectant and water are mixed, their units must be the same.

Carol may make a jug or a bucket of disinfectant. As long as she mixes it in the correct ratio, it will be effective.



Example 1 **Self Tutor**

Express as a ratio: 3 km is to 5 km

"3 km is to 5 km" means 3 : 5

EXERCISE 14A

1 Express as a ratio:

- **a** \$4 is to \$5
- **b** 15 mL is to 8 mL
- **c** 1 tonne is to 4 tonnes
- **d** 8 m is to 7 m
- **e** 9 kg is to 5 kg
- **f** 2 mm is to 11 mm

2 Write a simple ratio to describe the following:

- **a** number of red balloons to number of blue balloons

- **b** number of teachers to number of students

- **c** number of cats to number of mice

- **d** number of basketballs to number of tennis balls

Example 2 🔊 **Self Tutor**

Express as a ratio: 7 minutes is to 2 hours

7 minutes is to 2 hours = 7 minutes : 120 minutes
= 7 : 120

3 Express as a ratio:

- **a** 17 cents is to \$1
- **b** 50 seconds is to 1 minute
- **c** 1 kg is to 150 g
- **d** 9 months is to 2 years
- **e** 12 minutes is to 3 hours
- **f** 400 kg is to 1 tonne

Example 3 🔊 **Self Tutor**

Write as a ratio: Keith spends two hours watching TV and three hours doing homework.

TV : homework = 2 : 3

4 Write as a ratio:

- **a** Jess is 152 cm tall and Carly is 164 cm tall.
- **b** At the cricket there are 2 female spectators for every 5 male spectators.
- **c** A farmer has 3 dogs for every 500 sheep.
- **d** There are 20 people skiing for every 12 people snow boarding.

5 Write as a ratio:

- **a** I spend £8 for every £5 I save.
- **b** Mix 200 mL of cordial concentrate with 800 mL of water.
- **c** For every 2 km I walk, I run 700 m.
- **d** A restaurant makes 350 g of chips for every 1 kg of meat served.

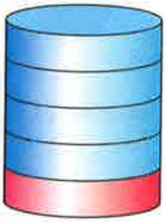
B WRITING RATIOS AS FRACTIONS

We can write ratios as fractions by considering the *total number of parts* in the ratio.

For example, Carol's mixture of hospital disinfectant combines disinfectant and water in the ratio 1 : 4.

The ratio contains $1 + 4 = 5$ parts in total. For every 5 parts of the mixture, 1 part is disinfectant and 4 parts are water.

So, $\frac{1}{5}$ of the mixture is disinfectant, and $\frac{4}{5}$ of the mixture is water.



Example 4 **Self Tutor**

The ratio of girls to boys in a class is 3 : 4.

What fraction of the class are:

a girls **b** boys?

The ratio contains $3 + 4 = 7$ parts in total. Of the 7 parts, 3 parts are girls and 4 parts are boys.

a $\frac{3}{7}$ of the class are girls. **b** $\frac{4}{7}$ of the class are boys.

EXERCISE 14B

1 The ratio of adults to children visiting a theme park is 3 : 5. What fraction of the visitors are:

a adults **b** children?

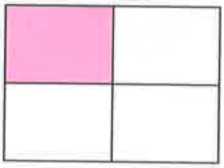
2 A fruit punch is made by mixing pineapple juice and orange juice in the ratio 2 : 3. What fraction of the fruit punch is:

a pineapple juice **b** orange juice?

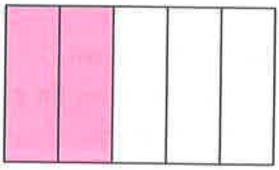
3 For each of the following figures, find:

- **i** the ratio of the shaded area to the unshaded area
- **ii** the fraction of the figure which is shaded
- **iii** the percentage of the figure which is shaded.

a

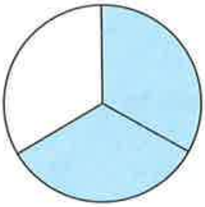


b

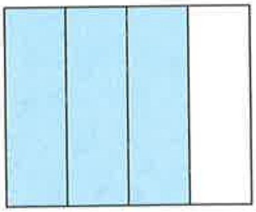


c

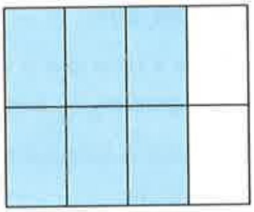
d



e



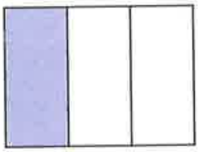
f



C EQUAL RATIOS

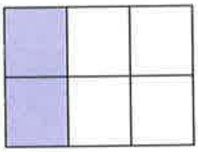
Consider the following diagrams. In each case the ratio shaded area : unshaded area is written below the figure.

A



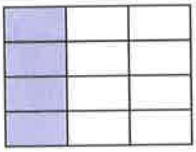
1 : 2

B



2 : 4

C



4 : 8

However, by looking at the diagrams we can see that the fraction of the total area which is shaded is the same in each case. The ratios for the shaded area to the unshaded area are therefore **equal**.

We can write that $1 : 2 = 2 : 4 = 4 : 8$.

We can see whether ratios are equal in the same way we see if fractions are equal.

Just as $\frac{1}{2} = \frac{2}{4} = \frac{4}{8}$, $1 : 2 = 2 : 4 = 4 : 8$.

If we multiply or divide both parts of a ratio by the same non-zero number, we obtain an **equal ratio**.

A ratio is in **simplest form** when it is written in terms of whole numbers with no common factors.

Example 5 **Self Tutor**

Express in simplest form:

a $8 : 16$ **b** $35 : 20$

a
$$\begin{aligned} &8 : 16 \\ &= 8 \div 8 : 16 \div 8 \\ &= 1 : 2 \end{aligned}$$

b
$$\begin{aligned} &35 : 20 \\ &= 35 \div 5 : 20 \div 5 \\ &= 7 : 4 \end{aligned}$$

To express a ratio in simplest form, divide by the **highest common factor**.

Two ratios are **equal** if they can be written in the same simplest form.

EXERCISE 14C

1 Write a ratio that is equal to $4:10$ by:

a multiplying both parts by 3

b dividing both parts by 2.

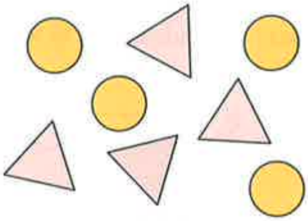
2 Express in simplest form:

a $2:6$ **b** $9:3$ **c** $2:10$ **d** $12:4$ **e** $40:50$

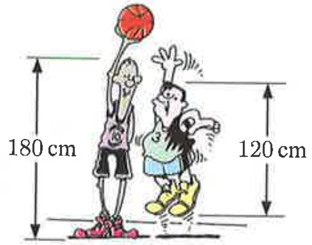
f $14:8$ **g** $10:25$ **h** $36:24$ **i** $12:18$ **j** $16:56$

3 Express as a ratio in simplest form:

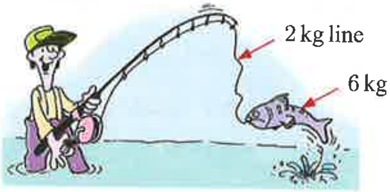
a the number of triangles to the number of circles



b the height of the tall player to the height of the short player

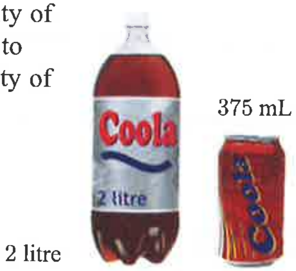


c the weight of the fish to the breaking strain of the fishing line

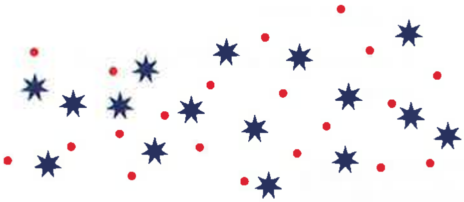


d the number of giraffes to the number of zebra

e the capacity of the bottle to the capacity of the can

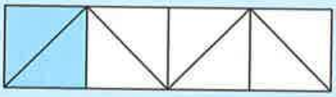


f the number of dots to the number of stars



Example 6 Self Tutor

Express in simplest form the ratio of shaded area : unshaded area.



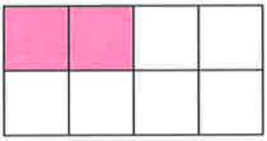
shaded area : unshaded area $= 2:6$

$= 2 \div 2 : 6 \div 2$ {HCF = 2}

$= 1:3$

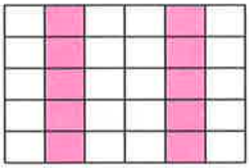
4 Express in simplest form the ratio of shaded area : unshaded area.

a

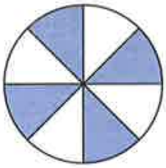


b

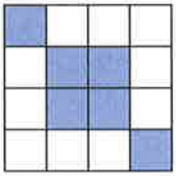
c



d



e



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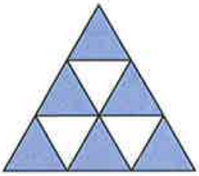


Example 7 **Self Tutor**

Express as a ratio in simplest form:

a 2 hours to 4 minutes

b 45 cm to 3 m

a 2 hours to 4 minutes
= 2 × 60 min to 4 min
= 120 min to 4 min
= 120 : 4
= 120 ÷ 4 : 4 ÷ 4 {HCF = 4}
= 30 : 1

b 45 cm to 3 m
= 45 cm to 300 cm
= 45 : 300
= 45 ÷ 15 : 300 ÷ 15
= 3 : 20

5 Express as a ratio in simplest form:

a 25 kg to 250 kg
b 30 cents to $1
c 14 cm to 28 cm
d 400 mL to 2 L
e 3 months to 1 year
f 200 m to 2 km
g 21 min to 14 min
h 350 g to 7 kg
i 2 hours to 30 min
j 5 days to 1 week
k 4 m to 80 cm
l 600 g to 1 kg
m 24 min to 1 hour
n 4 seconds to 1 min
o 30 seconds to 1 hour
p 280 mL to 0.32 L

Remember to convert to the same units!

Example 8 **Self Tutor**

Show that the ratio 4 : 6 is equal to 20 : 30.

4 : 6
= 4 ÷ 2 : 6 ÷ 2
= 2 : 3

and

20 : 30
= 20 ÷ 10 : 30 ÷ 10
= 2 : 3

∴ 4 : 6 = 20 : 30

Two ratios are equal if they have the same simplest form.

6 Which of the following pairs of ratios are equal?

a $3:2,\ \ 9:6$ **b** $4:8,\ \ 12:24$ **c** $7:3,\ \ 12:6$ **d** $2:6,\ \ 6:18$

e $4:7,\ \ 16:21$ **f** $3:9,\ \ 4:16$ **g** $20:50,\ \ 4:10$ **h** $32:24,\ \ 45:30$

Example 9 **Self Tutor**

Find $\square$ in $2:5=6:\square$

$$2:5=6:\square$$

$$\therefore\ \underset{\times 3}{\overset{\times 3}{2:5=6:\square}}$$

$$\therefore\ \square=5\times 3=15$$

Look at the first number in each ratio. We multiply by 3 to get from 2 to 6. We do the same with the second number.

7 Find the missing number:

a $2:3=4:\square$ **b** $3:1=9:\square$ **c** $2:11=6:\square$

d $3:8=\square:40$ **e** $2:3=\square:27$ **f** $9:2=36:\square$

g $15:20=3:\square$ **h** $8:12=\square:3$ **i** $32:8=4:\square$

j $20:\square=2:1$ **k** $\square:21=4:3$ **l** $8:\square=2:5$

D PROBLEM SOLVING USING RATIOS

In many situations we know two quantities have a certain ratio. If we know one of the quantities, we can work out the other quantity.

Example 10 **Self Tutor**

The student to leader ratio at a youth camp must be $9:2$. If there are 63 students enrolled, how many leaders are needed?

students : leaders $=9:2$

$$\therefore\ \underset{\div 7}{\overset{\div 7}{63:\square=9:2}}$$

$$\therefore\ \square\div 7=2$$

$$\therefore\ \square=14$$

So, 14 leaders are needed.

EXERCISE 14D

1 A hospital employs nurses and doctors in the ratio $10:3$.

- **a** If there are 120 nurses, find the number of doctors.
- **b** If there are 45 doctors, find the number of nurses.

2 The ratio of teachers to students in a school is $1:18$. If there are 360 students, find the number of teachers.

3 A store sells 8 DVDs for every 3 CDs sold. If the store sold 56 DVDs yesterday, how many CDs did it sell?

4 A car manufacturer produces station wagons and sedans in the ratio 2 : 5. Last month they made 140 sedans. How many station wagons did they make?

5 Consider the **Opening Problem** on page **290**.

a Find the ratio of chocolate topping to milk when Joel makes a chocolate milkshake.

b If Joel only has 100 mL of milk, how much chocolate topping should he use?

6 After school, Sasha likes to make a snack by mixing raisins and nuts in the ratio 3 : 5. When she checked the cupboard today, there were only 60 g of raisins and 75 g of nuts left. What is the largest snack Sasha can make while still using the correct ratio of ingredients?

E USING RATIOS TO DIVIDE QUANTITIES

If we are given a quantity to be divided in a certain ratio, we can use **fractions** to determine the size of each portion.

Example 11 **Self Tutor**

I wish to divide \$100 in the ratio 2 : 3 to give to my children Petra and Sam. How much does each one receive?

The ratio contains $2+3=5$ parts in total.

Petra gets $\frac{2}{5}$ of the money, and Sam gets $\frac{3}{5}$ of the money.

$\therefore$ Petra gets $\frac{2}{5}$ of \$100 $= \frac{2}{5}\times \$100 = \$40$

and Sam gets $\frac{3}{5}$ of \$100 $= \frac{3}{5}\times \$100 = \$60$

Check: $\$40 + \$60 = \$100$ ✓

EXERCISE 14E

1 A bag of 18 chocolates is divided between Nick and Petrov in the ratio 2 : 1.

a What fraction of the chocolates does: **i** Nick **ii** Petrov receive?

b How many chocolates does: **i** Nick **ii** Petrov receive?

2 Christina makes beetroot dip by combining beetroot and yoghurt in the ratio 5 : 3. How much of each ingredient will she need to make:

a 200 g of dip **b** 600 g of dip?

3 In a recipe for punch, the ratio of pineapple juice to orange juice is 2 : 3. How many mL of each juice is needed to make:

a a 400 mL glass of punch **b** a 1 L jug of punch?

4 Divide: **a** £30 in the ratio 1 : 5 **b** \$28 in the ratio 5 : 2

5 €600 is divided in the ratio 4 : 1. What is the larger share?

6 ¥160 000 is divided in the ratio 3 : 7. What is the smaller share?

DISCUSSION

- Have a look at your school timetable. Work out how much time is allocated to each subject in a week. Discuss the ratio of times allocated to:
  - Mathematics compared with Science
  - English compared with Art.
- If you were to divide your school week between sciences and humanities in the ratio 5 : 4, how much time would be allocated to sciences?

F SCALE DIAGRAMS

When designing a house, it would be ridiculous for an architect to draw a full-size plan.

Instead, the architect draws a smaller diagram in which all measurements have been divided by the same number or **scale factor**.

For house plans a scale factor of 100 would be suitable, since a length which is 3 m in reality would be drawn as 3 m ÷ 100 = 3 cm on the diagram.

A map of Brazil must accurately show the shape of the country. All distances are therefore divided by the same scale factor. In this case the scale factor is 80 000 000.

In a **scale diagram**:

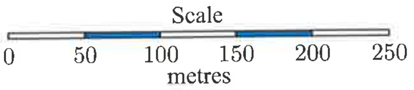
- all lengths are divided by the same **scale factor**
- all angles are unaltered.

To properly use a scale diagram, we need to know the scale used.

Scales are commonly given in the following ways:

- *Scale*: 1 cm represents 50 m.

This tells us that 1 cm on the scale diagram represents 50 m on the real thing.

- 

  This scale tells us that 1 cm on the scale diagram represents 50 m on the real thing.

- *Scale*: 1 : 5000

  This ratio tells us that 1 unit on the scale diagram represents 5000 of the same units on the real thing.
  For example:
  1 cm would represent 5000 cm or 50 m,
  1 mm would represent 5000 mm or 5 m.

Scales are written in ratio form as drawn length : actual length.

We usually simplify the scale to an equal ratio of the form 1 : the scale factor.

Example 12 **Self Tutor**

On a scale diagram, 1 cm represents 20 m.

a Write the scale as a ratio. **b** What is the scale factor?

a 1 cm to 20 m
= 1 cm to (20 × 100) cm
= 1 cm to 2000 cm
= 1 : 2000

b The ratio simplifies to 1 : 2000 so the scale factor is 2000.

Example 13 **Self Tutor**

Interpret the ratio 1 : 5000 as a scale.

1 : 5000 means 1 cm represents 5000 cm
$\therefore$ 1 cm represents (5000 ÷ 100) m {100 cm = 1 m}
$\therefore$ 1 cm represents 50 m

EXERCISE 14F.1

1 Write each scale as a ratio and state the corresponding scale factor:

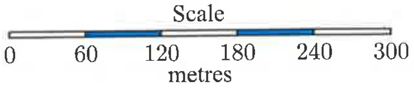
a 1 cm represents 1 m
b 1 cm represents 1 km
c 1 cm represents 30 m
d 1 mm represents 2 km
e 1 mm represents 250 m
f 1 cm represents 200 km

2 Interpret each ratio as a scale, explaining what 1 cm represents:

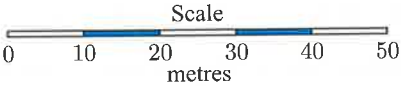
a 1 : 250
b 1 : 4000
c 1 : 500
d 1 : 25 000
e 1 : 150 000
f 1 : 22 000 000

3 Write each scale as a ratio:

a



b



USING SCALES AND RATIOS

A landscape gardener needs a scale diagram of a rectangular area 12 m by 5 m. The lengths on the scale diagram must be much less than the actual lengths so that we can fit them on paper. The landscaper wants us to use a scale of $1:50$.

We need to **divide** the actual length by the scale factor to obtain the drawn length.

drawn length = actual length ÷ scale factor

Example 14 **Self Tutor**

An object 12 m long is drawn with the scale $1:50$. Find the drawn length of the object.

$$\begin{aligned}\text{drawn length} &= \text{actual length} \div \text{scale factor}\\ &= 12\text{ m} \div 50\\ &= 0.24\text{ m}\\ &= (0.24\times 100)\text{ cm} \qquad \{1\text{ m} = 100\text{ cm}\}\\ &= 24\text{ cm}\end{aligned}$$

Now suppose we have a map with the scale $1:500\,000$. We measure the distance between towns A and B to be 15 cm. We can calculate the actual distance between towns A and B using the formula:

actual length = drawn length × scale factor

Example 15 **Self Tutor**

For a scale of $1:500\,000$, find the actual length represented by a drawn length of 15 cm.

$$\begin{aligned}\text{actual length} &= \text{drawn length} \times \text{scale factor}\\ &= 15\text{ cm}\times 500\,000\\ &= 7\,500\,000\text{ cm}\\ &= (7\,500\,000 \div 100)\text{ m} \qquad \{1\text{ m} = 100\text{ cm}\}\\ &= 75\,000\text{ m}\\ &= (75\,000 \div 1000)\text{ km} \qquad \{1\text{ km} = 1000\text{ m}\}\\ \therefore\ \text{actual length} &= 75\text{ km}\end{aligned}$$

EXERCISE 14F.2

1 Consider the scale $1:20$. Find the length which should be drawn to represent an actual length of:

a 10 m **b** 2.6 m **c** 480 cm **d** 5.6 m

2 Consider the scale $1:10\,000$. Find the actual length represented by a drawn length of:

a 3.5 mm **b** 16 cm **c** 5.2 cm **d** 6.4 mm

3 Make a scale drawing of:

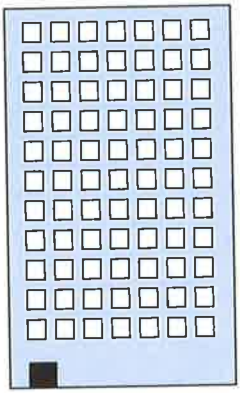
- **a** a square with sides 25 m using the scale 1 cm represents 10 m
- **b** a rectangle 3 km by 6 km using the scale 1 cm represents 2 km
- **c** a triangle with sides 10 m, 24 m, and 26 m using the scale 1 : 500
- **d** a circle of diameter 5 km using the scale 1 : 250 000.

4 Select an appropriate scale and draw a scale diagram of:

- **a** a rectangular house block 13 m by 29 m
- **b** a garage door 4.5 m by 2.2 m
- **c** a triangular park with sides 45 m, 60 m, and 75 m.

5 A scale diagram of a building is shown with scale 1 : 1000.

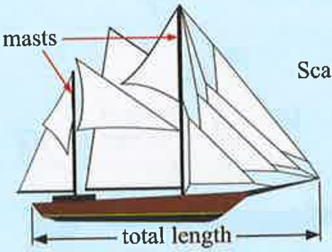
- **a** If the height is 5 cm and width is 3 cm on the drawing, find the actual height and width of the building in metres.
- **b** If the height of the windows on the drawing is 2.5 mm, how high are the actual windows?
- **c** If the actual height of the entrance door is 3.2 m, what is its height on the scale drawing?



Example 16 **Self Tutor**

Use your ruler and the given scale to determine:

- **a** the total length of the ship
- **b** the height of the taller mast
- **c** the distance between the masts.

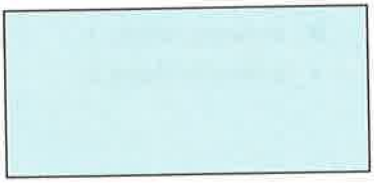


Scale: 1 : 1000

- **a** The measured length of the ship is 3.8 cm.
  So, the actual length is 3.8 cm × 1000 = 3800 cm = 38 m.
- **b** The measured height of the taller mast is 2.5 cm.
  So, the actual height is 2.5 cm × 1000 = 2500 cm = 25 m.
- **c** The measured distance between the masts = 1.4 cm.
  So, the actual distance is 1.4 cm × 1000 = 1400 cm = 14 m.

6 Consider the scale diagram of a rectangle.

- **a** Use your ruler to find the actual dimensions given that the scale is 1 : 800.
- **b** Which of the following could the rectangle represent?
  - **A** a $10 note
  - **B** a domino
  - **C** a tennis court area
  - **D** a chopping board?



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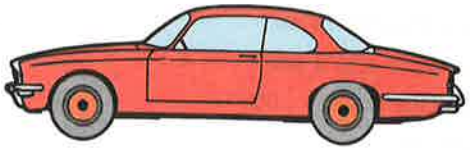


Scale: 1 : 80

Using the scale diagram, find:

a the length of the vehicle

b the diameter of a tyre

c the height of the top of the vehicle above ground level

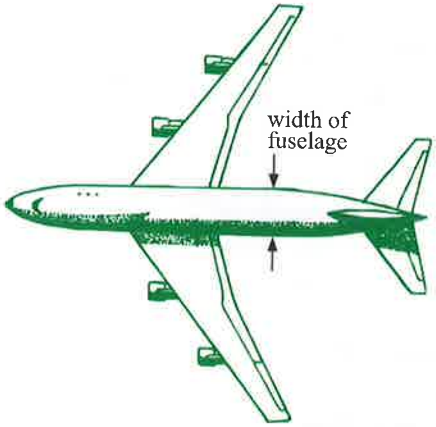
d the width of the bottom of the door.

8 Using this map of the USA, find the actual distance in a straight line between:

a New York and New Orleans **b** El Paso and Miami **c** Seattle and Denver.

Scale: 1 : 50 000 000

9



The actual length of the aeroplane in the scale drawing is 70 m.

Find:

a the scale used in the drawing

b the actual wingspan of the aeroplane

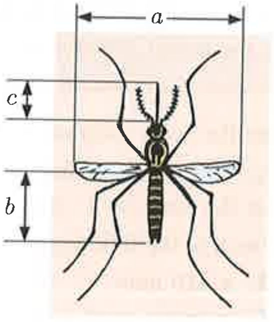
c the actual width of the fuselage.

10 The diagram is an enlargement of a mosquito, drawn to a scale of 1 : 0.25. Find the actual lengths of the dimensions marked:

a wingspan, a

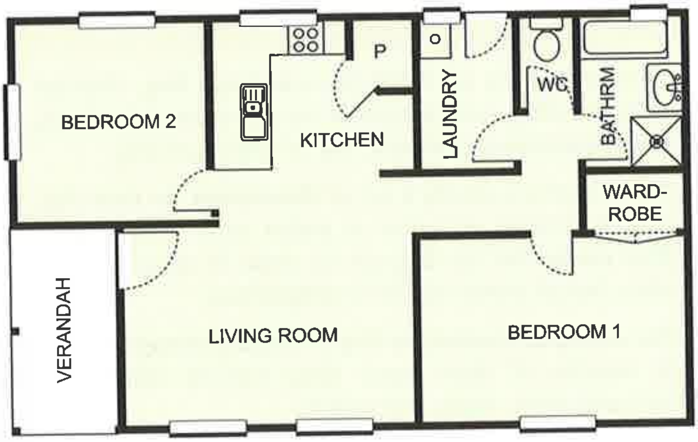
b abdomen length, b

c proboscis length, c.



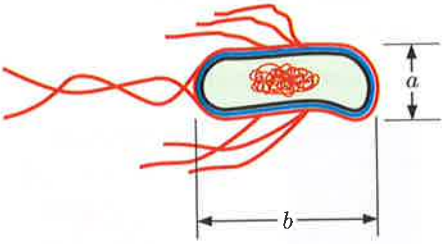
11 The floor plan of this house has been drawn using the scale 1 : 120. Find:

- **a** the external dimensions of the house, including the verandah
- **b** the dimensions of the verandah
- **c** the cost of covering each of the bedroom floors with wooden floorboards at £127.50 per square metre.



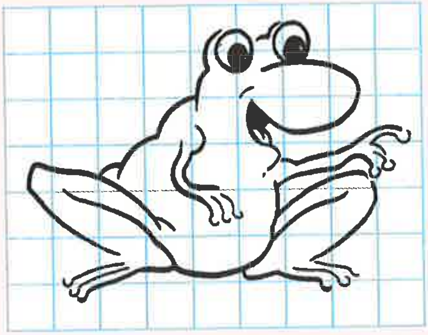
12 The diagram given shows a microscopic organism enlarged using the scale 8000 : 1.
Find the actual length of the dimensions marked:

- **a** cell width, a
- **b** cell length, b.



ACTIVITY 1 SCALE DIAGRAMS

One way to make a scale drawing is to draw a grid over the picture to be enlarged or reduced. We then copy the picture onto corresponding positions on a larger or smaller grid. Click on the icon to obtain grid paper. You could use a photocopier to further enlarge or reduce it.



GRID PAPER

ACTIVITY 2 HOUSE PLANS

What to do:

1 Use a measuring tape or ruler to find the dimensions of the rooms in your house.

2 Using an appropriate scale, draw a plan of your house like the one shown on page 298. Do not forget to include the scale on your plan.

ACTIVITY 3

FLAG RATIOS

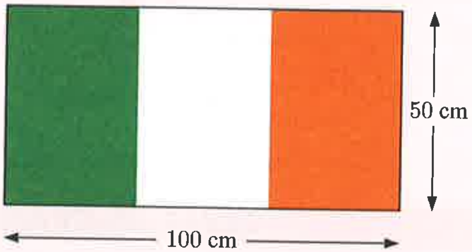
Every country in the world has a **national flag**. National flags are often seen in schools, on government buildings, at international conferences, and at sporting events.

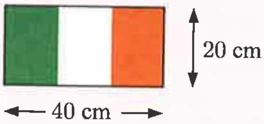
Most countries specify a set of **dimensions** for their flag. The dimensions are given as **ratios** rather than lengths. This means that the flag can be made in many different sizes, but all copies will be in **proportion**.

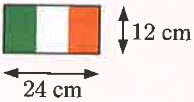
For example, consider the flag of **Ireland** alongside. It consists of three equal sized vertical stripes coloured green, white, and orange.

The **height to length ratio** of the Irish flag is 1 : 2. This means that the flag must be twice as long as it is high.

So, acceptable dimensions of the Irish flag include:







If we know the ratios of a flag and one of its dimensions, we can calculate the remaining dimensions.

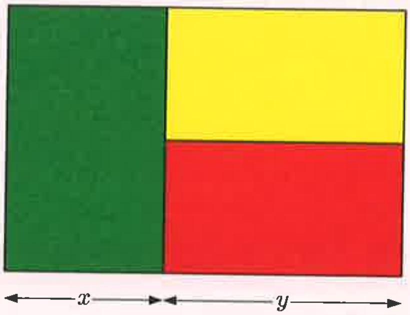
What to do:

1 An Irish flag is 90 cm high. Find the:

- **a** length of the flag
- **b** width of each of the vertical stripes.

2 The flag of **Benin** is shown alongside. The flag consists of two equal horizontal stripes coloured yellow and red, and a vertical green stripe.

The height to length ratio of the flag is 2 : 3. The width of the green stripe is in proportion such that $x : y = 2 : 3$.



A Beninese flag is 40 cm high.

- **a** Show that the flag is 60 cm long.
- **b** Find the dimensions of the:
  - **i** green stripe
  - **ii** yellow and red stripes.
- **c** Find the area of the:
  - **i** green
  - **ii** yellow
  - **iii** red portions of the flag.
- **d** What percentage of this flag is:
  - **i** green
  - **ii** yellow
  - **iii** red?

e Are the percentages you found in **d** true for all Beninese flags, or only this one?

f Find the dimensions, in centimetres, of the smallest Beninese flag such that each of the stripes have whole number dimensions.

Global context

click here

Nutrition

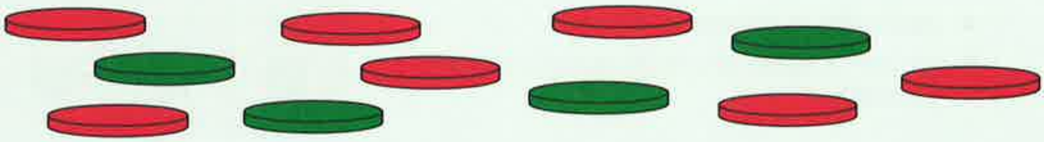
Statement of inquiry:	Understanding the ratios in which we should eat certain food groups can improve our health and well-being.
Global context:	Identities and relationships
Key concept:	Relationships
Related concepts:	Equivalence, Simplification
Objective:	Applying mathematics in real-life contexts
Approaches to learning:	Research, Self-management

KEY WORDS USED IN THIS CHAPTER

- equal ratios
- scale factor
- ratio
- simplest form
- scale diagram

REVIEW SET 14A

1 Find the ratio of red discs to green discs.



2 Express as a ratio:

a \$9 is to \$4 **b** 5 m is to 2 m **c** 11 g is to 5 g

3 Express in simplest form:

a $2:8$ **b** $24:15$ **c** $16:44$

4 Which of the following pairs of ratios are equal?

a $2:5,\ 6:15$ **b** $5:8,\ 20:36$ **c** $18:8,\ 27:12$

5 At a school, the ratio of right handed students to left handed students is $13:2$. What fraction of the students are:

a right handed **b** left handed?

6 Find the missing number:

a $7:2=21:\square$ **b** $18:10=\square:5$ **c** $\square:35=9:7$

7 A commercial vehicle yard has vans and trucks in the ratio $5:3$. If there are 35 vans in the yard, how many trucks are there?

8 When Craig exercises, he does push-ups and sit-ups in the ratio 5 : 4.
In one session Craig completed 60 push-ups.
How many sit-ups did he complete?

9 €300 is divided between Courtney and Wendy in the ratio 3 : 7.

- **a** What fraction of the money does Courtney receive?
- **b** How much money does Courtney receive?

10 Draw a scale diagram of a triangular block of land with side lengths 18 m, 24 m, and 30 m. Use the scale 1 : 500.

REVIEW SET 14B

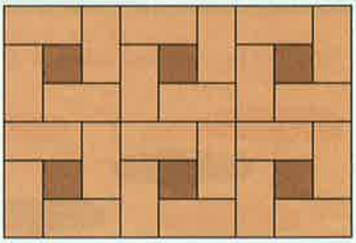
1 Write as a ratio: "I saw seven white cars for every two red cars."

2 Express as a ratio in simplest form:

- **a** 53 minutes to 2 hours
- **b** 3 cm to 9 mm
- **c** 600 mL to 4 L

3 Write a ratio that is equal to 32 : 12 by:

- **a** multiplying both parts by 2
- **b** dividing both parts by 4.



4 This tile pattern is made from a combination of square tiles and rectangular tiles.
Find, in simplest form, the ratio of:

- **a** square tiles to rectangular tiles used
- **b** the area of square tiles to the area of rectangular tiles used.

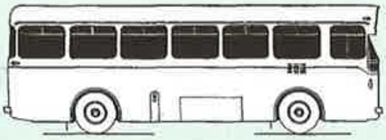
5 The width to height ratio of a television screen is 16 : 9. If the screen is 48 cm wide, find the height of the screen.

6 During the netball season, a club's win-loss ratio was 4 : 3. If the team lost 12 matches, how many did they win?

7 Water and vinegar are mixed in the ratio 4 : 5 to make cleaning liquid for a coffee pot. How much vinegar is needed to make 720 mL of cleaning liquid?

8 A fruit grower plants apple trees and pear trees in the ratio 6 : 5. If he plants a total of 1320 trees, how many of each type does he plant?

9 A 1 : 500 scale model of the Golden Gate Bridge is being built for a museum. If the actual length of the bus is 1300 metres, how long will the model be?



10 The actual length of the bus shown alongside is 10 m. Find:

- **a** the scale used for this diagram
- **b** the actual height of the windows
- **c** the actual height of the bus.

Chapter 15

Probability

Contents:

OPENING PROBLEM

Anna and her three sisters each want to sit in the front seat of the car.

Their mother places each of their names into a hat, and selects a name at random.

Things to think about:

- **a** Are each of the sisters equally likely to be selected?
- **b** How likely is it that Anna will be selected?
- **c** How likely is it that Anna will *not* be selected?

A DESCRIBING PROBABILITY

We often hear statements involving probability.
For example:

"We will probably buy a new television soon."
"It is likely that the storms will damage the crops."
"I am almost certain that I passed the exam."
"It is unlikely that our team will win today."

The key words in these statements are:
probably, *likely*, *almost certain*, and *unlikely*.

Each of these words describes probability.

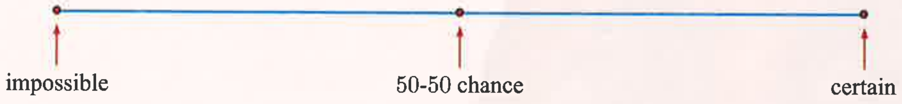
Probability deals with the likelihood or chance of events occurring.

DISCUSSION

Many words are used to describe probability. They indicate how likely an event is to occur.

For example:
possible, likely, impossible, unlikely, maybe, certain, uncertain, no chance, little chance, good chance, highly probable, probable, improbable, doubtful, often, rarely, and '50-50' chance.

Discuss what each of these expressions mean. As a class, place them on the following line, in order of how likely the event is. A few have already been placed to get you started.



Example 1

Self Tutor

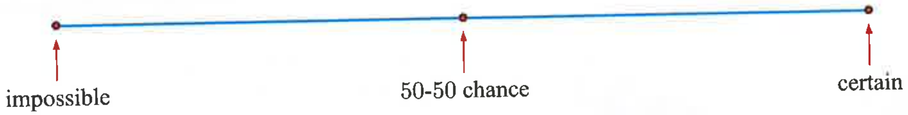
Describe, using a word or phrase, the probability of the following happening:

a A woman will be playing Olympic hockey at the age of 60.
b Sam, who is now 13, will be alive in 12 months' time.
c The next person to cross the street will be female.

a highly unlikely **b** highly likely **c** a '50-50' chance

EXERCISE 15A

1 Copy the line below, then add the following words using arrows in appropriate positions:



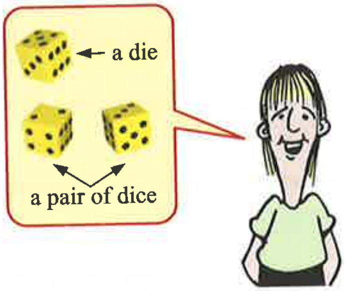
a highly likely **b** very rarely **c** almost certain
d doubtful **e** a little more than even chance **f** unlikely

2 Describe, using a word or phrase, the probability of the following happening:

a Ken, who is now 13, will live to the age of 100 years.
b You will win the major prize in Lotto in your lifetime.
c There will be water at the beach.
d Everyone in your class will have a computer.
e You will be struck by lightning next year.
f When tossed once, a coin will land on heads.
g You will get homework tonight.
h In your next car trip through the city, you will be stopped at a red light.

3 Describe the following as either *certain*, *possible*, or *impossible*:

a When rolling a die, a 6 results.
b When rolling a pair of dice, a sum of 7 results.
c When rolling a pair of dice, a sum of 14 results.
d When tossing a coin, a head results.
e When tossing a coin, it falls on its edge.
f When tossing a coin twelve times, it lands tails every time.
g When twirling each of the given square spinners, a 1 results:



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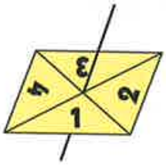

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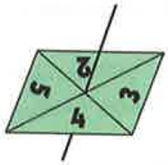

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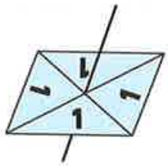


4 A bag contains 1 pink ticket and 99 purple tickets. A ticket is randomly chosen from the bag.

a Describe how likely it is that the ticket is purple.

b Is it certain that the ticket will be purple?

c True or false? "There is a 1 in 99 chance that the ticket will be pink."

5 A box contains 4 blue marbles and 7 red marbles. One marble is randomly selected from the box.

a Is it more likely that the marble is blue or red? Explain your answer.

b True or false? "There is a 4 in 11 chance that the marble will be blue."

6

A black cat has 7 kittens. 2 kittens are white and 5 are black. 3 kittens are female and 4 are male.

a If one of the eight cats is selected at random, describe the chance that it is:

i black **ii** female.

b Can you determine the chance that a randomly selected cat is black *and* female? Explain your answer.

B ASSIGNING NUMBERS TO PROBABILITIES

When we talk about probability, we can pretend we are running an experiment.

The **outcome** of an experiment is the result we obtain in one trial of the experiment.

An **event** occurs when we obtain an outcome with a particular property.

For example, suppose the die alongside is rolled. The possible outcomes are 1, 2, 3, 4, 5, and 6.

The event *an even number* occurs if we get one of the outcomes 2, 4, or 6.

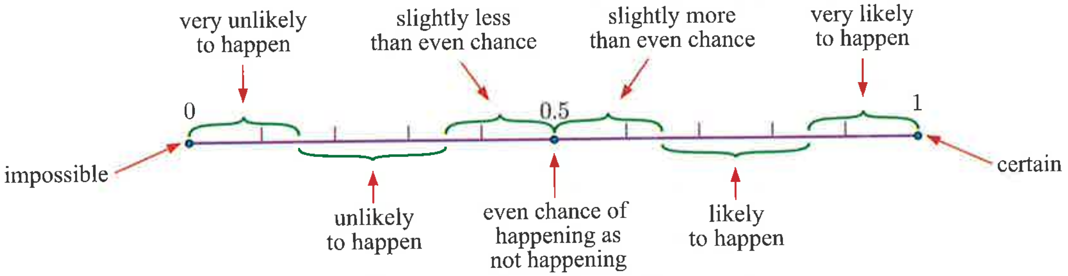
If an event is impossible, it **cannot occur**. We assign it the probability 0 or 0%.

If an event is **certain to occur** we assign it the probability 1 or 100%.

The chance of any event occurring must lie between the two extremes of impossible and certain. So, the probability of any event occurring lies between 0 and 1, or 0% and 100% inclusive.

Events which may occur or not occur with equal chance are assigned the probability 0.5 or $\frac{1}{2}$ or 50%.

This number line shows how we could describe different probabilities:



Example 2 **Self Tutor**

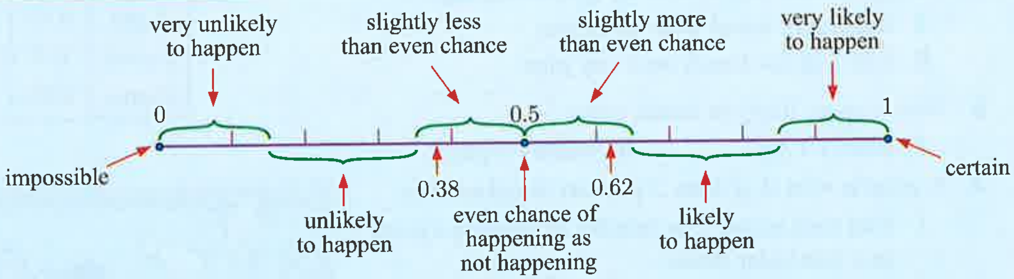
Jill and Vicki are playing a game of squash. Jill has probability 0.62 of winning, and Vicki has probability 0.38 of winning.

a Write a word or phrase to describe the probability that:

i Jill will win **ii** Vicki will win.

b Which player is more likely to win the game?

a



i There is a *slightly more than even chance* that Jill will win.

ii There is a *slightly less than even chance* that Vicki will win.

b The probability that Jill will win is higher than the probability that Vicki will win, so Jill is more likely to win the game.

EXERCISE 15B

1 Write a word or phrase to describe the probability value:

a 0.97 **b** 0.3 **c** 0.5 **d** 1 **e** 0.56

2 People entering a charity raffle are told they have a $\frac{1}{5}$ chance of winning a prize.

a Write this probability as a decimal.

b Write a word or phrase to describe the probability that a particular person will win a prize.

3 A weather forecast reported a 45% chance of snow on Saturday, and an 80% chance of snow on Sunday.

a Write a word or phrase to describe the probability of snow on:

i Saturday **ii** Sunday.

b Is it more likely to snow on Saturday or on Sunday?

4 A container holds 5 black discs and 5 white discs. One disc is selected randomly from it.

a Find the probability that the disc is black.

b All of the white discs are now removed, and another disc is randomly selected.
Find the probability that this disc is: **i** black **ii** white.

5 Each morning, Naomi catches the train to school. She catches the 8:00 am train $\frac{1}{10}$ of the time, the 8:10 am train $\frac{1}{2}$ of the time, and the 8:20 am train $\frac{2}{5}$ of the time.

a Write each of these probabilities as a decimal.

b On any particular morning, which train is Naomi most likely to catch?

c Find the probability that, on any particular morning, Naomi catches *either* the 8:00 am *or* the 8:10 am train.

d Find the sum of the probabilities for the three trains. Explain your answer.

6 Lily and Ralph enjoy playing a sideshow game where they must try to knock over a set of 3 pins with a ball.
The table alongside shows the probability of each player knocking over 0, 1, 2, or 3 pins with a particular throw.

	Lily	Ralph
0 pins	0.2	0.05
1 pin	0.45	0.35
2 pins	0.3	0.5
3 pins	0.05	0.1

a For a particular throw, find the probability that:

i Ralph will knock over all 3 pins

ii Lily will not knock over any pins.

b Who is more likely to knock over:

i exactly 1 pin **ii** exactly 2 pins?

c A prize is won if at least 2 pins are knocked over.

i Find each player's probability of winning a prize on a particular throw.

ii Use a word or phrase to describe each player's probability of winning a prize.

d Who would you say is better at the game? Explain your answer.

e Find the sum of the probabilities for each player. Can you explain the answers you obtain?

C SAMPLE SPACE

In games of chance we often use coins, dice, and spinners. We use these items because there is an equal chance of their different outcomes occurring on each throw or spin.

A **sample space** is the set of possible outcomes of an experiment.

COINS

When a **coin** is tossed there are two possible sides which could show upwards: the *head* (H) which is usually the head of a monarch, president, or leader, and the *tail* (T) which is the other side of the coin.

When tossing one coin, the sample space is {H, T}.

head

tail

DICE

The most commonly used **dice** are small cubes with the numbers 1, 2, 3, 4, 5, and 6 marked on them using dots.

The sample space when rolling one die is $\{1, 2, 3, 4, 5, 6\}$.

SPINNERS

A **spinner** can be made from a regular polygon or a circle divided into equal sectors, with a needle spinning at the centre.

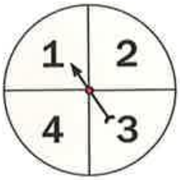
The sample space for this spinner is $\{1, 2, 3, 4, 5, 6, 7, 8\}$.

EXERCISE 15C

1 List the sample space for:

- **a** flipping a disc with a smile on one side and a frown on the other
- **b** choosing a day of the week
- **c** selecting a two digit number from 10 to 19
- **d** choosing a month of the year
- **e** twirling each of these spinners:

i



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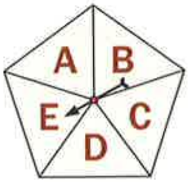


iii

2 State the number of possible outcomes for each experiment in **1**.

Example 3

Self Tutor

List the sample space for:

- **a** the different orders for the genders of two children in a family
- **b** two spins of this spinner.

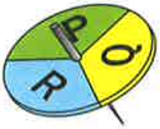
a Let B represent a boy and G represent a girl.
The sample space is $\{BB, BG, GB, GG\}$.

b Let AB represent a result of A with the first spin and B with the second spin. The sample space is $\{AA, AB, AC, AD, BA, BB, BC, BD, CA, CB, CC, CD, DA, DB, DC, DD\}$.

3 List the sample space for:

- **a** tossing a 5-cent and a 10-cent coin
- **b** the different ways in which 3 students Anna, Barry, and Catherine may line up
- **c** tossing 3 different coins simultaneously
- **d** the 8 different orders for the genders of 3 kittens in a litter

e two spins of the spinner alongside



f rolling two dice simultaneously

g the different orders in which 4 alphabet blocks W, X, Y, and Z may be placed in a line.

4 State the number of possible outcomes for each experiment in **3**.

5 Consider a ball in the middle of a flat, square table. It could roll off any edge of the tabletop with equal chance. Explain how you could write down the sample space of possible outcomes.

D THEORETICAL PROBABILITY

If the outcomes of an experiment are equally likely, we can use symmetry to generate a **mathematical** or **theoretical** probability for an event. This probability is based on what we theoretically expect to occur.

TOSSING A COIN

When a coin is tossed there are two possible outcomes. From the symmetry of the coin, we expect each of these results to occur 50% of the time, or 1 time in every 2.

We say that the probability of getting a *head* with one toss is $\frac{1}{2}$, and we write $P(H) = \frac{1}{2}$.

We read this as "the probability of a head occurring is one half".

Likewise, the probability of getting a *tail* is $\frac{1}{2}$, and we write $P(T) = \frac{1}{2}$.

ROLLING A DIE

When a die is rolled there are six possible outcomes $\{1, 2, 3, 4, 5, 6\}$ that form our sample space. The different outcomes are equally likely.

The probability of each outcome occurring is $\frac{1}{6}$, and we write, for example, $P(\text{a } 5) = \frac{1}{6}$.

For some events there may be more than one outcome.

For example:

- $P(\text{a 5 or a 6}) = \frac{2}{6}$ since 2 of the 6 outcomes correspond to the event.
- $P(\text{result is } \textit{even}) = P(\text{a 2, a 4, or a 6})$
  $= \frac{3}{6}$ since 3 of the 6 outcomes correspond to the event.

THEORETICAL PROBABILITY

In general, when we are dealing with an event in a sample space containing a finite number of equally likely outcomes:

$$P(\text{an event}) = \frac{\textbf{number of outcomes corresponding to the event}}{\textbf{total number of possible outcomes}}.$$

Example 4

Self Tutor

Tickets numbered 1 to 9 are placed in a hat, and one is drawn at random. Find the probability that the number drawn is:

a 3 or 7 **b** greater than 5.

a There are 9 possible outcomes, and there are 2 outcomes (3 and 7) corresponding to the event.

$\therefore$ P(a 3 or a 7) $= \frac{2}{9}$

b There are 4 outcomes (6, 7, 8, and 9) greater than 5.

$\therefore$ P(greater than 5) $= \frac{4}{9}$

Example 5

Self Tutor

Two blue and three white discs are placed in a bag, and one disc is randomly selected from it.

What is the probability of selecting:

a a blue disc **b** a white disc?

There are 5 discs which could be selected with equal chance.

a 2 discs are blue, so there is a 2 in 5 chance of selecting a blue disc.

$\therefore$ P(a blue disc) $= \frac{2}{5}$

b 3 discs are white, so there is a 3 in 5 chance of selecting a white disc.

$\therefore$ P(a white disc) $= \frac{3}{5}$

In a random selection, each disc has the same chance of being selected.

EXERCISE 15D

1 A six-sided die is rolled once.

- **a** List the sample space of possible outcomes.
- **b** How many possible outcomes are there?
- **c** Determine the probability that the result is:
  - **i** a 4 **ii** greater than 3 **iii** not a 6.

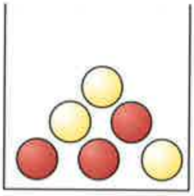
2 A letter of the alphabet is chosen at random.

- **a** How many possible outcomes are there?
- **b** Find the probability of choosing:
  - **i** T **ii** A, B, C, or D **iii** a letter contained in the word CHOCOLATE.

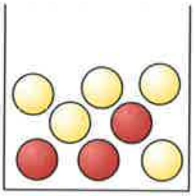
3 Yellow and red marbles are placed in a container. One marble is randomly selected from it. For each of the following containers of marbles, answer these questions:

- **i** How many of each colour marble are in the container?
- **ii** What is the probability of selecting a yellow marble?
- **iii** What is the probability of selecting a red marble?

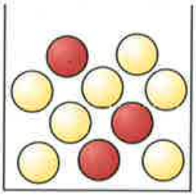
a



b

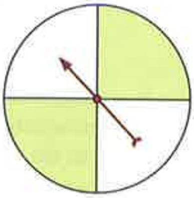


c



4 Determine the probability that the spinning needle will finish on green:

a



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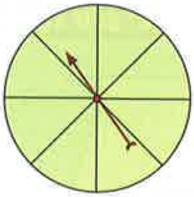

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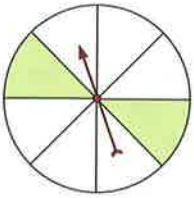

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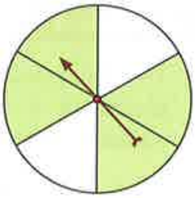


5 The illustrated spinner is a regular octagon.
If the spinner is spun once, find the probability of getting:

- **a** a 7
- **b** a 2 or 4
- **c** an even number
- **d** a result less than 1
- **e** a result greater than 3.

6 A month of the year is chosen at random. Find the probability that the chosen month:

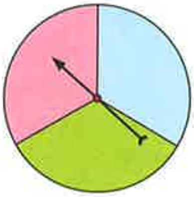
- **a** is July
- **b** starts with an 'A'
- **c** contains the letter 'r'
- **d** has 30 days
- **e** is later in the year than May.

7 A bag contains 1 blue, 2 yellow, and 5 green discs. One disc is randomly selected from it. Find the probability that the disc is:

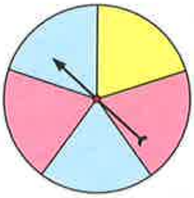
- **a** blue
- **b** yellow
- **c** green
- **d** red
- **e** not blue
- **f** not yellow
- **g** neither yellow nor blue
- **h** blue, yellow, or green
- **i** neither blue, nor yellow, nor green.

8 If each of the following spinners is spun, which is most likely to end on pink?

A

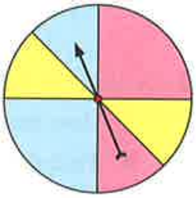


B



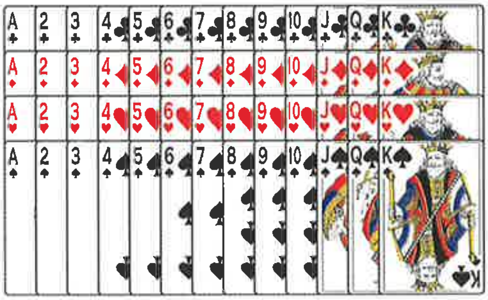
C

D



9 There are 52 cards in a pack of playing cards. They are divided into four suits: the two red suits are Hearts and Diamonds, and the two black suits are Spades and Clubs. In each suit there is an ace, the numbers 2 to 10, and three picture cards called the Jack, Queen, and King.

Frank shuffles a pack of cards thoroughly, places them face down on the table, then picks one card at random.



Determine the chance of getting:

a the Queen of ♥ **b** a club ♣ **c** an 8
d a red 10 **e** a picture card **f** a red card
g a black 2 **h** a red picture card **i** a 10 or an ace.

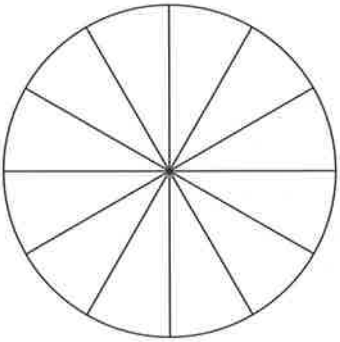
10 For this question you will need to draw spinners with 12 sectors like the one alongside.

Draw a coloured spinner so that the probability of spinning:

a red is 25% and blue is 50%

b green is $\frac{1}{3}$ and blue is $\frac{1}{4}$

c red is $\frac{2}{3}$, black is $\frac{1}{4}$, and yellow is $\frac{1}{12}$.



PRINTABLE SPINNERS

Example 6 **Self Tutor**

3 coins are tossed simultaneously. Find the probability of getting:

a exactly one head **b** at least two heads.

The possible outcomes are {HHH, HHT, HTH, THH, HTT, THT, TTH, TTT}, so there are 8 possible outcomes.

a 3 of the outcomes (HTT, THT, TTH) have exactly one head.

$\therefore$ P(exactly one head) $= \frac{3}{8}$

b 4 of the outcomes (HHT, HTH, THH, HHH) have at least two heads.

$\therefore$ P(at least two heads) $= \frac{4}{8} = \frac{1}{2}$

11 2 coins are tossed simultaneously.

a List the sample space of possible outcomes.

b How many possible outcomes are there?

c Find the probability of getting:

i two heads **ii** two tails **iii** exactly one head **iv** at least one head.

12 **a** List the 8 possible 3-child families according to gender.

b Assuming that each of them is equally likely to occur, determine the probability that a randomly chosen 3-child family consists of:

i all boys
ii all girls
iii boy, then girl, then girl
iv two girls and a boy
v a girl for the eldest
vi at least one boy.

13 Three seats are placed in a row. Three children A, B, and C enter the room and sit down randomly, one on each chair. Determine the probability that:

a A sits on the leftmost chair
b they sit in the order BCA from left to right
c C sits in the middle
d B does not sit in the middle.

14 This spinner is spun twice. Find the probability of getting:

a two reds
b no greens
c a black and a red
d at least one black.

DISCUSSION

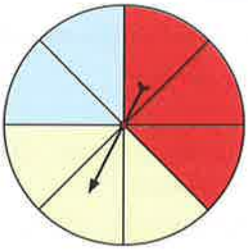
When you roll an ordinary die, each outcome 1, 2, 3, 4, 5, or 6 is equally likely.

Suppose you rolled two dice and added the numbers.

- What different results could you get?
- Are the different possible results equally likely?
- What are the probabilities for each possible result?

E COMPLEMENTARY EVENTS

For the spinner alongside:



$$\text{P(spinning a red)} = \frac{3}{8}$$

$$\begin{aligned}\text{P}(\textit{not}\text{ spinning a red}) &= \text{P(spinning a blue or yellow)}\\ &= \frac{2+3}{8}\\ &= \frac{5}{8}\end{aligned}$$

Notice that $\text{P(spinning a red)} + \text{P}(\textit{not}\text{ spinning a red}) = \frac{3}{8} + \frac{5}{8} = 1$

We should expect these probabilities to sum to 1, because when we spin the spinner, either it will land on red, or it will not. One of these events is *certain* to occur.

We say that these are **complementary events**.

Two events are complementary if exactly one of them *must* occur.

If E is an event, then the **complementary event** of E is the event that E does *not* occur.

The complementary event of E is written as E'.

For any event E, $\mathbf{P}(E') = 1 - \mathbf{P}(E)$.

Example 7 **Self Tutor**

Balls numbered 1 to 60 are placed in a box, and one is selected at random. Determine the probability of:

a selecting a multiple of 8 **b** not selecting a multiple of 8.

a There are 60 possible outcomes.
The 7 outcomes 8, 16, 24, 32, 40, 48, and 56 are multiples of 8.

$\therefore$ P(selecting a multiple of 8) $= \frac{7}{60}$

b P(not selecting a multiple of 8)

$= 1 - \frac{7}{60}$ {this event is complementary to the event in **a**}

$= \frac{53}{60}$

EXERCISE 15E

1 For each of these events, write down the complementary event:

a Terry will go to school tomorrow.

b Jennifer has at least 3 pets.

c When selecting a ball from this bag, the result is either red or blue.

2 Suppose $P(A) = 0.7$ and $P(B) = 0.12$. Find $P(A')$ and $P(B')$.

3 A day of the week is selected at random. Determine the probability that the selected day is:

a Wednesday **b** not Wednesday.

4 A six-sided die is rolled once. Which two of these events are complementary?

A rolling a prime number
B rolling a number less than 4
C rolling a 1, 5, or 6
D rolling a number greater than 4
E rolling a 2, 3, or 4
F rolling a composite number

5 Suppose F is the event that Fran will forget to get her diary signed tonight. $P(F) = 0.23$.

a State F', the complementary event of F. **b** Find $P(F')$.

c Which is more likely to occur, F or F'?

6 A class contains 29 students, 5 of whom suffer from asthma. What is the probability that a randomly chosen student does not suffer from asthma?

7 Tickets numbered 1 to 50 are placed in a box, and one is selected at random. Find the probability of:

a selecting a multiple of 9 **b** not selecting a multiple of 9.

8 A coin is tossed 4 times.

a List the sample space of possible outcomes.

b Find the probability of tossing:

i all heads **ii** at least one tail.

9 A number from 1 to 10 is selected at random. Let A be the event that a factor of 10 is selected, and B be the event that a number greater than 4 is selected.

a Find $P(A)$ and $P(B)$.

b Show that $P(A) + P(B) = 1$.

c Does this mean that A and B are complementary events? Explain your answer.

ACTIVITY EXPERIMENTAL PROBABILITY

What to do:

1 Place three red, five blue, and two green counters in a bag. If you do not have these colours, use your own colours.

2 Shake the bag, then without looking, take out one counter and record its colour. Then put the counter back in the bag. Repeat this 100 times.

3 In a table like the one below, write down:

a the total number of times you took each colour out of the bag

b the experimental probability of obtaining each colour, using:

$$\text{experimental probability} = \frac{\text{number of that colour counter selected}}{\text{total number of counters selected}}$$

c the theoretical probability of obtaining each colour.

Colour	*Number of times selected*	*Experimental probability*	*Theoretical probability*
Red			
Blue			
Green			

4 Were your experimental results the same as your theoretical probabilities?

5 Repeat the experiment and calculate the probabilities again. Are the results the same? Explain your results.

6 **a** Run the computer simulation so you can experiment with much larger samples. Try taking a counter out 10 000 and 100 000 times.

b Are the experimental results in general closer to the theoretical probabilities?

c Discuss your results.

DEMO

KEY WORDS USED IN THIS CHAPTER

- chance
- coin
- complementary event
- dice
- die
- event
- outcome
- sample space
- spinner
- theoretical probability

REVIEW SET 15A

1 Describe, using a word or phrase, the probability of the following happening:

a The next person to cross the street will be older than 70 years of age.

b The house next to yours will still be standing in 1 year's time.

2 For each of the following probability values, choose the word or phrase from the list alongside which best describes it:

a 0.2 **b** 0.97 **c** 0

d 0.5 **e** 1 **f** 75%

probable
certain
impossible
extremely likely
'50-50' chance
unlikely

3 List the sample space for choosing:

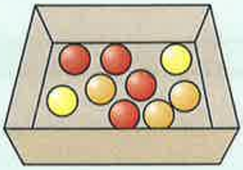
a a sweet from a bag containing peppermints, caramels, jelly babies, and marshmallows

b a prime number between 20 and 40.

4 A box contains 4 red, 3 orange, and 2 yellow marbles. If one marble is randomly selected from the box, determine the chance that the marble is:

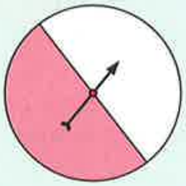
a red **b** orange **c** not yellow

d yellow or red **e** neither yellow nor red.

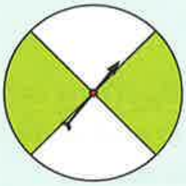


5 Find the probability that the spinning needle will land on white:

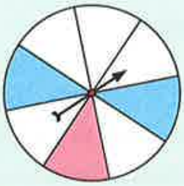
a



b



c



d

6 Raffle tickets numbered 1 to 100 are placed in a big bag. A ticket is selected at random. Find the probability that the ticket number is:

a 40 **b** an odd number **c** closer to 90 than to 20.

7 Draw a coloured spinner for which the probability of spinning blue is $\frac{5}{8}$, green is $\frac{1}{4}$, and red is $\frac{1}{8}$.

8 The letters of the alphabet are each written on pieces of paper and placed in a hat. One of the letters is selected at random.
Determine the probability that the letter is:

a A **b** a vowel **c** in the word MATHEMATICS.

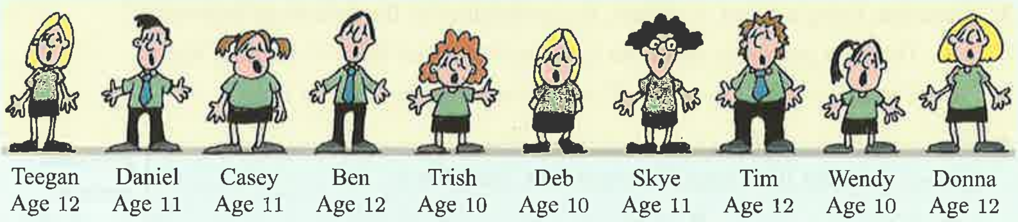
9 Suppose $P(A) = 0.94$.

a Find $P(A')$.

b Describe, using a word or phrase, the probability of:

i A occurring **ii** A' occurring.

10 The members of a school choir are shown below:



A girl from the choir will be selected at random to perform a solo.

a List the sample space for this selection.

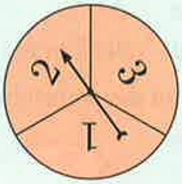
b Determine the probability that the selected girl:

i has a name that starts with T **ii** is 10 years old.

c A 10 year old is randomly selected from the choir to introduce the conductor at a concert. Find the probability that the chosen person is male.

REVIEW SET 15B

1 The spinner alongside is spun once. Describe the following events as either *certain*, *possible*, or *impossible*:



a an even number

b a number less than 4

c a composite number.

2 Use a word or phrase to describe the probability of these events:

a There is a 95% chance that I will have to stand on the bus on my way to work.

b There is a 0% chance that Steve will quit his job.

c There is a 52% chance that Melina's first child will be a girl.

3 List the sample space for:

a selecting a planet in our solar system

b choosing one letter and one number from this number plate.

4 The numbers 1 to 40 are marked on separate cards and placed in a hat. Determine the probability that a randomly chosen card is a multiple of 8.

5 A flower bulb packet contains 6 daffodil bulbs, 9 tulip bulbs, 4 iris bulbs, and 1 amaryllis bulb. Clive picks a bulb at random and plants it. Determine the probability that the resulting flower is not a daffodil.

6 Is it possible to make a spinner with the following probabilities:

$$\text{P(red)} = \frac{3}{8}, \quad \text{P(blue)} = \frac{1}{4}, \quad \text{P(green)} = \frac{2}{5}?$$

Give a reason for your answer.

7 For the given spinner, find:

a the probability that blue will be spun

b the colour that is most likely to be spun

c the two colours that are equally likely to be spun.

8 Jacqueline and Richard will compete in a tennis match tomorrow.
E is the event that Jacqueline will win, and $P(E) = 0.6$.

a State E', the complementary event of E.

b Find $P(E')$.

9 In a board game, a 20-sided die is rolled to select a letter of the alphabet. The die contains all the letters of the alphabet except Q, U, V, X, Y, and Z.

a Construct a sample space of outcomes when this die is rolled.

b Are each of the outcomes equally likely?

c The die is rolled once. Find the probability of rolling:

i a B

ii a vowel

iii an M or an N.

10 A tetrahedral die with sides numbered 1, 2, 3, and 4 is rolled twice.

a List the sample space of possible outcomes.

b Find the probability that:

i the first roll is 3 and the second roll is 1

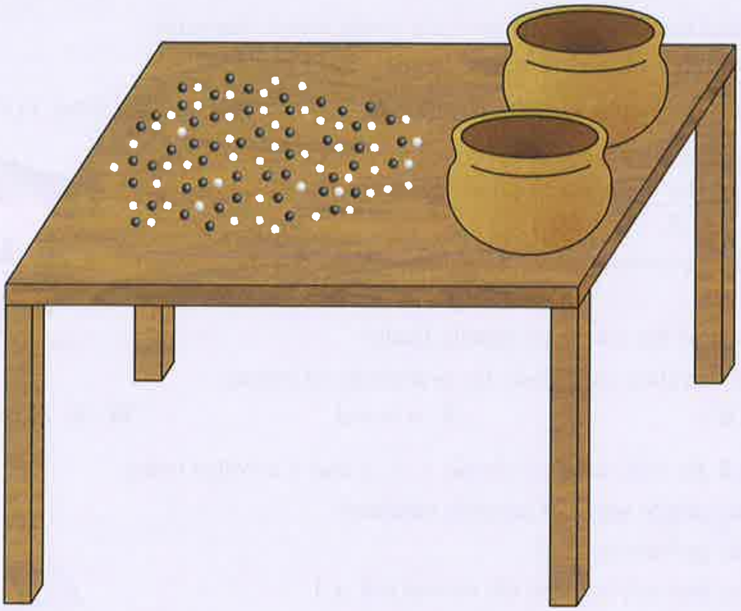
ii both rolls are the same number

iii at least one 4 is rolled.

PUZZLE THE EMPEROR'S PROPOSITION

A prisoner has been sentenced to death. However, the Emperor decides to give the prisoner a chance to live.

The prisoner is given 50 black marbles, 50 white marbles, and two empty bowls. He may place the marbles in the bowls in any combination he likes, but all of the marbles must be used, and neither bowl may be empty.



The prisoner is blindfolded, and the bowls are placed on a table in front of the prisoner so he does not know which is which. The prisoner must choose a bowl at random, and then select one marble from that bowl. If the marble is black, the prisoner will be executed. If the marble is white, the prisoner will be allowed to live.

1 How should the prisoner divide the marbles between the bowls, to maximise the probability that he will live?

2 What is the probability that the prisoner will live?

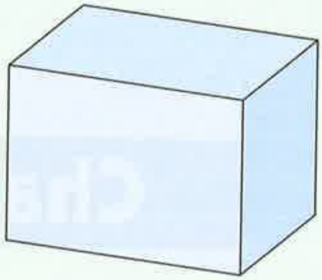
Chapter 16

Solids

Contents:

OPENING PROBLEM

Have a look at this diagram:



Things to think about:

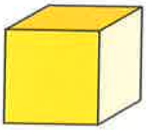
- **a** What object does the diagram represent?
- **b** Is the object 2-dimensional or 3-dimensional?
- **c** Is the *diagram* 2-dimensional or 3-dimensional?
- **d** Can you draw a 2-dimensional shape which could be folded to create this object?
- **e** How can we illustrate how the object looks from different directions?

In this chapter we will study three-dimensional **solids**. We will consider how they can be represented on a two-dimensional page, and two-dimensional **nets** which can be folded to create them.

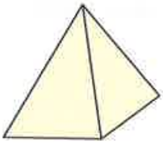
A SOLIDS

A **solid** is a three-dimensional body which occupies space.

The diagrams below show a collection of solids. Each solid has three dimensions: *length*, *width*, and *height*.

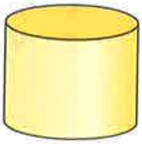


cube

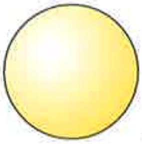


square-based pyramid

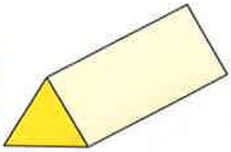
cone



cylinder

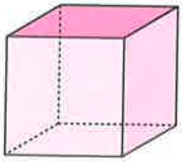


sphere

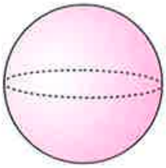


triangular prism

The boundaries of a solid are called **surfaces**. These may be flat surfaces, curved surfaces, or a mixture of both.



A cube is bounded by six flat surfaces.



A sphere is bounded by one curved surface.

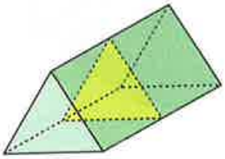
A cylinder is bounded by two flat surfaces and one curved surface.

When we draw solids, we often use dashed lines to show edges which are hidden at the back of the solid. The dashed lines remind us these edges are there, even if we cannot normally see them. Dashed lines can also help us to appreciate the three-dimensional nature of the solids.

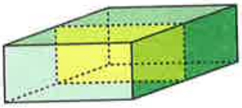
DEMO

PRISMS

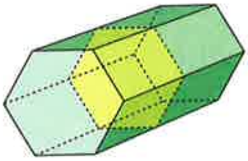
A **prism** is a solid with a uniform cross-section that is a polygon.



triangular prism



rectangular prism

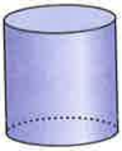


hexagonal prism

A **polygon** is a closed 2-dimensional shape with straight edges.

CYLINDERS

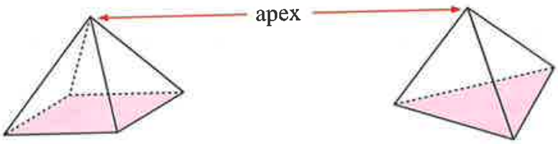
A **cylinder** is a solid with a uniform cross-section that is a circle.



PYRAMIDS

A **pyramid** is a solid with a polygonal base, and triangular faces which come from the edges of the base to meet at a point called the **apex**.

A triangular-based pyramid is also called a **tetrahedron**.

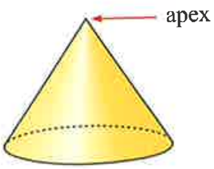


square-based pyramid

triangular-based pyramid

CONES

A **cone** is a solid with a circular base, and a curved surface from the base to a point called the **apex**.



DISCUSSION

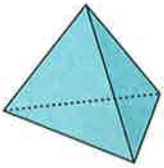
- Is a cylinder a prism?
- Is a cone a pyramid?

EXERCISE 16A

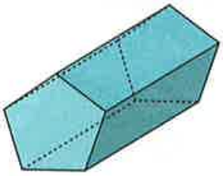
1 Name the following solids:

a

b



c



2 Draw a diagram to represent:

a a cone
b a rectangular-based pyramid
c a sphere
d an octagonal prism
e a hexagonal-based pyramid.

3 Name the solid which best resembles:

a a can of soup
b a marble
c a cereal box
d a witch's hat
e a four-sided die
f a coin.

4 What shape are the side faces of a:

a prism
b pyramid?

5 Draw a solid which has:

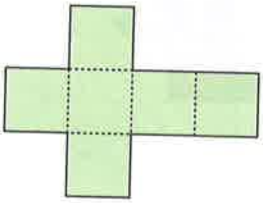
a only a curved surface
b a curved and a flat surface
c two flat surfaces and one curved surface.

B NETS OF SOLIDS

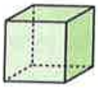
A **net** is a two-dimensional shape which may be folded to form a solid.

For example, the following nets may be cut out and folded along the dotted lines to form common solids:

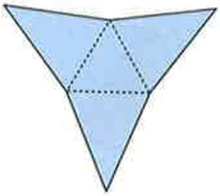
PRINTABLE NETS



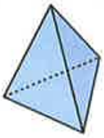
becomes



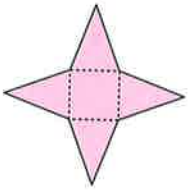
a **cube**



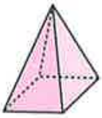
becomes



a **triangular-based pyramid**



becomes



a **square-based pyramid**

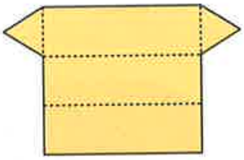
Click on the icon to view demonstrations of how the nets form the solids.

DEMO

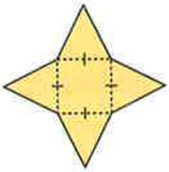
EXERCISE 16B

1 For each of the following nets, draw and name the corresponding solid:

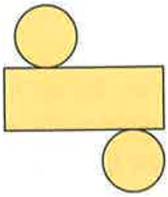
a



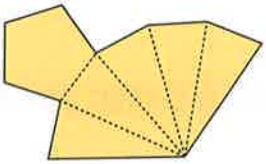
b



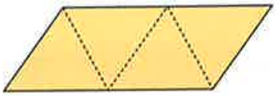
c



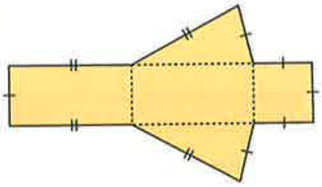
d



e

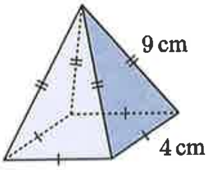


f

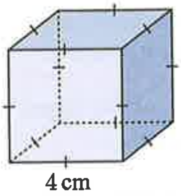


2 Draw nets for each of the following three-dimensional solids, clearly marking the lengths of the sides:

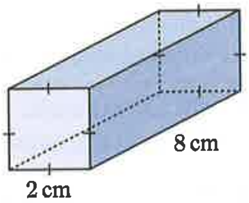
a



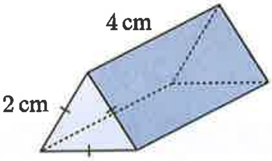
b



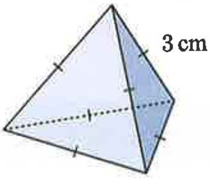
c



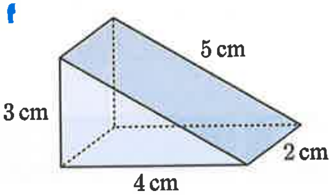
d



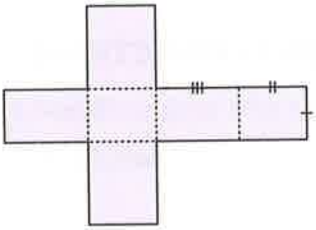
e



f

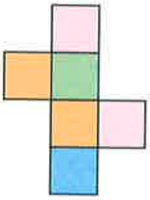


3 Copy this net of a rectangular prism. Place tick marks on the remaining lines to indicate the sides of equal length.

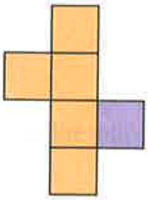


4 Which of the following nets can be used to make this cube?

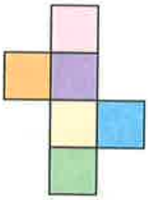
A



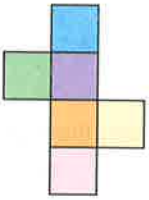
B



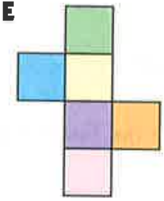
C



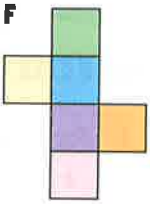
D



E

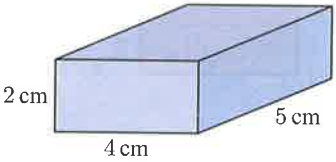


F



5 **a** Draw the net for this prism, clearly marking the lengths of the sides.

b Find the area of the net.



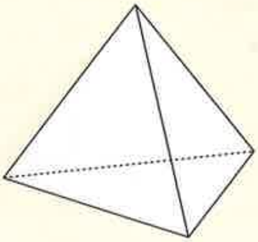
INVESTIGATION TRIANGULAR-BASED PYRAMIDS

In this Investigation we will use an acute angled triangle as a net to form a triangular-based pyramid.

PRINTABLE NETS

What to do:

1. Click on the icon and print the different nets. Fold the nets along the dashed lines to form triangular-based pyramids.
2. On separate sheets of paper, draw acute angled triangles of different shapes. Cut them out with scissors.
3. For each triangle, use three folds of the paper to attempt to construct a triangular-based pyramid.
4. Will your method work if the original triangle is:
   a right angled **b** obtuse angled?



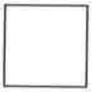
C DRAWING RECTANGULAR SOLIDS

There are two different methods we can use to draw rectangular solids. These methods are called **projections** because we *project* the image of the three-dimensional solid onto the two-dimensional paper.

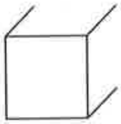
OBLIQUE PROJECTIONS

To draw a cube using an **oblique projection** we use the following steps:

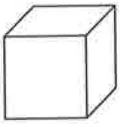
Step 1: Draw a square for the front face.



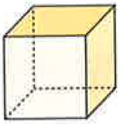
Step 2: Draw edges back from the front face at 45°, and shorter than those of the front face.



Step 3: Complete the cube.



Step 4: If appropriate, draw in dashed lines to show the hidden edges.

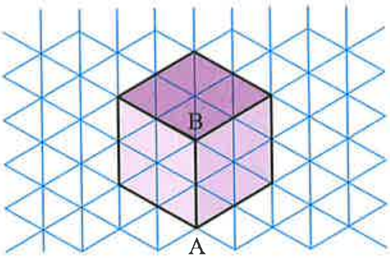


ISOMETRIC PROJECTIONS

When drawing a rectangular solid using an **isometric projection**, we use **isometric graph paper** which is made up of equilateral triangles.

We start with a vertical edge of the solid. The horizontal edges are drawn inclined at 30°.

The diagram alongside shows the isometric projection of a cube. Notice that all the edges drawn have the same length. The edge [AB] appears closest to us. This is often the **starting edge** of the figure, or first edge drawn.



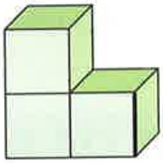
ISOMETRIC GRAPH PAPER

EXERCISE 16C

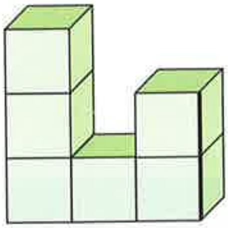
1 Draw an oblique projection of a box which has sides 2 units by 2 units by 1 unit. Start with a 2 unit by 1 unit rectangle as the front face.

2 Draw the following solids on isometric paper. Use the darker lines as the starting edges.

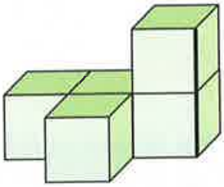
a



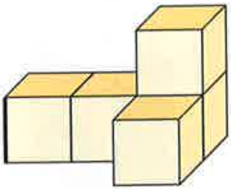
b



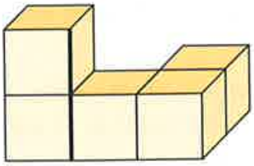
c



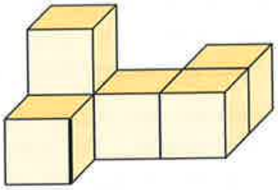
d



e

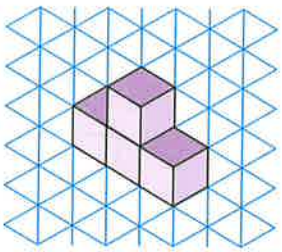


f

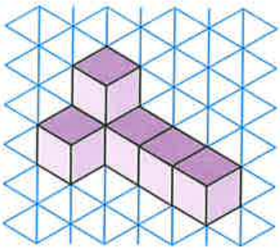


3 Redraw these isometric projections as oblique projections.

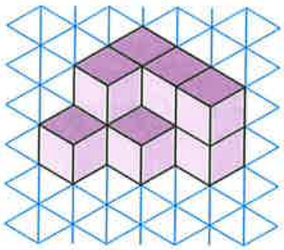
a



b



c

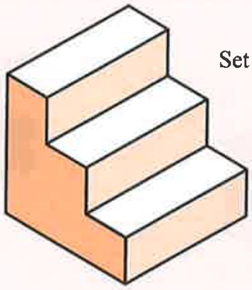


ACTIVITY 1

Can you use isometric paper to draw these figures?

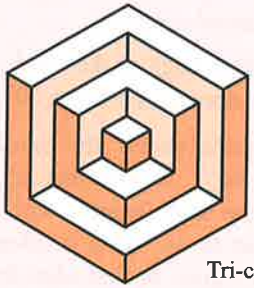
ISOMETRIC GRAPH PAPER

1



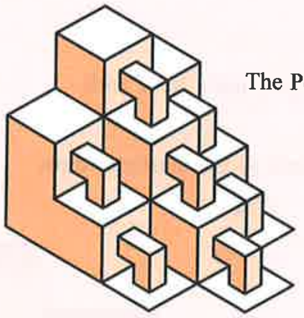
Set of steps

2



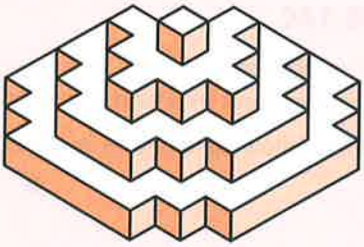
Tri-cube

3



The Power Station

4



Fancy Pyramid

D VIEWS OF SOLIDS

When drawing a three-dimensional solid, we cannot show the details on all of the faces at the same time, because many of the faces are hidden from view.

Instead we can create several drawings of the solid from different angles. We normally draw the solid as it appears from the **front**, **top**, **left**, **right**, and **back**.

front

top

left

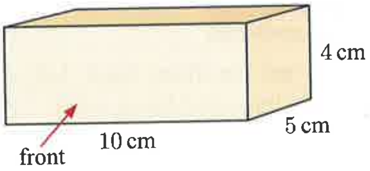
right

back

EXERCISE 16D.1

1 A rectangular prism is 10 cm long, 5 cm wide, and 4 cm high.
Sketch, including dimensions, how the prism would look from the:

a front **b** top **c** left
d right **e** back.

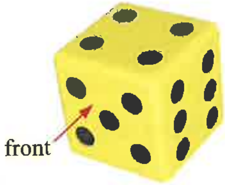


2 The numbers on a die are arranged so that the sum of each pair of opposite faces is seven.

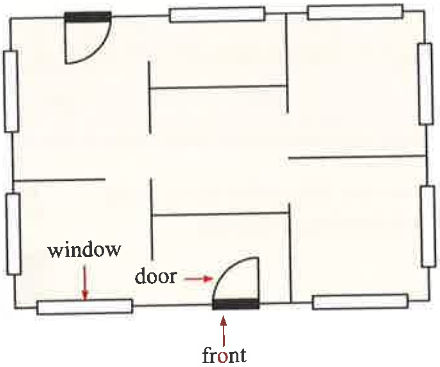
a Which number is on the bottom of the die?

b Sketch the die alongside from the:

i front **ii** top **iii** right
iv left **v** back.



3

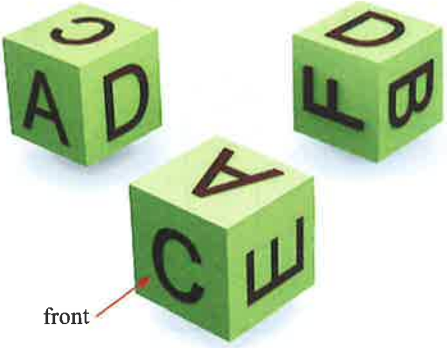


The diagram shows an architect's plan for a building. The doors and windows are indicated on the plan. Sketch how the building would look when viewed from the:

a front **b** back
c left **d** right.

4 A cube has the letters A, B, C, D, E, and F painted on its faces. Three different views of the cube are shown alongside.
Using the bottom view of the cube as a basis, draw how the cube would look from the:

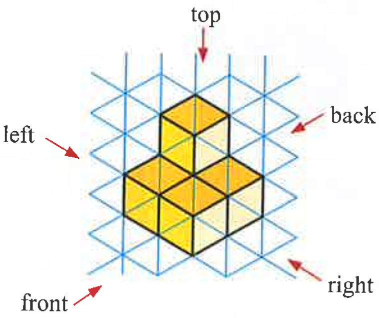
a front **b** right
c left **d** top
e back.



BLOCK SOLIDS

Drawings of block solids on isometric graph paper can also be viewed from different angles.

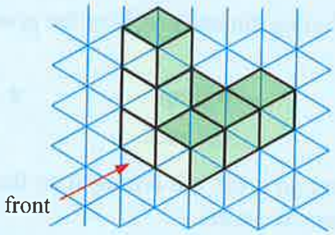
We assume that on the isometric graph paper, the view from the bottom left corner is the front view.



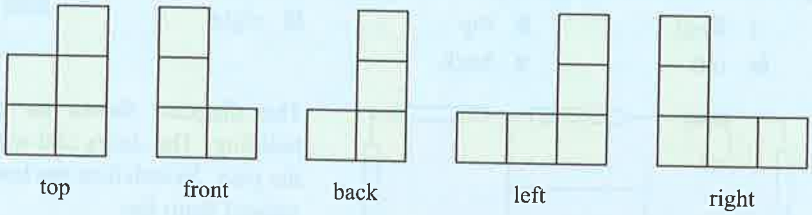
Example 1

Self Tutor

Draw the top, front, back, left, and right views of the illustrated block solid.



The views are:



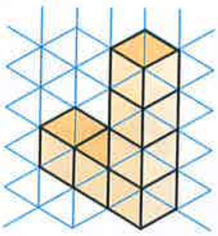
Click on the icon to run the Blockbuster software. You can use this software to help you visualise block solids and answer the questions in the following **Exercise**.

BLOCK BUSTER

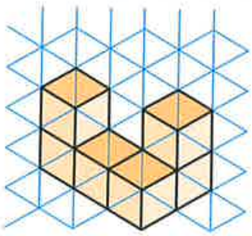
EXERCISE 16D.2

1 Draw the top, front, back, left, and right views of these block solids:

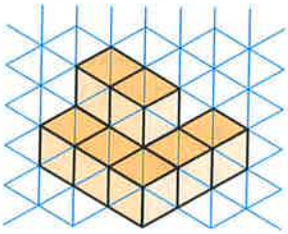
a



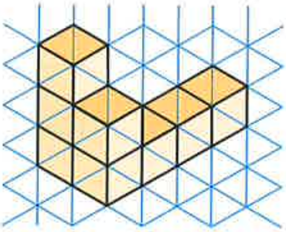
b



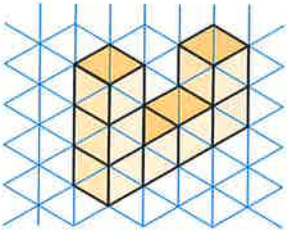
c



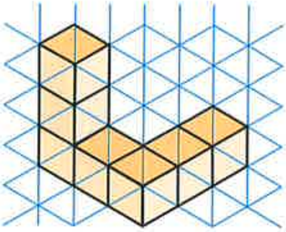
d



e

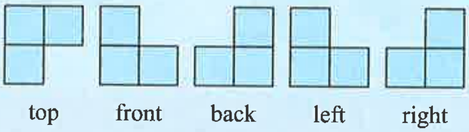


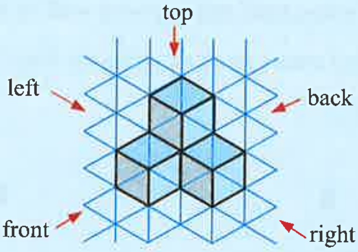
f



Example 2 **Self Tutor**

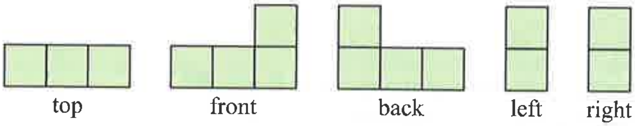
These diagrams show the different views of a block solid. Draw the object on isometric paper.



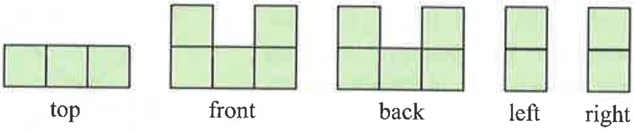


2 Draw the block solid which has these views:

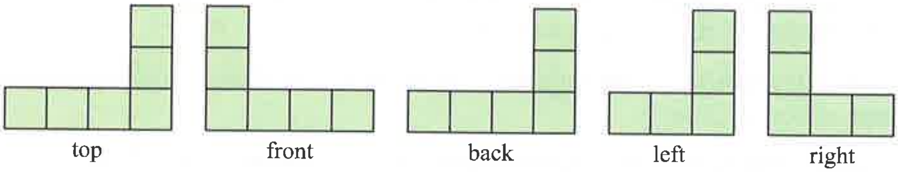
a



b

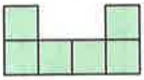


c



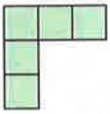
3 Match the following views with the correct block solid:

a



front

b

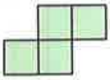


top

c

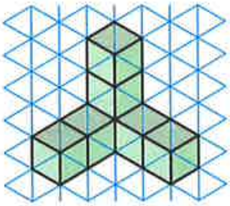
right

d

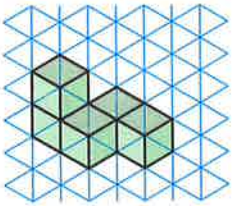


top

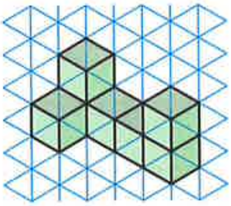
A



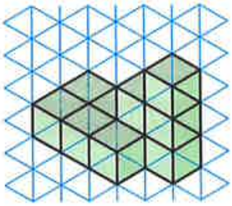
B



C



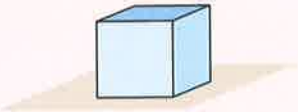
D



ACTIVITY 2 HUMBLE HOUSES

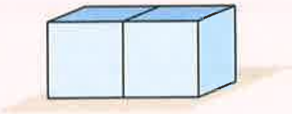
The Humble House factory manufactures cubic living quarters for countries where the conditions are mostly dry and hot. Heat enters every roof and exposed wall at the same rate.

For a house made of **one cube**, heat enters in equal amounts from 5 sides, but not the floor.

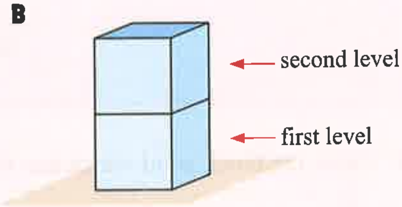


There are two possible house designs made from **two cubes** placed together with faces touching:

A



B

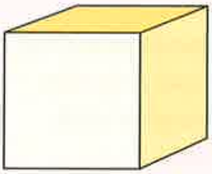


What to do:

1. How many exposed faces are there in each of the designs **A** and **B**? Which one would be more suitable for hot conditions?
2. Draw the four different possible housing arrangements using three adjacent cubes. The blocks must touch face to face and the building must be free-standing. For example, columns for support are not acceptable, since we do not want heat to come in through the floor.

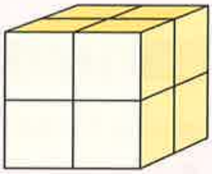
3. From your models, determine the two 'best' 3-cube structures which would allow the least amount of heat to come in.
4. Investigate the possible 4-cube buildings, and determine the model which would allow the least amount of heat to come in.
5. How many different possible 5-cube buildings are there? Which one is 'best'?
6. Write some general conclusions about how these buildings should be designed to minimise the amount of heat coming in.

ACTIVITY 3 PAINTED CUBES



A cube is painted and then cut into 8 smaller cubes.

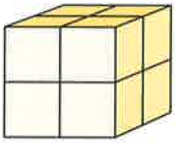
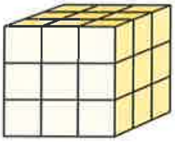
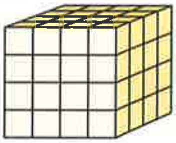
On dismantling the $2 \times 2 \times 2$ cube, we see that all 8 cubes have paint on exactly 3 faces.



In this Activity we consider how many cubes are painted the same when the cube is cut into a $3 \times 3 \times 3$ cube and a $4 \times 4 \times 4$ cube.

What to do:

1 Copy and complete:

Cube cut	3 *faces painted*	2 *faces painted*	1 *face painted*	*No faces painted*
	8	0	0	0
				
				

2 From the results in your table, what patterns do you notice?

Global context

click here

Papercraft and polygon models

Statement of inquiry:	Real-world objects can be represented by polygons.
Global context:	Personal and cultural expression
Key concept:	Form
Related concepts:	Model, Representation
Objective:	Applying mathematics in real-life contexts
Approaches to learning:	Research, Thinking

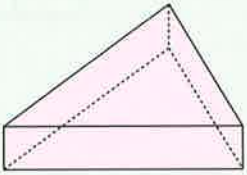
KEY WORDS USED IN THIS CHAPTER

- apex
- cone
- isometric projection
- oblique projection
- rectangular solid
- surface
- back view
- cylinder
- left view
- prism
- right view
- top view
- block solid
- front view
- net
- pyramid
- solid

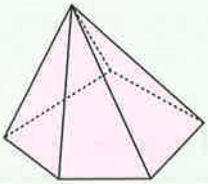
REVIEW SET 16A

1 Name the following solids:

a

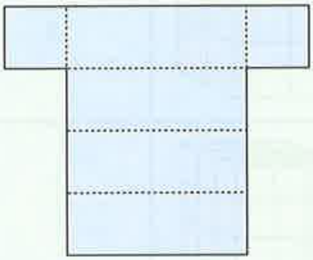


b



2 Draw a net for a triangular-based pyramid.

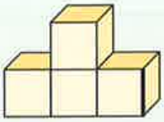
3 For the net shown, name the corresponding solid.



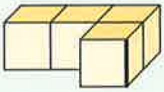
4 Draw an oblique projection of a rectangular prism which is 5 cm long, 3 cm wide, and 2 cm high. Start with a 5 cm by 2 cm rectangle as the front face.

5 Draw the following as isometric projections. Use the darker lines as the starting edges.

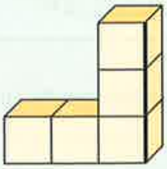
a



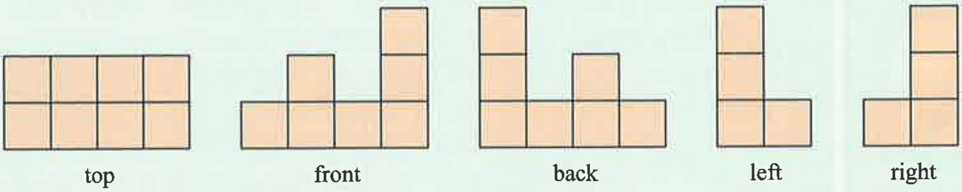
b



c

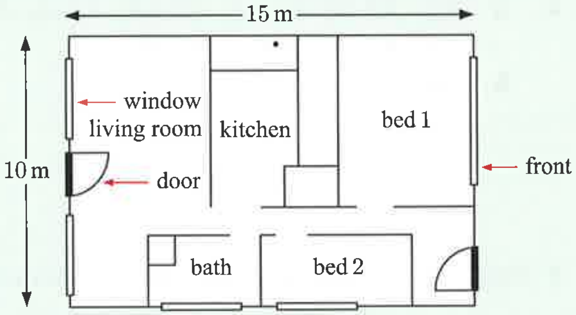


6 Draw the block solid with these views:



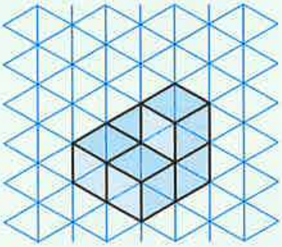
7 The plan of a house is given alongside. The house is 3 metres high. Sketch, including dimensions, how the house looks from the:

a front **b** back
c left **d** right.

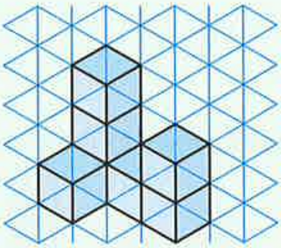


8 Draw the top, front, back, left, and right views of:

a



b



REVIEW SET 16B

1 Draw a diagram to represent:

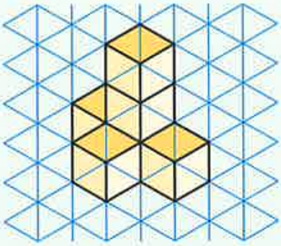
a a cylinder **b** a square-based pyramid.

2 Draw a net for making a 5 cm by 3 cm by 1 cm rectangular prism.

3 Name the solid which best resembles a six-sided die.

4 Draw these isometric projections as oblique projections:

a



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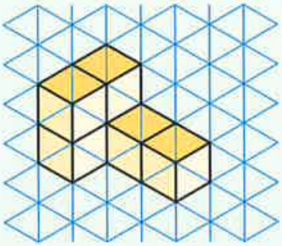
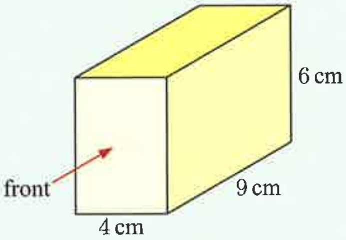


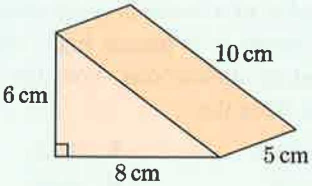
5 Sketch, including dimensions, how this rectangular prism would look from the:

a front **b** top
c left **d** right
e back.

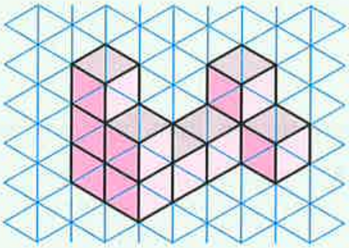


6 **a** Draw the net for this triangular prism, clearly marking the lengths of the sides.

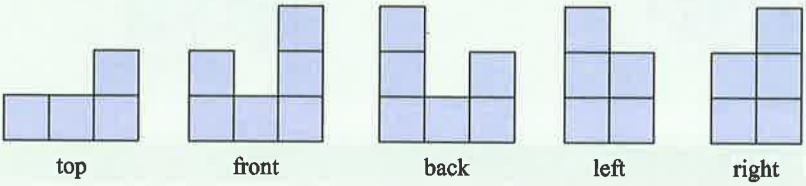
b Find the area of the net.



7 Draw the top, front, back, left, and right views of:



8 Draw the block solid with these views:



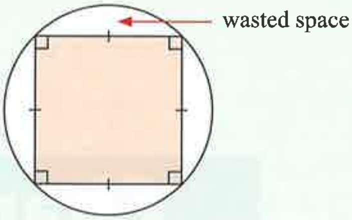
Chapter 17

Circles

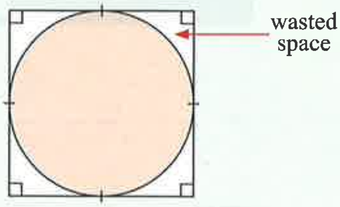
Contents:

OPENING PROBLEM A SQUARE PEG IN A ROUND HOLE

Steve's Square Peg Store makes square pegs which are packed in round containers.



Rod's Round Peg Retailers makes round pegs which are packed in square containers.



Things to think about:

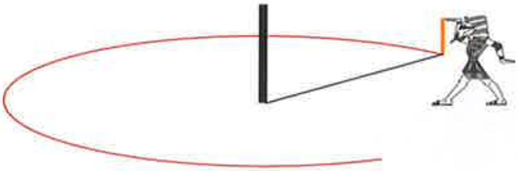
- **a** How can we calculate the area of each peg?
- **b** Which of the designs is more wasteful?

Suppose we make a loop at the end of a length of rope. We place it over a fixed spike in the ground. The rope is made taut, and a stick is placed at the opposite end to the fixed spike.

By keeping the rope taut and moving the stick around the spike, a **circle** is produced.

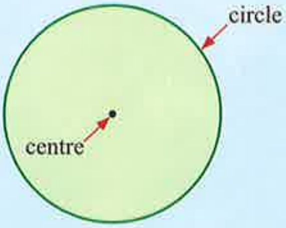
The fixed spike is the circle's **centre**.

This method of drawing a circle was known and used by builders in ancient Egypt.



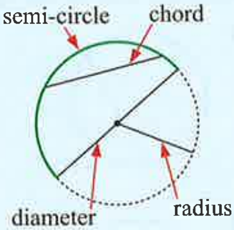
A CIRCLES

A **circle** is a two-dimensional shape. All points on the circle are the same distance from a fixed point called the **centre** of the circle.

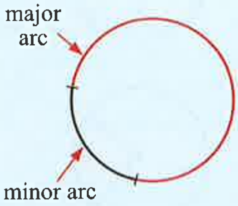


These are some words which are used to describe different parts of a circle:

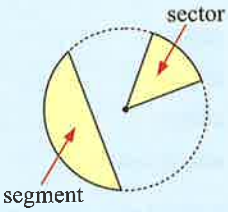
- A **chord** of a circle is a line which joins any two points of the circle.
- A **diameter** of a circle is a chord which passes through the circle's centre.
- A **radius** of a circle is a straight line segment which joins the circle's centre to any point on the circle. **Radii** is the plural of radius.
- A **semi-circle** is a half of a circle.



- An **arc** is a part of a circle. It joins any two different points on the circle.
  For any two points, we can define a **minor arc** and a **major arc** which are the shorter and longer arcs around the circle respectively.

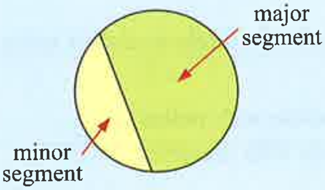


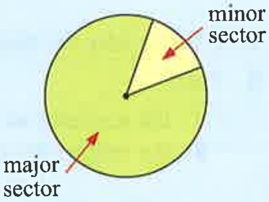
- A **segment** of a circle is the region between a chord and the circle.



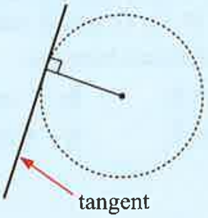
A **sector** of a circle is the region between two radii and the circle.

We can define minor and major segments and sectors just as we did for arcs.

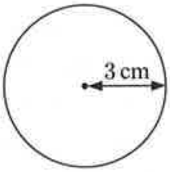




- A **tangent** to a circle is a line which *touches* the circle but does not enter it. A tangent is always at right angles to the radius at that point.



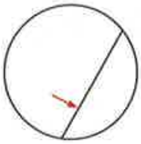
It is common to refer to the radius of a circle as the length of any of its radii, and the diameter of a circle as the length of any of its diameters. For example, we say that the radius of this circle is 3 cm.



EXERCISE 17A

1 Match the part of the figure indicated to the phrase which best describes it:

a



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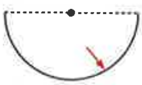

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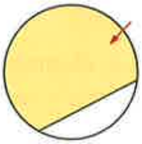

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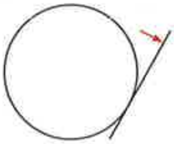

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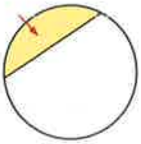

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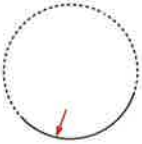

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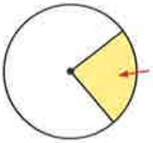

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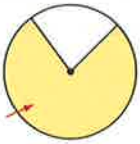


k

l

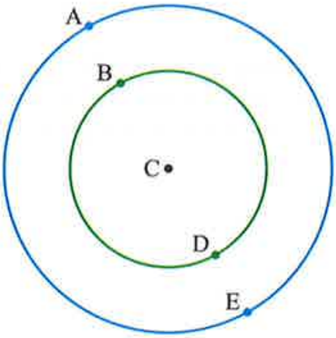
A semi-circle
B radius
C minor arc
D major arc
E diameter
F chord
G minor segment
H centre
I major segment
J major sector
K minor sector
L tangent

2 What name can be given to the longest chord that you can draw in a circle?

3 **a** Explain why the diameter of a circle is always twice as long as its radius.
b Find:
  i the diameter of a circle with radius 4 cm
  ii the radius of a circle with diameter 12 cm.

4 The circles shown both have centre C. The larger circle has radius 5 cm, and the smaller circle has radius 3 cm. Points A, B, C, D, and E are collinear.
Find the distance between:

a C and B
b C and A
c B and D
d A and E
e A and B
f E and B.



5 **a** Use a compass to draw a circle with radius 23 mm.
b Find the diameter of the circle.
c On the circle, draw a chord [AB] with length 4 cm.
d Label the major arc of the circle with endpoints A and B.
e Shade the minor segment of the circle which can be formed using points A and B.

PUZZLE

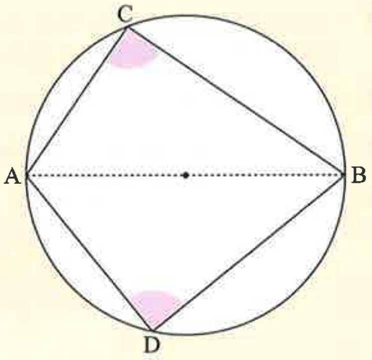
You are given a circular disc of paper which does not have the circle's centre marked.

Explain how you could find the centre of the circle by folding the paper.

INVESTIGATION 1 THE ANGLE IN A SEMI-CIRCLE

In this Investigation we consider the *size* of an angle in a semi-circle.

What to do:

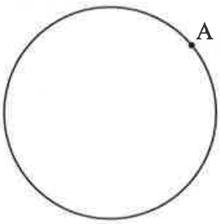


1. Draw a circle of radius greater than 5 cm.
2. Draw any diameter [AB] of the circle, which divides the circle into two semi-circles.
3. Choose any point C on one of the semi-circles.
4. Measure the angle ACB.
5. Now choose any point D on the second semi-circle.
6. Measure the angle ADB.
7. What do you suspect about the angle in a semi-circle?
8. Click on the icon to run software for measuring the angle in a semi-circle.
9. Comment on the statement: *The angle in a semi-circle is always a right angle.*

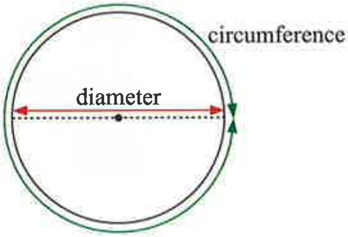
GEOMETRY PACKAGE

B CIRCUMFERENCE

The **circumference** of a circle is its perimeter.



If an ant starts at point A on the circle and walks around it until it gets back to A, then the total distance walked by the ant is the circumference of the circle.

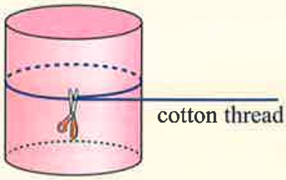


It is clear that for any circle, the circumference is longer than the diameter. But how much longer is it? Is it twice as long, or 3 times as long, or something else?

INVESTIGATION 2 CIRCUMFERENCE

In this Investigation we look for a connection between the circumference of a circle and the length of its diameter.

We can measure the circumference of a circular object by wrapping a cotton thread around it exactly once, then cutting the thread. We then measure the length of cotton with a ruler to find the circumference.



What to do:

1 Gather some cylinders such as a drink can, a tin can, a toilet roll, and a length of water pipe.

2 Use the cotton thread method to find the circumference of each object.

3 Use a ruler to find the diameter of each object.

4 Construct a table to record the circumference and diameter of each object.

Object	*Circumference*	*Diameter*	$\dfrac{\textit{Circumference}}{\textit{Diameter}}$
⋮			

5 Fill in the last column by calculating the fraction $\dfrac{\text{circumference}}{\text{diameter}}$ for each object.

6 What do you notice from the results in **5**?

From the **Investigation** you should have found that the fraction $\dfrac{\text{circumference}}{\text{diameter}}$ has the same value for any circle. This value lies between 3.1 and 3.2 .

In fact, the fraction $\dfrac{\text{circumference}}{\text{diameter}}$ is an exact number which we write as π, the **Greek** letter "pi".

The exact value of π cannot be written down because its decimal places go on forever without repeating. Its value is about 3.141 592 653 589 79 ....

So, $\dfrac{\text{circumference}}{\text{diameter}} = \pi$, which we can write as

$$\frac{C}{d} = \pi \quad \text{or} \quad C = \pi d \qquad \{\text{multiplying both sides by } d\}$$

The circumference of a circle is approximately 3.14 times as long as its diameter.

The **circumference** of a circle is given by $C = \pi d$ where d is the **diameter** of the circle
or $C = 2\pi r$ where r is the **radius** of the circle.

In practice we use $\pi \approx 3.14$ or the $\boxed{\pi}$ key on our calculator.

Example 1 **Self Tutor**

Use $\pi \approx 3.14$ to find the circumference of a circle with:

a diameter 10 cm **b** radius 2 m.

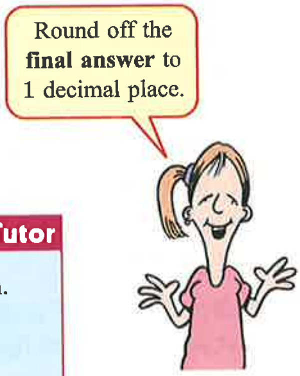
a
$$C = \pi d$$
$$\therefore \; C \approx 3.14 \times 10 \text{ cm}$$
$$\therefore \; C \approx 31.4 \text{ cm}$$

b
$$C = 2\pi r$$
$$\therefore \; C \approx 2 \times 3.14 \times 2 \text{ m}$$
$$\therefore \; C \approx 12.6 \text{ m}$$

EXERCISE 17B

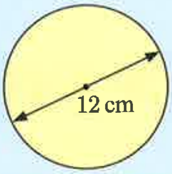
1 Use $\pi \approx 3.14$ to find the circumference of a circle with:

a diameter 5 cm
b diameter 7 cm
c radius 3 m
d radius 7 cm
e diameter 11 m
f radius 9 cm.



Example 2 **Self Tutor**

Use a calculator to find the circumference of this circle:



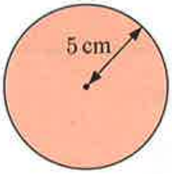
The circle has diameter 12 cm.

$$C = \pi d$$
$$\therefore \; C = \pi \times 12 \text{ cm}$$
$$\therefore \; C \approx 37.7 \text{ cm}$$

Calculator: $\boxed{\pi}$ $\boxed{\times}$ 12 $\boxed{=}$

2 Use your calculator to find the circumference of:

a



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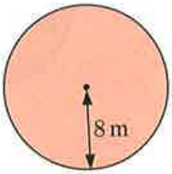
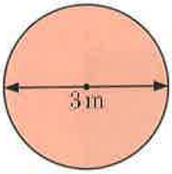


c

d



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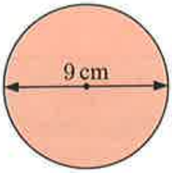

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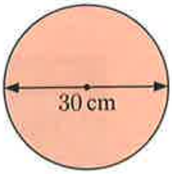

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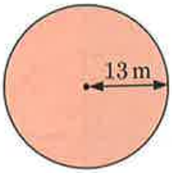

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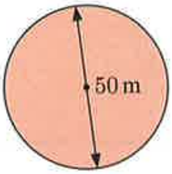

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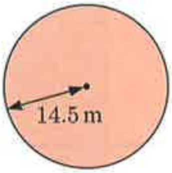


3 A cylindrical barrel has radius 40 cm. What is the circumference of its base?

4 A circular pond has diameter 8 m and needs to be fenced for the protection of children.

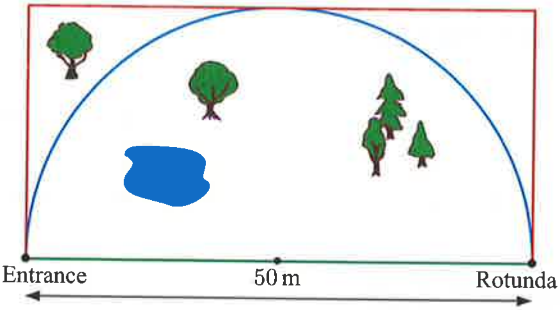
- **a** What length of fencing is required?
- **b** Fencing comes in 1 m lengths. How many lengths are needed?
- **c** Find the total cost of the fencing if each length costs €25.

5 A car wheel has radius 35 cm.

- **a** Find the circumference of the wheel.
- **b** If the wheel rotates 100 000 times, how far does the car travel?

6 The map alongside shows some walking trails in a botanical garden. To get from the entrance to the rotunda, you can either take the green path, the red path, or the semi-circular blue path.

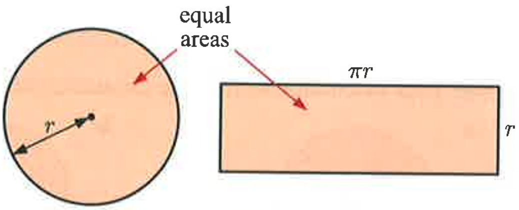
- **a** Write the paths in order, from shortest to longest.
- **b** Explain why the length of the blue path is between 50 m and 100 m.
- **c** Use your calculator to find the length of the blue path.



7 The circumference of a cylindrical rubbish bin is 1.45 m. Find the radius of the bin, giving your answer to 2 significant figures.

C AREA OF A CIRCLE

We can show using geometry that the area of a circle with radius r is the same as the area of a rectangle with length πr and width r. You can view a demonstration of this by clicking on the icon.



So, the area of a circle $A = \pi r \times r$

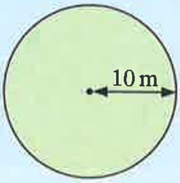
Hence $\boxed{A = \pi r^2}$

DEMO

Example 3 Self Tutor

Find the area of a circle with radius 10 m.

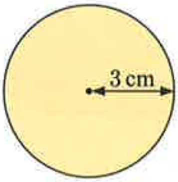
$$A = \pi r^2$$
$$\therefore \ A = \pi \times 10^2 \text{ m}^2$$
$$\therefore \ A \approx 314.2 \text{ m}^2$$



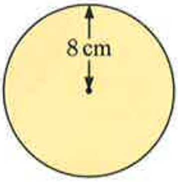
EXERCISE 17C

1 Find the area of:

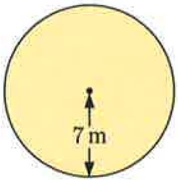
a



b



c



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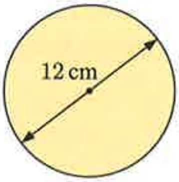

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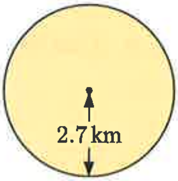

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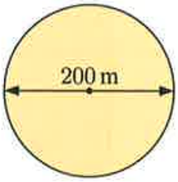


2 A sprinkler sprays water over a field. The radius of the spray is 11 m. What area of the field is being watered?

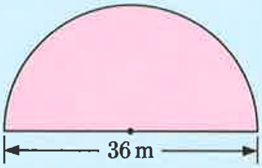
3 Find the area of a circular plate with diameter 24 cm.

Example 4

Self Tutor

Find the area of:

a



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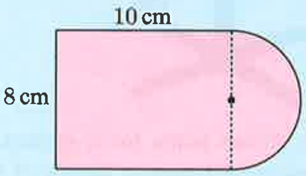


a Area

$= \frac{1}{2}$ of the area of the whole circle

$= \frac{1}{2} \times \pi r^2$

$= \frac{1}{2} \times \pi \times 18^2 \text{ m}^2$

$\approx 508.9 \text{ m}^2$

b Area

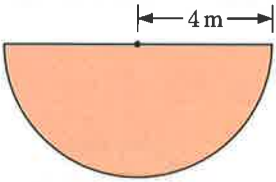
$=$ area of rectangle $+$ area of semi-circle

$= 10 \times 8 \text{ cm}^2 + \frac{1}{2} \times \pi \times 4^2 \text{ cm}^2$

$\approx 105.1 \text{ cm}^2$

4 Find the shaded area:

a



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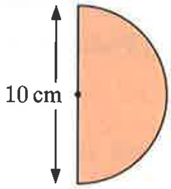

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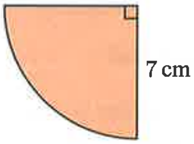

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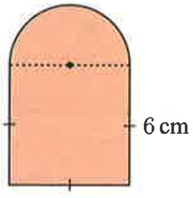

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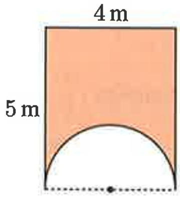

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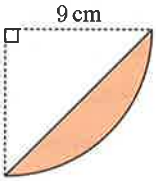
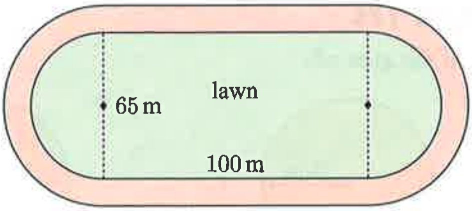


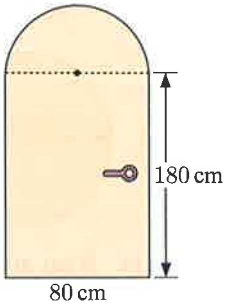
5 The inner part of an athletics track is lawn. Find the area of the lawn.



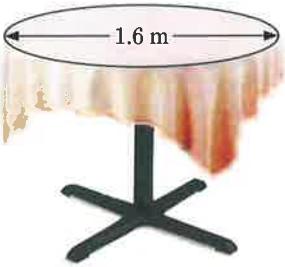
6 A door has the dimensions shown.

a How high is the door at its highest point?

b Find the area of the door, in square metres.



7 A circular table top has diameter 1.6 m. A 2 m by 2 m rectangular tablecloth is placed over the table top. What area of the tablecloth overlaps the table?



8 The diagram shows plans for a garden which is 40 m by 40 m. It consists of 4 quarter circles of lawn, with a flower bed in the middle as shown. Find:

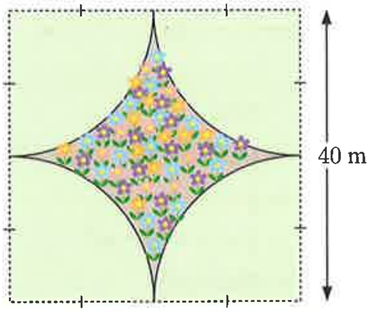
a the perimeter of the garden

b the total area of the garden

c the total area of lawn

d the area of the flower bed

e the length of edging around the flower bed.



INVESTIGATION 3 A SQUARE PEG IN A ROUND HOLE

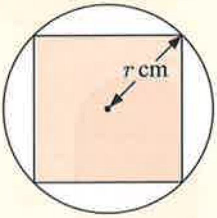
In this Investigation we explore the **Opening Problem** on page **342**.

What to do:

1 Consider a square peg packed in a round container of radius r cm.

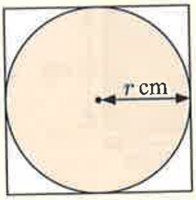
a By dividing the square into two triangles, show that the area of the square is $2r^2$ cm^2.

b Hence, show that the square occupies $\frac{2}{\pi}$ of the container. Express this fraction as a percentage.



2 Now consider a round peg of radius r cm packed in a square container.

- **a** Show that the area of the square is $4r^2$ cm^2.
- **b** Find the percentage of the container that is occupied by the round peg.



3 Which design is more wasteful?

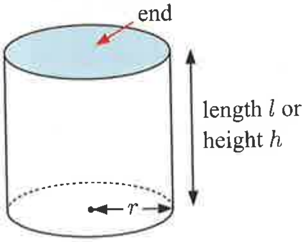
D VOLUME OF A CYLINDER

A **cylinder** is a solid with a circular uniform cross-section.

Since a cylinder has a uniform cross-section,

$$\begin{aligned} \text{Volume} &= \text{area of end} \times \text{length} \\ &= \text{area of circle} \times \text{length} \\ &= \pi r^2 \times l \end{aligned}$$

$$\boldsymbol{V = \pi r^2 l} \quad \text{or} \quad \boldsymbol{V = \pi r^2 h}$$

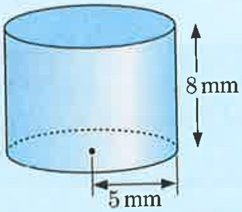


Example 5

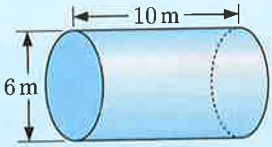
Self Tutor

Find, to 1 decimal place, the volume of each cylinder:

a



b



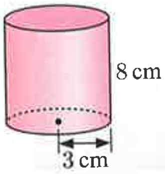
a $V = \pi r^2 h$
$= \pi \times 5^2 \times 8$ mm^3
≈ 628.3 mm^3

b The base has diameter 6 m, so the radius is 3 m.
$V = \pi r^2 l$
$= \pi \times 3^2 \times 10$ m^3
≈ 282.7 m^3

EXERCISE 17D

1 Find the volume of each cylinder:

a



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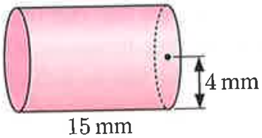

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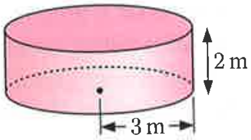

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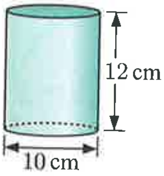

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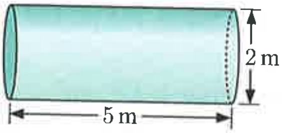

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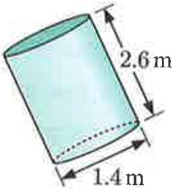
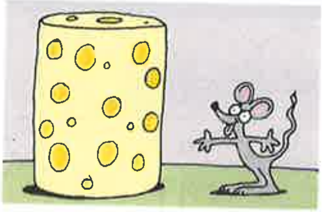
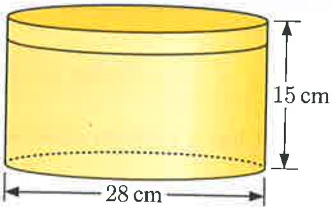


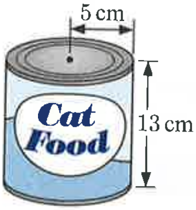
2 A round of cheese is 18 cm high and has radius 12 cm. Find the volume of cheese in this round.



3 A cylindrical biscuit barrel is 15 cm high and has diameter 28 cm. Find the volume of the biscuit barrel.



4 Find the volume of cat food in the cylindrical can shown.



5 A steel bar is 2.2 m long and has diameter 5 cm. Find the volume of the bar in cm^3.

Example 6 **Self Tutor**

A cylindrical rainwater tank has a base radius of 2 m and a height of 2.5 m. Find the capacity of the tank in kL.

$$\begin{aligned} \text{Volume} &= \pi r^2 h \\ &= \pi \times 2^2 \times 2.5 \text{ m}^3 \\ &\approx 31.4 \text{ m}^3 \end{aligned}$$

$\therefore$ capacity ≈ 31.4 kL $\{1 \text{ m}^3 \equiv 1 \text{ kL}\}$

6 A cylindrical rainwater tank has radius 1.5 m and height 2 m. Find the capacity of the tank.

7 A cylindrical drinking glass is 10 cm high and 6 cm wide. It is filled with juice to a height 2 cm below the rim of the glass. How much juice is in the glass? Give your answer in mL.

8 Frank is on a driving vacation. His car has just run out of petrol. The next petrol station is 60 km away, but fortunately Frank has a small can of petrol in his car boot. The can is cylindrical with the dimensions shown. If Frank's car can travel 15 km for each litre of petrol, will he be able to reach the petrol station?

KEY WORDS USED IN THIS CHAPTER

- chord
- circle
- circumference
- cylinder
- diameter
- major arc
- major sector
- major segment
- minor arc
- minor sector
- minor segment
- radius
- semi-circle
- tangent

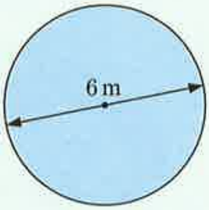
REVIEW SET 17A

1 Clearly define, with the aid of diagrams, the meaning of:

a an *arc* of a circle **b** a *sector* of a circle **c** a *chord* of a circle.

2 Find the circumference of:

a



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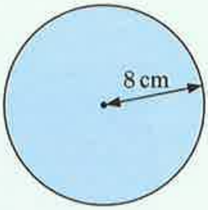
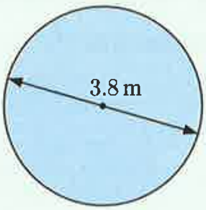


c



3 A circular hoop has a radius of 40 cm. Find the length of tubing needed to make the hoop.

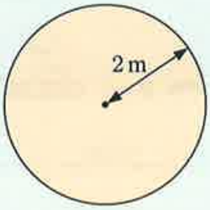
4 Are the following statements true or false? Explain your answers.

a A minor arc of a circle is always shorter than a semi-circle.

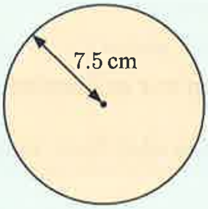
b A chord of a circle is always longer than the radius of the circle.

5 Find the area of:

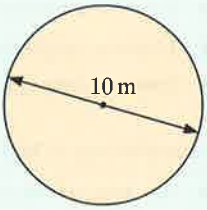
a



b



c



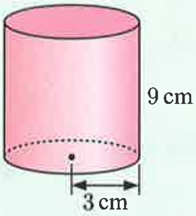
6 A gardener is making a path using 8 cylindrical concrete pavers. Each paver has a radius of 20 cm, and is 5 cm thick.

a Find the area of the top of each paver.

b Find the total volume of the pavers.

7 Find the volume of:

a



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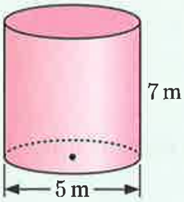
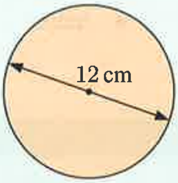


8 A coin is 20 mm in diameter and 2 mm high. Find the volume of the coin.

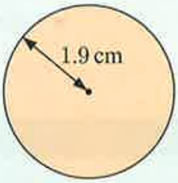
REVIEW SET 17B

1 Find the circumference of:

a



b



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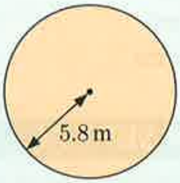
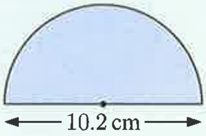


2 A circle has diameter 13 cm. Find:

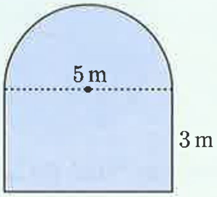
- **a** the radius of the circle
- **b** the circumference of the circle
- **c** the area of the circle.

3 Find the area of:

a



b



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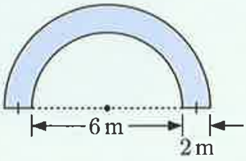
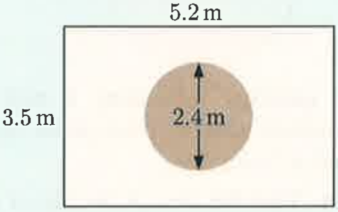


4 The London Eye Ferris wheel is 120 metres in diameter. What distance does a passenger travel in each revolution of the wheel?

5
- **a** Use a compass to draw a circle with radius 2 cm.
- **b** Draw a diameter [PQ] on the circle.
- **c** Draw a tangent to the circle at P and at Q.
- **d** Explain why the tangents drawn in **c** are parallel.

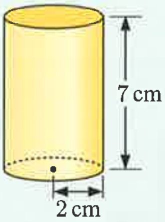
6 A circular rug is laid on a rectangular tiled floor. Find:

- **a** the area of the rug
- **b** the visible area of the tiled floor.

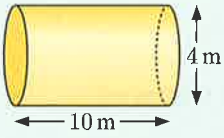


7 Find the volume of:

a



b



8 How many litres of water can this saucepan hold?

Chapter 18

Statistics

Contents:

OPENING PROBLEM

Zach and Ed both enjoy going fishing with their father. They record how many fish they catch each time they go fishing over the holidays:

Zach: 5 8 4 6 9 7 9 6 9 9
Ed: 8 7 10 5 4 8 4 6

Things to think about:

- By just looking at these values, is it easy to tell who catches more fish?
- Would it be fair to compare the boys by finding the total number of fish they caught?
- How could we determine which boy generally catches more fish?

When we collect facts or information about something, the information we collect is called **data**.

For example, the data in the **Opening Problem** are the numbers of fish caught by each boy.

Statistics is the study of solving problems and answering questions by collecting, organising, and analysing data.

Governments, businesses, sports organisations, manufacturers, and scientific researchers all use statistics to examine things.

For example, an athletics club may want to know whether a new training method has improved the speed of its athletes.

The club could collect data about the speed of the athletes before and after the change in training method. If the speeds of most athletes have improved since the change, it could indicate that the new method is effective.

In statistical work we use **tables**, **graphs**, and **diagrams** to represent data.

The process of **statistical enquiry** or **investigation** includes the following steps:

Step 1: Examine a problem which may be solved using data. Determine appropriate questions you wish to answer.
Step 2: Collect data.
Step 3: Organise the data.
Step 4: Summarise and display the data.
Step 5: Analyse the data and make a conclusion.
Step 6: Write a report.

HISTORICAL NOTE

The collection and analysis of data has been important to people for thousands of years.

- Before 3000 BC, the **Babylonians** recorded yields for their crops on small clay tablets.
- Pharaohs in **Ancient Egypt** recorded their wealth on walls of stone.
- Censuses were conducted by the **Ancient Greeks** so that taxes could be collected.
- After **William the Conqueror** invaded and conquered England in 1066, his followers overtook estates previously occupied by Saxons. Confusion reigned over who owned what.

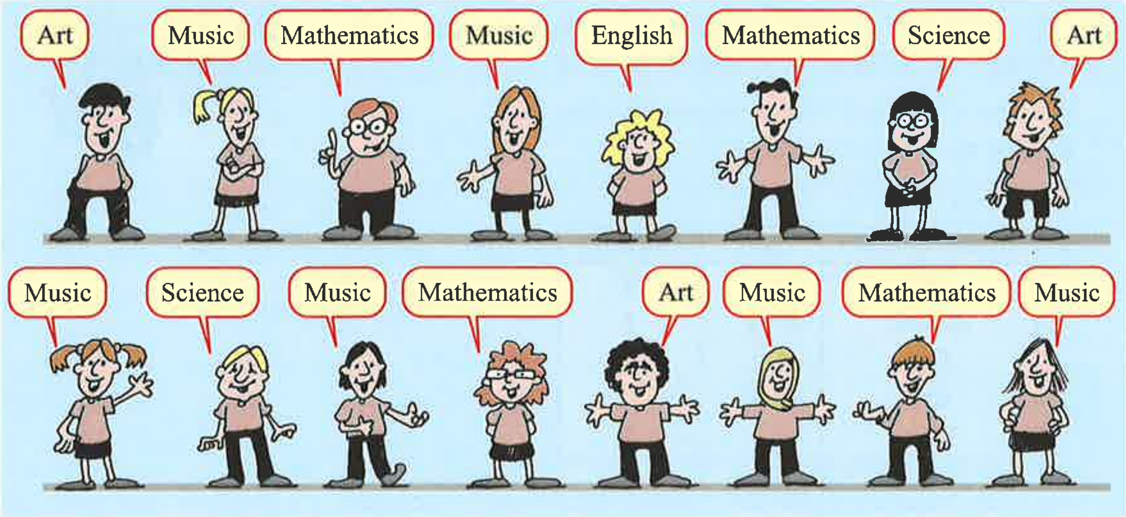
  In 1086 William ordered that a census be conducted to record population, wealth, and land ownership. A person's wealth was recorded in terms of land, animals, farm implements, and the number of peasants on the estate. All this information was collated in what is now called the **Domesday Book**. Regarded as the greatest public record of Medieval Europe, the Domesday Book is displayed in the National Archives in Kew.

William the Conqueror

A CATEGORICAL DATA

Categorical data is data which can be placed in categories.

For example, suppose the students in Alan's class are asked to name their favourite subject. The data collected is categorical data. The possible categories may include Mathematics, Art, Science, Music, and English.



TALLY AND FREQUENCY TABLES

We can organise data on favourite subjects using a **tally and frequency table**.

For each student we place a tick mark in the tally for his or her favourite subject.

The **frequency** of a category is the number of data in that category.

Favourite subject	*Tally*	*Frequency*
Mathematics	||||	4
Art	|||	3
Science	||	2
Music	𝍸 |	6
English	|	1
	Total	16

Each group of five is represented as 𝍸 .

From this table we can identify features of the data.

For example, Mathematics is the favourite subject for $\frac{4}{16} \times 100\% = 25\%$ of the students.

THE MODE

The **mode** is the most frequently occurring category.

For this data set, the mode is Music.

Example 1

Self Tutor

The data below records how students in a class travel to school on a particular day.

W = walk, Bi = bicycle, Bu = bus, C = car, T = train

The data is:

W Bi Bu T C Bi C W Bi Bu Bi C C Bi Bu W Bu Bu T C
Bi Bi Bu T C C Bi C C C W W Bu T C

a Draw a frequency table to organise the data.

b Find the mode of the data.

a

Method of travel	*Tally*	*Frequency*
Walk	𝍸	5
Bicycle	𝍸 |||	8
Bus	𝍸 ||	7
Car	𝍸 𝍸 |	11
Train	||||	4
	Total	35

b The mode is 'car' as this category occurs most frequently.

EXERCISE 18A.1

1 Students in a science class obtained the following levels of achievement:

D C C A A C C D C B C C C D
B C C C C E B A C C B C B C

a Complete a tally and frequency table for this data.

b Use your table to find the:

i number of students who obtained a C

ii fraction of students who obtained a B.

c What is the mode of the data?

2 A group of children at a summer camp were asked which sport they wanted to play. The choices were: T = tennis, S = swimming, C = cricket, B = basketball, and A = athletics.

The data was: AACTC CSAST TTBAS AACSA TATBC

a Draw a tally and frequency table for the data.

b Find the mode of the data.

3 People visiting the local show were asked whether they preferred the side shows (S), the farm animals (F), the ring events (R), the dogs and cats (D), or the wood chopping (W).

The results were: SRWSS WFDDS RSFWS RSRWS SRRRF

a Draw a tally and frequency table for the data.

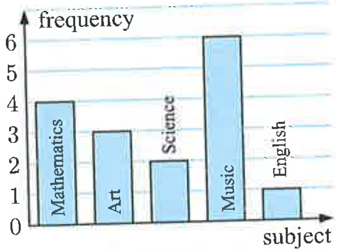
b Find the mode of the data.

GRAPHS TO DISPLAY CATEGORICAL DATA

Categorical data may be displayed using:

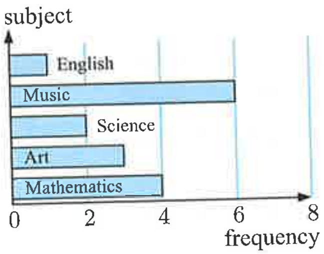
- a vertical column graph
- a horizontal bar chart
- a pie chart.

Vertical column graph



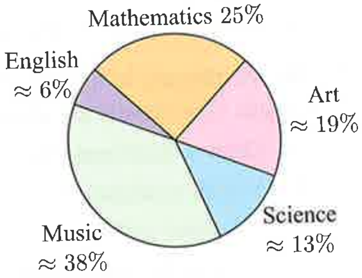
The heights of the columns indicate the frequencies.

Horizontal bar chart



The lengths of the bars indicate the frequencies.

Pie chart



The angles at the centre are calculated using the frequencies for each class.

Example 2

Self Tutor

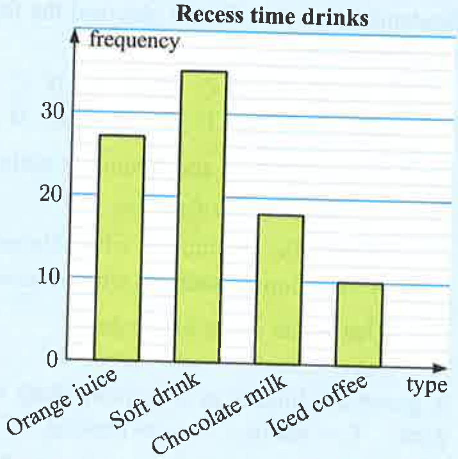
The vertical column graph shows the types of drink purchased by students at recess time.

a What is the least popular drink?

b What is the mode of the data?

c How many students bought orange juice?

d What percentage of students bought chocolate milk?



a Iced coffee {shortest column}

b 'Soft drink' is the mode.

c 27 students bought orange juice.

d The total number of students purchasing drinks $= 27 + 35 + 18 + 10$

$= 90$

So, the percentage of students who bought chocolate milk is $\frac{18}{90} \times 100\% = 20\%$

EXERCISE 18A.2

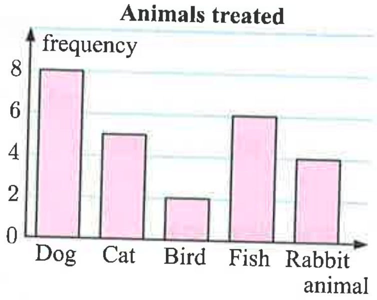
1 A Hong Kong vet clinic kept a record of the animals they treated on Wednesday. The results are displayed in the column graph.

a How many cats were treated?

b How many animals were treated?

c What percentage of the animals treated were rabbits?

d Find the mode of the data.



2 The 20 players in a football team voted to decide who should be their captain. The results are given in the table alongside.

a Draw a horizontal bar chart to display the data.

b Which candidate received the:

i most votes **ii** least votes?

c What percentage of the team voted for:

i Luke **ii** Greg or Steve?

Candidate	*Votes*
Cameron	3
Greg	7
Luke	4
Steve	6

3 At a school camp, the students selected their favourite ice cream flavour out of chocolate (C), strawberry (S), vanilla (V), and lime (L).

The results were: CVCSS VLSCV CVSLV SCCVV CSLCV
VCLSC CCVLS SLVCV CLCSC LCVLC

a Organise this data into a tally and frequency table.

b How many students chose vanilla?

c What percentage of the students chose lime?

d Find the mode of the data.

e Draw a vertical column graph to display the data.

Example 3

Self Tutor

The table opposite shows the results when the Year 7 students at a school were asked "What is your favourite fruit?"

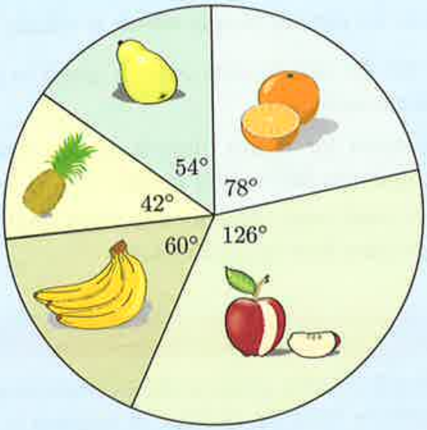
Construct a pie chart to display this data.

Fruit	*Frequency*
Orange	13
Apple	21
Banana	10
Pineapple	7
Pear	9
Total	60

There are 60 students in the sample, so each student represents $\frac{1}{60}$th of the pie chart.

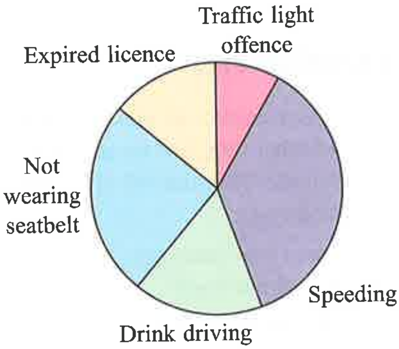
$\frac{1}{60}$th of 360° is 6°, so we can calculate the sector angles on the pie chart:

$13 \times 6° = 78°$ for the orange sector
$21 \times 6° = 126°$ for the apple sector
$10 \times 6° = 60°$ for the banana sector
$7 \times 6° = 42°$ for the pineapple sector
$9 \times 6° = 54°$ for the pear sector.



4 The pie chart shows the different types of traffic fines handed out by a police officer over one month. Determine whether the following statements are true or false:

- **a** The most common fine is for drink driving.
- **b** Fines for not wearing a seatbelt account for about one quarter of all fines.
- **c** More than half of the fines were either for speeding or drink driving.
- **d** There were more traffic light offence fines than expired licence fines.



5 A survey of eye colour in a group of 30 teenagers revealed these results:

Eye colour	*Blue*	*Brown*	*Green*	*Grey*
Number of students	9	12	2	7

- **a** Illustrate these results on a pie chart.
- **b** What percentage of the group have: **i** green eyes **ii** blue or grey eyes?

B COMPARING CATEGORICAL DATA

To understand the significance of the results we collect, we often need to compare two data sets.

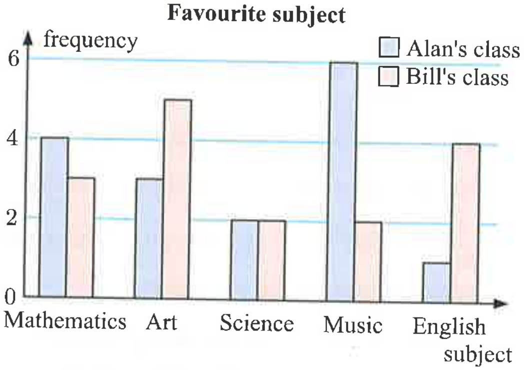
For example, at the start of the chapter we studied the favourite subjects of students in Alan's class. We now also consider the students in Bill's class, whose favourite subjects are shown in the table alongside.

Favourite subject	*Frequency*
Mathematics	3
Art	5
Science	2
Music	2
English	4
Total	16

To compare the results from Alan's class and Bill's class, we can draw a column graph for each data set on the same axes. This is known as a **side-by-side column graph**. A different colour is used for each data set so we can see clearly which is which.

We can use the side-by-side column graph to make observations such as:

- The mode for Alan's class is Music, whereas the mode for Bill's class is Art.
- There were more students who liked English in Bill's class than in Alan's class.



DISCUSSION

- Would it make sense to do a comparison like this if the number of students in Bill's class was different from the number of students in Alan's class?
- In this case, what could we do with the data so that a valid comparison could be made?

EXERCISE 18B

1 The students in class A and class B were asked whether they live north, east, south, or west of their school. This side-by-side column graph shows their responses.

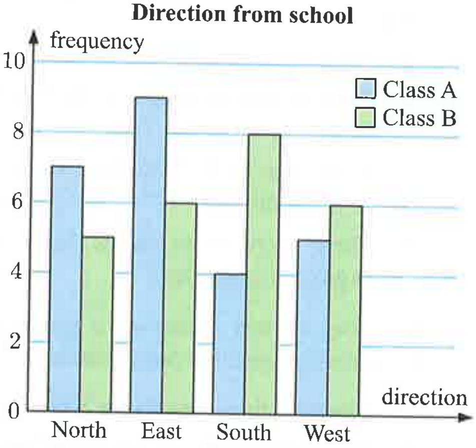
a How many students from class A live south of the school?

b How many students from class B live west of the school?

c Find the mode for:

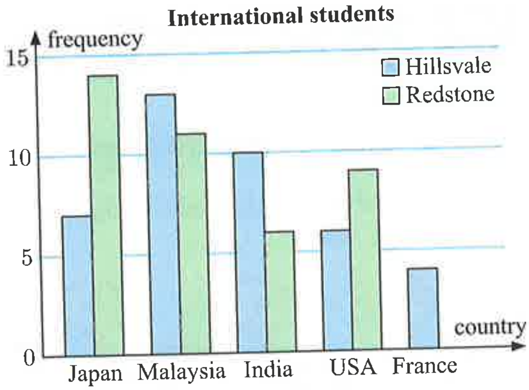
i class A **ii** class B.

d In which class are there more students who live north of the school?



2 Hillsvale School and Redstone School each have 40 international students. This side-by-side column graph shows the countries that these international students come from.

- **a** How many of Hillsvale's international students come from India?
- **b** Which school does not have any students from France?
- **c** Which school has more students from:
  - **i** Japan
  - **ii** Malaysia?



3 30 children and 30 adults were asked which section of the newspaper they enjoyed the most.

Children

Section	*Frequency*
News	5
Sport	7
Comics	10
Puzzles	8

Adults

Section	*Frequency*
News	10
Sport	9
Comics	4
Puzzles	7

- **a** Draw a side-by-side column graph to display the data.
- **b** Find the mode of each data set.
- **c** Which sections have the most difference in popularity between children and adults? Discuss your answer.

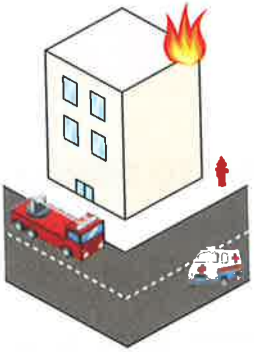
4 On a particular day, a fire truck and an ambulance each received 20 call-outs. The data below shows the location of each call-out, using the categories house (H), apartment (A), office (O), and factory (F).

Fire truck

F	O	A	H	F
H	F	O	F	H
O	H	F	O	F
F	A	H	O	H

Ambulance

H	F	O	H	A
A	H	F	H	H
F	A	H	F	H
O	H	F	H	A



- **a** Draw a tally and frequency table for each set of data.
- **b** Draw a side-by-side column graph to display the data.
- **c** Find the mode of each data set.
- **d** Which vehicle was called out to more offices?

ACTIVITY 1 USING TECHNOLOGY

STATISTICS PACKAGE

Click on the icon to load a statistical package which can construct a variety of graphs for a given data set.

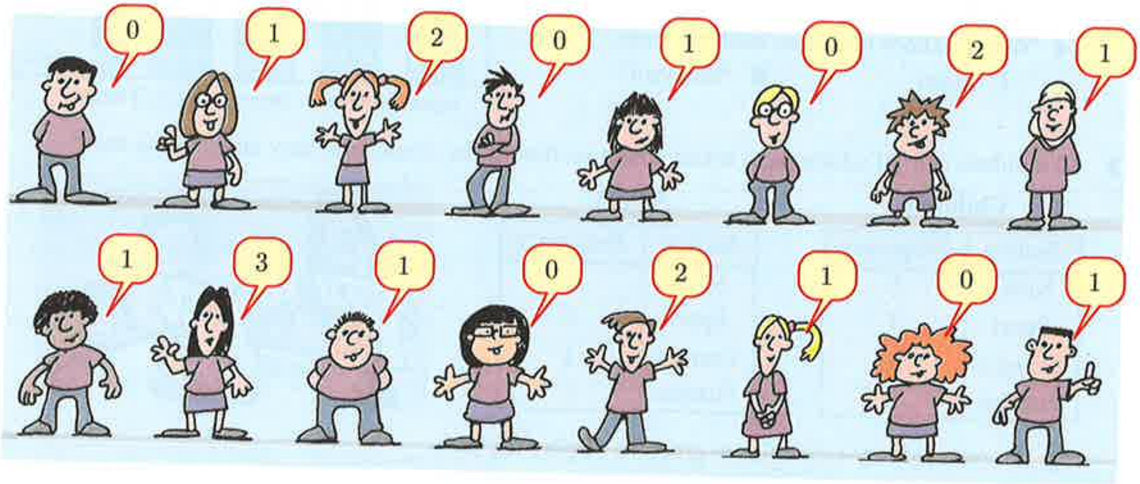
Change to a different graph by clicking on a different tab. You can also change the labels on the axes and the title of the graph.

Use the software or a spreadsheet to reproduce some of the statistical graphs in the previous **Exercises**. You can also use this software in any statistical project.

C NUMERICAL DATA

Numerical data is data which is given in number form.

The number of musical instruments that students in a class can play is an example of numerical data. It can take the values 0, 1, 2, ...



As with categorical data, numerical data can be organised using a tally and frequency table:

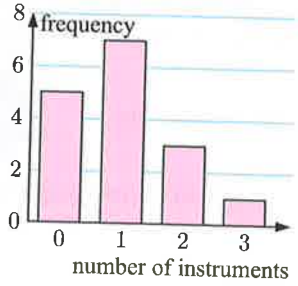
Number of instruments	*Tally*	*Frequency*
0	𝍸	5
1	𝍸 𝍷𝍷	7
2	𝍷𝍷𝍷	3
3	𝍷	1
	Total	16

GRAPHS TO DISPLAY NUMERICAL DATA

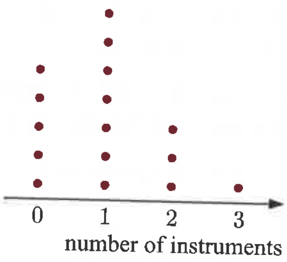
Numerical data can be displayed using:

- a **column graph**
- a **dot plot**
- a **stem-and-leaf plot**.

The **column graph** is the same as for categorical data, but with numbers on the horizontal axis instead of categories.



Dot plots are used when we have a small amount of data, and not many possible values for the data. Each dot represents a data value.



STEM-AND-LEAF PLOTS

A **stem-and-leaf plot** displays a set of data in order of size.

For example, the numbers of photographs taken by tourists on a bus tour were:

21	33	41	17	24	38	40	12	26	39
15	43	23	35	72	29	19	47	38	21
20	35	12	46	37	40	25	32	18	24

For each data value, the units digit is used as the **leaf**, and the digits before it determine the **stem** on which the leaf is placed.

So, the stem labels are 1, 2, 3, 4, 5, 6, 7 and they are written under one another in ascending order.

We now look at each data value in turn. We remove the last digit to find the stem, then write the last digit as a leaf in the appropriate row.

Once we have done this for all the data values, we have an **unordered stem-and-leaf plot**.

We can then **order** the stem-and-leaf plot by writing each set of leaves in ascending order.

Unordered stem-and-leaf plot

1	7 2 5 9 2 8
2	1 4 6 3 9 1 0 5 4
3	3 8 9 5 8 5 7 2
4	1 0 3 7 6 0
5	
6	
7	2

Scale: 1 | 7 means 17

Ordered stem-and-leaf plot

1	2 2 5 7 8 9
2	0 1 1 3 4 4 5 6 9
3	2 3 5 5 7 8 8 9
4	0 0 1 3 6 7
5	
6	
7	2

Scale: 1 | 7 means 17

DEMO

So, 4 | 0 0 1 3 6 7 represents the values 40, 40, 41, 43, 46, and 47.

Notice how the value 72 is separated from the rest of the data. Values such as this are called **outliers**.

Example 4

Self Tutor

A tennis player has won the following numbers of matches in tournaments during the last two years:

1	2	0	1	3	1	4	2	1	2	3	4
0	0	1	2	2	3	2	1	6	3	2	1
1	1	1	2	2	0	3	4	1	1	2	3
0	2	3	1	4	1	2	0	3	1	2	1

a Organise the data to form a frequency table.

b Draw a column graph of the data.

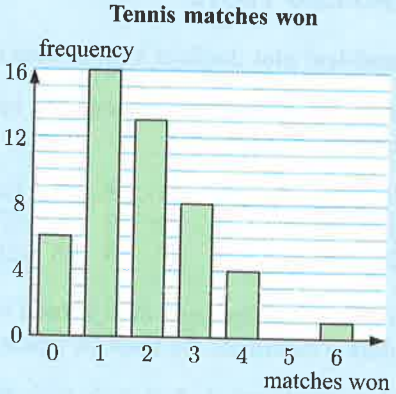
c How many times did the player advance past the second match of a tournament?

d On what percentage of occasions did the player win less than 2 matches?

a

Wins	*Tally*	*Frequency*
0	𝍸 |	6
1	𝍸 𝍸 𝍸 |	16
2	𝍸 𝍸 |||	13
3	𝍸 |||	8
4	||||	4
5		0
6	|	1
	Total	48

b



c The player won at least 2 matches on $13 + 8 + 4 + 1 = 26$ occasions.

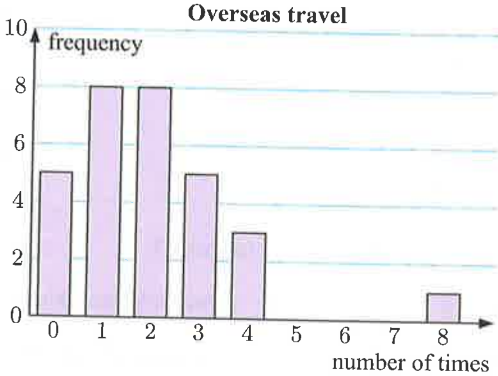
So, the player advanced past the second match of a tournament 26 times.

d The player won less than 2 matches on $6 + 16 = 22$ occasions.

This corresponds to $\frac{22}{48} \times 100\% \approx 45.8\%$ of the tournaments.

EXERCISE 18C

1 Workers in an office were asked how many times they had travelled overseas. The responses are displayed in the column graph alongside.



- **a** How many workers were surveyed?
- **b** How many workers have never been overseas?
- **c** What percentage of workers have been overseas at least three times?
- **d** Identify the outlier in the data.

2 Students at a school ran as many laps of the school athletics track as they could in one hour. The results are recorded on this frequency table.

- **a** Draw a column graph for this data.
- **b** What was the most common number of laps completed?
- **c** How many students completed 12 laps or less?
- **d** What fraction of the students completed at least 14 laps?

Number of laps	*Students*
10	1
11	2
12	4
13	6
14	3
15	10
16	17
17	8
18	13
19	2

3 Yvonne counted the number of chocolate chips in each biscuit of a packet, and obtained these results:

4, 7, 5, 5, 6, 4, 7, 8, 2, 5, 6, 6, 5, 5, 7, 5, 7, 5, 3, 6

- **a** Draw a dot plot of her results.

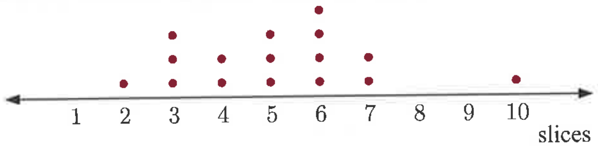
b What was the most frequent number of chocolate chips?

c What was the highest number of chocolate chips in a biscuit?

d How many biscuits contained five chocolate chips?

e What percentage of biscuits contained less than five chocolate chips?

4 A birthday party was held at an all-you-can-eat pizza restaurant. The number of slices eaten by each person is shown in the dot plot below.



a How many people attended the party?

b What was the least number of slices eaten?

c How many people ate six slices of pizza?

d Are there any outliers in the data?

5 A group of schools in a city were surveyed to find how many Grade 7 students they had. The results are displayed in a stem-and-leaf plot.

4	8
5	4 4 7
6	0 2 5 6 8
7	2 4 7
8	0 1 1
9	5 2
10	5 8
11	1

Scale: 4 | 8 means 48 students

a How many schools were surveyed?

b How many schools had 54 Grade 7 students?

c What was the highest number of Grade 7 students a school had?

d How many schools had at least 80 Grade 7 students?

6 The numbers of runs scored by a batsman over a 30 game season were:

27, 7, 12, 74, 30, 11, 42, 19, 29, 51, 62, 14, 49, 22, 2,
35, 43, 12, 62, 22, 28, 37, 59, 40, 5, 13, 69, 32, 16, 21

a Construct an unordered stem-and-leaf plot of the data. Make sure you include a scale.

b Construct an ordered stem-and-leaf plot of the data.

c How many times did the batsman score more than 25 runs?

d Find the batsman's: **i** lowest **ii** highest score.

7 Walter recorded the number of pages in the daily newspaper for 4 weeks:

86 94 78 108 96 112 100 122 92 88 100 96 80 112
78 92 104 124 88 160 116 92 86 94 106 82 114 116

a Construct a stem-and-leaf plot to display the data.

b How many newspapers contained at least 100 pages?

c What percentage of the newspapers contained less than 95 pages?

d Are there any outliers in the data?

DISCUSSION

When displaying numerical data, when is it best to use:

- a dot plot
- a column graph
- a stem-and-leaf plot?

ACTIVITY 2

CONDUCT YOUR OWN SURVEY

What to do:

1. Decide on a question about your class you would like to investigate. For example:
   "What is the most common method of travelling to school?"
   "What type of pet is most common?"
   "What type of TV show is the most watched?"
   "How many pets have you owned?"
2. Collect the questions from the students in the class, and use them to make a survey for everyone to do.
3. Collect the data for your question from each of your classmates.
4. Is your data categorical or numerical?
5. Organise your data into a table.
6. Display your data using an appropriate graph.
7. Share your findings with your class.

D

MEASURING THE CENTRE AND SPREAD

When we analyse numerical data, we need to understand how the numbers are distributed. We need a measure of its **centre**, and also how the data is **spread** on either side of this centre.

MEASURING THE CENTRE

There are three different numbers which are commonly used to measure the **middle** or **centre** of a set of numerical data. These are the **mean** or **average**, the **median**, and the **mode**.

THE MEAN

The **mean** or **average** is the sum of all data values divided by the number of data values.

$$\textbf{mean} = \frac{\textbf{sum of data values}}{\textbf{number of data values}}$$

Example 5 Self Tutor

Find the mean of this data set: 5, 13, 10, 13, 15, 9, 17, 14

There are 8 data values.

$$\text{mean} = \frac{\text{sum of data values}}{\text{number of data values}}$$
$$= \frac{5+13+10+13+15+9+17+14}{8}$$
$$= \frac{96}{8}$$
$$= 12$$

THE MEDIAN

The **median** of a set of data is the *middle* value of the ordered set of data values.

To find the median of a set of data, we follow these steps:

Step 1: Write the data in order from smallest to largest.

Step 2: Starting at the ends, cross out the data values in pairs, working inwards until you reach the middle.

Step 3:
- If there is an *odd number* of data values, there will be one middle value. This value is the median.
- If there is an *even number* of data values, there will be two middle values. The median is the average of these two values.

Example 6 Self Tutor

Find the median of:

a 9, 7, 6, 14, 10, 4, 11 **b** 2, 5, 9, 4, 12, 3, 7, 4, 10, 7

a The ordered data set is: ~~4, 6, 7,~~ 9, ~~10, 11, 14~~

$\therefore$ median = 9

b The ordered data set is: ~~2, 3, 4, 4,~~ 5, 7, ~~7, 9, 10, 12~~

$$\therefore \text{ median} = \frac{5+7}{2} = 6$$

THE MODE

The **mode** is the score in the data set which occurs most often.

For example, the mode of the data set 0, 2, 3, 3, 4, 5, 5, 5, 6, 7, 9 is 5 since 5 occurs most frequently.

If two data values occur most frequently, the data is **bimodal** and we list both values as modes.

If more than two data values occur most frequently, we do not use the mode as a measure of the centre of the data set.

Example 7

Self Tutor

An exceptional footballer scores the following goals for her school during a season:

1 3 2 0 4   2 1 4 2 3   0 3 3 2 2   5 2 3 1 2

For this data, find the:

a mean   **b** median   **c** mode.

a mean $= \dfrac{\text{sum of all scores}}{\text{number of matches}}$

$= \dfrac{45}{20} = 2.25$ goals

b The ordered data set is:

~~0 0 1 1 1 2 2 2 2~~ 2 2 ~~2 3 3 3 3 3 4 4 5~~

$\therefore$ median $= \dfrac{2+2}{2} = 2$ goals

c mode $= 2$ goals {2 occurs most often}

EXERCISE 18D.1

1 Find the mean of the following data sets:

a 7, 10, 4, 11

b 12, 9, 6, 11, 17

c 3, 1, 5, 4, 4, 7

d 7, 5, 0, 3, 0, 6, 0, 9, 1, 4

e 2.1, 4.5, 5.2, 7.1, 9.3

f 5, 2.4, 6.2, 8.9, 4.1, 3.4

2 Find the median of the following data sets:

a 2, 4, 5, 8, 10, 11, 13

b 5, 8, 10, 11, 13, 16, 19, 20

c 2, 1, 1, 3, 4, 3, 2, 1, 5, 4, 3, 3, 0

d 5, 9, 2, 4, 6, 6, 7, 6, 11

e 1.2, 1.9, 2.2, 2.6, 2.9

f 0.5, 5.6, 3.8, 4.9, 2.7, 4.4

3 Consider the data set: 7, 8, 0, 3, 0, 6, 0, 11, 1.

a For this data, find the:

i mean   **ii** median   **iii** mode.

b Is the mode a suitable measure of the "centre" of this data set? Explain your answer.

4 Margaret played 10 games of Scrabble in a tournament, and obtained the following scores:

206 120 108 185 219 168 245 295 195 307

For these scores, find the:

a mean   **b** median.

5 The number of text messages that Jim received each day for the last 15 days were:

2 3 9 13 4 3 12 1 6 15 3 4 10 2 3

For this data, find the:

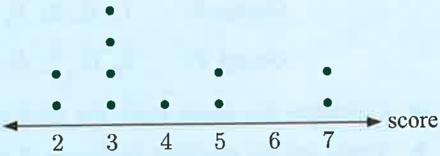
a mean   **b** median   **c** mode.

Example 8

Self Tutor

For the data represented by this dot plot, find the:

a mode **b** mean **c** median.



The data values are 2, 2, 3, 3, 3, 3, 4, 5, 5, 7, 7

a The value 3 occurs most often, so the mode is 3.

b $$\text{mean} = \frac{2+2+3+3+3+3+4+5+5+7+7}{11}$$
$$= \frac{44}{11}$$
$$= 4$$

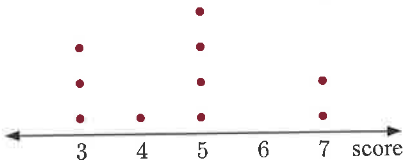
c The ordered data set is: ~~2, 2, 3, 3, 3,~~ 3, ~~4, 5, 5, 7, 7~~

$\therefore$ median = 3

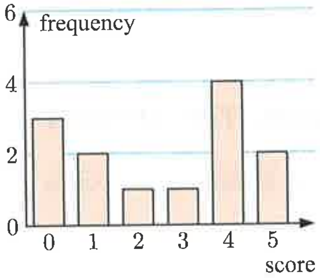
6 For the data in each of the following graphs, find the:

i mode **ii** mean **iii** median.

a



b

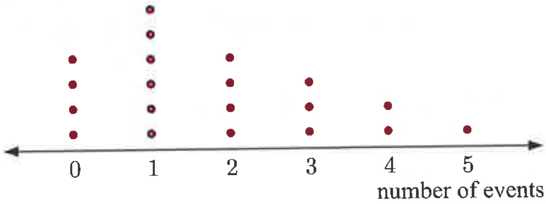


c

1	3 5
2	0 1 2 7
3	4 6 6
4	3 5 9

Scale: 1 | 3 means 1.3

7 The students in a class were asked how many events they competed in at the school's swimming carnival. The results are displayed on a dot plot.



a How many students did not compete in any events?

b For this data, find the: **i** mode **ii** median **iii** mean.

c Copy the graph, and locate on it the mode, median, and mean.

8 Consider the performances of two groups of students in the same mental arithmetic test out of 10 marks.

Group X: 7, 6, 6, 8, 6, 9, 7, 5, 4, 7

Group Y: 9, 6, 7, 6, 8, 10, 3, 9, 9, 8, 9

- **a** Calculate the mean mark for each group.
- **b** There are 10 students in *Group X* and 11 in *Group Y*. Is it unfair to compare the mean scores for these groups?
- **c** Which group performed better at the test?

9 Consider the data in the **Opening Problem** on page **356**.

- **a** Calculate the mean and median for each boy.
- **b** Who generally catches more fish? Discuss your answer.

10 Josh and Eugene each own a hot dog stand. They record the number of hot dogs they sell every day for two weeks. The results are:

Josh: 33, 40, 28, 43, 38, 32, 24, 35, 47, 29, 31, 36, 27, 38

Eugene: 39, 47, 32, 51, 48, 55, 61, 35, 49, 58, 52, 67, 55, 43

- **a** What was the highest number of hot dogs that Josh sold in one day?
- **b** Calculate the mean and median for each data set.
- **c** Who generally sells more hot dogs? Discuss your answer.

MEASURING THE SPREAD

In addition to measuring the centre of a set of data, it is also important to consider how the data is **spread**.

The simplest measure of spread is the **range**.

The **range** of a data set is the difference between the **maximum** or largest data value, and the **minimum** or smallest data value.

range = maximum value − minimum value

Example 9 **Self Tutor**

17 students were asked how many days they had stayed home sick from school so far this year. The results were:

2, 1, 5, 5, 3, 4, 3, 6, 2, 9, 4, 2, 3, 5, 6, 2, 3

Find the range of this data set.

The minimum value is 1 and the maximum value is 9.

So, the range = 9 − 1 = 8 days.

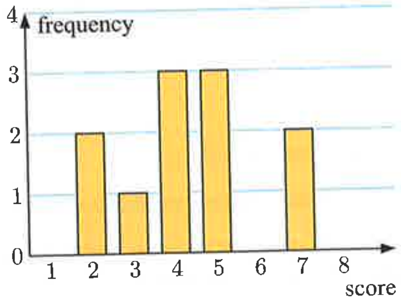
EXERCISE 18D.2

1 Find the range of the following data sets:

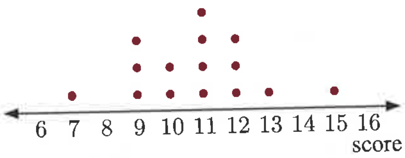
- **a** 2, 4, 4, 5, 6, 8, 9, 10, 11, 11, 13
- **b** 6, 6, 6, 6, 7, 7, 7, 7, 8
- **c** 7, 9, 12, 9, 4, 8, 11, 6, 10
- **d** 6, 8, 15, 4, 11, 18, 14, 10
- **e** 19, 33, 27, 38, 46, 17, 39
- **f** 8.5, 4.2, 7.6, 7.2, 9.3, 9.1, 5.6

2 Find the range of the data represented by each graph:

a



b



c

2	5 7 9
3	0 1 2 6 8
4	3 5 6 9
5	0 3 8

Scale: 5 | 0 means 50

3 The numbers of items bought by customers at a convenience store were:

3 5 5 8 5 3 5 9 7 4 5 8 7 7 6

- **a** Draw a dot plot to display the data.
- **b** Calculate the mean, median, and range of the data, and indicate these values on your dot plot.

4 The table alongside shows the maximum temperatures, in °C, in some capital cities for one week.

- **a** Calculate the range for each city.
- **b** Which city had the:
  - **i** most variation
  - **ii** least variation in maximum temperature during the week?

City	*Mon*	*Tue*	*Wed*	*Thu*	*Fri*	*Sat*	*Sun*
Beijing	33	32	22	29	32	32	29
Berlin	24	23	24	23	18	20	21
Cairo	34	35	36	36	36	35	36
Lima	18	17	18	17	18	18	18
Moscow	27	30	25	24	28	22	21
Ottawa	30	22	24	17	16	17	20
Reykjavik	19	16	15	13	12	12	11
Wellington	13	13	12	12	10	12	13

RESEARCH

With most goods we buy, we can read the amount we are buying on the packaging. For example, we might buy 35 g of sultanas, 20 m of baking paper, or 600 mL of water. But how do we know the manufacturer is telling the truth?

Choose a bulk packet that has several of the same item in it. For example:

- a bag containing 8 balls of wool, each 75 m long
- a packet containing 12 bags of chips, each 22 g
- a 6 pack of fruit juice cartons, each 175 mL.

Your task is to analyse whether the manufacturer has made a truthful claim about how much is in their product.

What to do:

1 Choose your item to analyse.

- **a** Describe exactly what you are trying to find out, and how you are going to test it.
- **b** What do you expect your results to be?

2 **a** Measure the mass, length, or volume of each item in your packet. Round your data appropriately. Construct a stem-and-leaf plot of your results.

- **b** For your data, find the:
  - **i** mode
  - **ii** median
  - **iii** mean.
- **c** Were the results in **b** what you expected? Explain your answer.

3 Calculate the percentage of items that were below the amount stated on the packaging.

4 From your results, can you form any conclusions about the amount of each item in your packet? Do you think the manufacturers are telling the truth? Explain your answer.

E DATA COLLECTION

For any statistical problem there is a **target population**. This is the group of things or people we are interested in finding information about. For example, we may want to know the colours of cats in a particular pet store, or the ages of people living in South Africa.

CENSUS OR SAMPLE

When we collect data, we can either perform a **census** or a **sample**.

A **census** involves collecting data about *every* individual in the target population.

For example, if we wanted to know the colours of cats in a particular pet store, we could visit the store and record the colour of every cat in the store. Analysing the data would provide exact information, such as the exact percentage of cats in the store which are black.

However, if the target population is very large, or if it is very expensive to collect information from the whole population, we may choose instead to take a **sample** from the population.

A **sample** involves collecting data about a *part* of the target population only.

For example, it would be time-consuming and expensive to survey every person living in South Africa, to ask his or her age. We would instead collect data from a selection of South Africans, and use this data to draw conclusions about the whole population.

Conclusions based on data from samples always involve some error. However, we can use the properties of the sample to **estimate** the properties of the population.

We can improve the quality of our estimate by choosing a sample that is **unbiased** and **sufficiently large**.

BIAS IN SAMPLING

For the properties of a sample to be a reliable estimate of the properties of the whole population, the sample we choose must be **representative** of the population.

For example, suppose you wanted to find the ages of people living in South Africa. If you asked the ages of a selection of students leaving a South African high school, the information you would obtain would not be representative of the whole population of South Africa. We would call this a **biased sample**.

To obtain more representative data, we could choose our sample from a location where there is less age bias, such as a shopping centre.

DISCUSSION

What other methods could we use to select an unbiased sample of people?

SAMPLE SIZE

For the results of a sample to be reliable, the sample must also be **sufficiently large**. For example, if we were to estimate the average age of South Africans by surveying only five people, we would not have a very reliable estimate.

A good sample is *unbiased* and *sufficiently large*.

Example 10 **Self Tutor**

Ellen wanted to know the mean height of the Year 8 students at her school. She measured the students in her Year 8 class, and obtained these results, in centimetres:

152	163	149	166	151	155	142	150	161	160
148	158	163	170	153	162	149	154	160	157
151	142	164	158	147	165	158	163	145	164

a Use this sample to estimate the mean height of Year 8 students at the school.

b Do you think this estimate will be accurate? Explain your answer.

a There are 30 students in the sample.

$$\therefore \text{ the mean of the sample} = \frac{152 + 163 + ... + 145 + 164}{30}$$

$$= \frac{4680}{30}$$

$$= 156 \text{ cm}$$

We estimate that the mean height of the Year 8 students at the school is 156 cm.

b The sample is of a reasonable size, and there is no reason to expect any height bias between the Year 8 classes.
So, this estimate should be accurate.

EXERCISE 18E

1 State whether a census or a sample would be used for these investigations:

- **a** the country of origin of the parents of students in your class
- **b** the number of people in Canada who are concerned about global warming
- **c** people's opinions about the public transport system in Lisbon
- **d** the heights of trees in a garden.

2 Explain any bias in the following samples:

- **a** To determine the proportion of English people who can swim, Bill surveys a selection of people at a local swimming pool.
- **b** To determine the average height of plants in her wheat crop, Jill measures the plants nearest to the barn.
- **c** To determine the average time workers in the United States take to travel to work, Gary surveys a group of workers in New York.

3 Maura is interested in finding the average membership size for gyms around New Zealand. She randomly selects four gyms, and finds they have the following membership numbers:

280 173 227 88

- **a** Use these results to predict the average number of members for gyms around New Zealand.
- **b** Comment on the reliability of your prediction.

4 Cindy wanted to know what percentage of her youth group prefer drinking tea to coffee. She randomly selected 80 fellow group members to survey. 56 preferred tea, and 24 preferred coffee.

- **a** Estimate the percentage of youth group members who prefer tea.
- **b** Do you think this estimate is reliable? Explain your answer.

5 A "Healthy Eating" group is trying to determine how many meals the average adult eats out per week. The group takes a sample of 30 adults who all live in the central business district. The results are given below:

2, 8, 7, 5, 1, 0, 2, 5, 9, 12, 10, 4, 3, 6, 1,
10, 5, 4, 3, 6, 0, 1, 2, 7, 4, 5, 0, 2, 2, 5

- **a** Find the mean of the data set.
- **b** Has the group used a good sample? Explain your answer.

KEY WORDS USED IN THIS CHAPTER

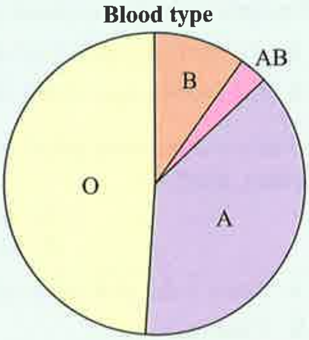
- bar chart
- census
- data set
- median
- pie chart
- spread
- tally and frequency table
- biased sample
- centre
- dot plot
- mode
- range
- statistics
- target population
- categorical data
- column graph
- mean
- numerical data
- sample
- stem-and-leaf plot

REVIEW SET 18A

1 A random sample of people were surveyed about their blood type. The results are displayed in the pie chart opposite.

Decide whether these statements are true or false, giving reasons for your answers.

- **a** The most common blood type is type O.
- **b** More than one quarter of the people surveyed have type B blood.
- **c** More than one half of the people surveyed do not have type O blood.



2 A survey of hair colour in a class of 40 students revealed the results in the table.

Hair colour	*Frequency*
Red	4
Brown	17
Black	11
Blond	8

- **a** Construct a horizontal bar chart to display this data.
- **b** For this group of students, which was the least common hair colour?
- **c** Could conclusions be made from this survey about the hair colour of all students? Explain your answer.

3 A supermarket puts 1 L cartons of milk on sale, and records the number of cartons bought by each customer over an hour. The results were:

0 0 1 1 1 2 1 0 3 1 2 4 0 1 2 7 1 1 0 2 3

- **a** Draw a dot plot to display this information.
- **b** Are there any outliers in the data?

4 Jillian recorded the number of pages in the weekly local newspaper over a period of time. The results are shown in the stem-and-leaf plot.

3	8
4	1 1 2 3 6 7 8 8
5	1 2 7 8 9
6	0 0 1 2
7	1

3 | 8 means 38

- **a** What percentage of newspapers contained at least 60 pages?
- **b** Find the median of the data.

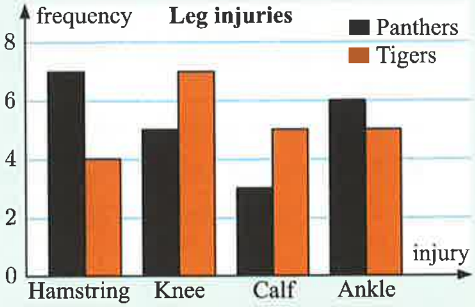
5 While practising at the driving range, Colin hit golf balls the following distances (in m):

186 229 234 192 235 229

Find the mean distance of Colin's shots.

6 This side-by-side column graph shows the leg injuries received by two rugby teams during a season.

- **a** How many calf injuries were received by:
  - **i** the Panthers
  - **ii** the Tigers?
- **b** Which team suffered the most ankle injuries?
- **c** Find the mode of each data set.



7 Find the range of the data set: 70 35 25 67 82 53 63 79 41

8 Determine whether a census or a sample would be appropriate for finding:

- **a** the average number of detentions given to the students of a Grade 7 class
- **b** the median age of full-time university students in Denmark.

9 Some children were asked how much pocket money they receive each week. The results, in dollars, were:

2, 4, 0, 10, 4, 0, 5, 5, 2, 4, 10, 5, 2,
0, 10, 0, 2, 5, 2, 8, 5, 10, 2, 0, 10

- **a** Draw a dot plot of the data.
- **b** Find the:
  - **i** mode
  - **ii** mean
  - **iii** median
  - **iv** range.
- **c** Indicate the values found in **b** on your dot plot.

10 Explain any bias in the following samples:

- **a** To find the favourite TV show of high school students, the Grade 7 students at the local high school are surveyed.
- **b** To determine the most popular dish at a restaurant, the diners at Sunday lunch are surveyed.

REVIEW SET 18B

1 Monica received £60 for her birthday. She spent £15 on a book, £20 on a necklace, and saved the remaining £25. Draw a pie chart to display this information.

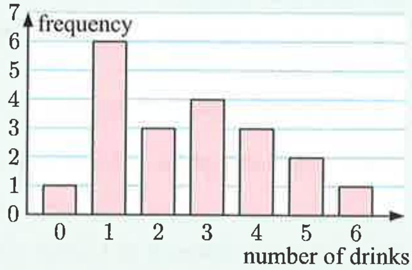
2 Consider the data set: 3, 4, 6, 6, 7, 9, 12, 13, 14, 17, 19. Find the:

- **a** mean
- **b** median
- **c** mode

3 Melanie is conducting a survey of her classmates about their favourite sport. Will the results be categorical or numerical data?

4 The numbers of drinks sold at tables in a café are displayed in this frequency column graph.

- **a** What percentage of customers ordered 4 or more drinks?
- **b** Find the mode of the data.



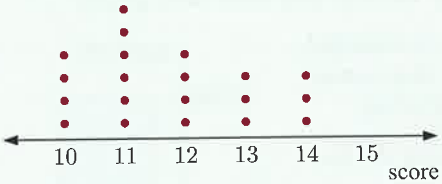
5 Ruth notes the number of biscuits she eats at work each day, and records them in the table opposite.

- **a** Copy and complete the table.
- **b** For how many days did Ruth record data?
- **c** Draw a column graph to display the data.

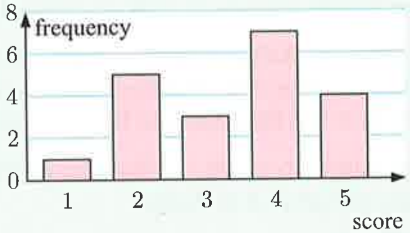
Number of biscuits	*Tally*	*Frequency*
0	|	
1		4
2	𝍸 |||	
3	𝍸 |	
4	||	
	Total	

6 Find the mean, median, mode, and range of the data represented in each graph:

a



b



7 During the 24 game netball season, Alyssa played in all games and scored 482 goals. Due to injury, Stephanie only played 19 games, and scored 335 goals. Which player received the award for the highest average goals scored per game?

8 A group of 20 children played a round of mini-golf. Their scores were:

43 32 59 35 60 26 39 41 53 67
39 54 28 46 65 30 45 23 32 65

a Draw a stem-and-leaf plot to display the data.

b How many children scored less than 40?

c What percentage of children scored more than 55?

d For this data, find the:

i mean **ii** median **iii** range.

9 A cinema wanted to find the average age of their patrons. At the end of a children's movie, 20 randomly selected audience members were asked their age. The responses were:

4 29 6 8 42 9 10 61 31 5 7 42 6 51 9 11 33 8 28 12

a Find the mean of the sample. **b** Find the median of the sample.

c Are the mean and the median good estimates of the average age of the cinema's patrons? Explain your answer.

10 40 boys and 40 girls were asked to name their favourite piece of playground equipment.

Boys

Equipment	*Frequency*
Slippery dip	13
Swings	8
Monkey bars	7
Flying fox	12

Girls

Equipment	*Frequency*
Slippery dip	6
Swings	9
Monkey bars	15
Flying fox	10

a How many boys chose the flying fox?

b How many girls chose the slippery dip?

c Draw a side-by-side column graph to display the data.

d Which piece of playground equipment was most popular with:

i boys **ii** girls?

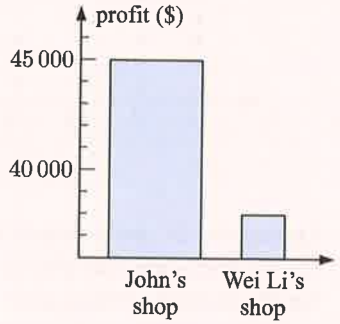
e Did more boys or girls choose the flying fox?

DISCUSSION — MISLEADING GRAPHS

Some people may try to trick or mislead others by the way they draw their graphs.

For example, Kelly owns two shops. One of them is managed by John, and the other by Wei Li. Last year John's shop earned a profit of $45 000, whereas Wei Li's profit was $38 000.

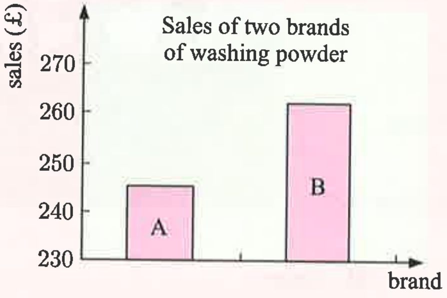
John draws this graph to show the profits earned by the two shops, and gives it to Kelly.



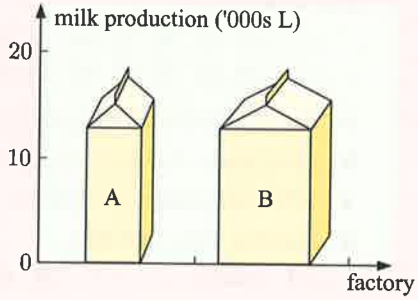
What to do:

1. Discuss the misleading features of John's graph.
2. Why do you think John has drawn the graph like this?
3. Discuss the misleading features of these graphs:

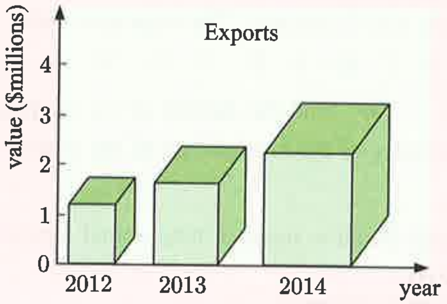
a



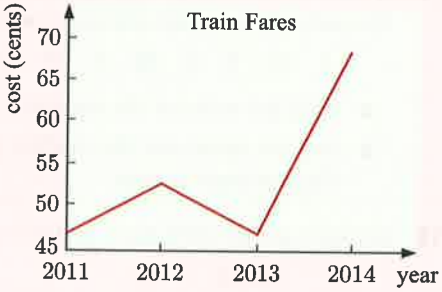
b



c



d



Chapter 19

Transformations

Contents:

OPENING PROBLEM

Consider the photograph alongside.

Things to think about:

How is the photograph *transformed* into each of the cases below?

a

b

c

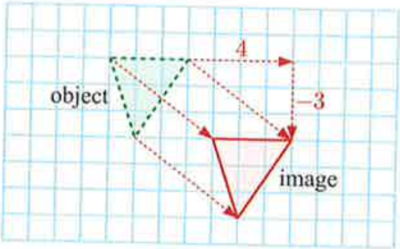
In this course we will consider the following **transformations**:

- translations
- reflections
- rotations.

When we perform a transformation, the original shape is called the **object**. The shape which results from the transformation is called the **image**.

A TRANSLATIONS

A **translation** of a figure occurs when every point on the figure is moved the same distance in the same direction.



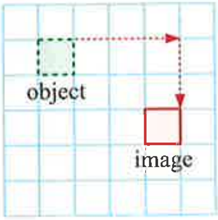
In the translation shown, the original figure has been translated 4 units right and 3 units down to give the image.

DEMO

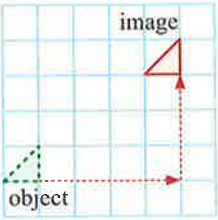
EXERCISE 19A

1 Describe each of these translations:

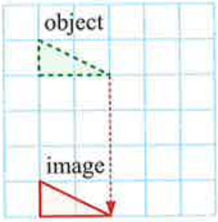
a



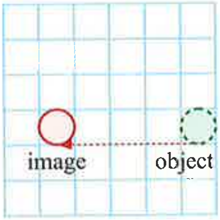
b



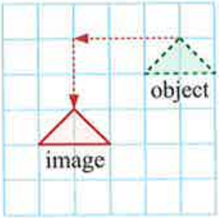
c



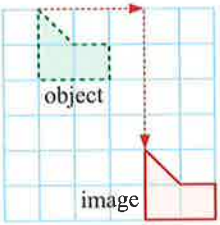
d



e

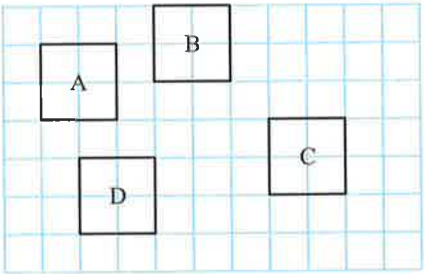


f



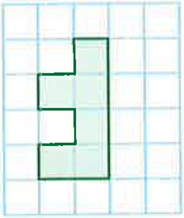
2 For the figures alongside, describe the translation from:

a A to B **b** B to A **c** B to C
d C to B **e** D to C **f** C to D
g B to D **h** D to B.

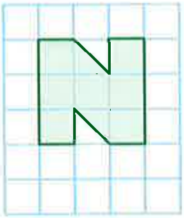


3 Copy these figures onto grid paper, then translate them according to the given directions:

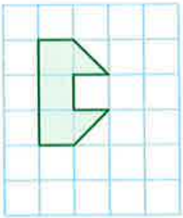
a 3 right, 4 down



b 6 left, 4 up



c 2 right, 5 up

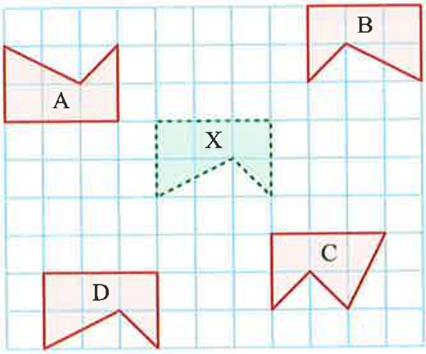


DEMO

PRINTABLE DIAGRAMS

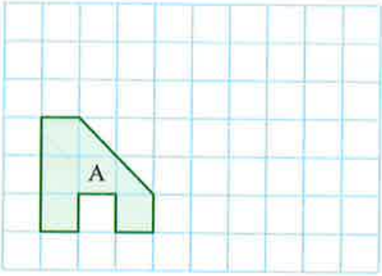
4 **a** Which of the figures A, B, C, or D is a translation of figure X?

b Describe the translation from X to this figure.



5 **a** Translate A 4 units right and 3 units up to give A'.

b What translation is needed to shift A' back to A?

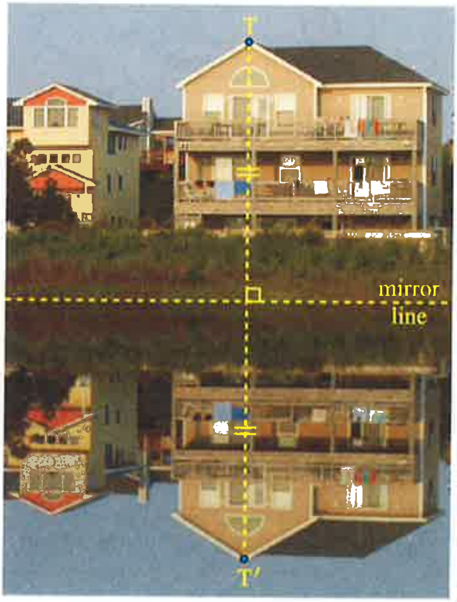


B REFLECTIONS AND LINE SYMMETRY

In the picture alongside, the house is **reflected** in the lake. The line where the lake meets the land is called the **mirror line**.

The point T at the top of the house is reflected to give the image point T'. T and T' are the same distance from the mirror line, and the line joining T and T' is perpendicular to the mirror line.

In general, to reflect an object in a mirror line, we draw lines at right angles to the mirror line which pass through key points on the object. The image of each point will be the same distance away from the mirror line as the point on the object, but on the opposite side of the mirror line.

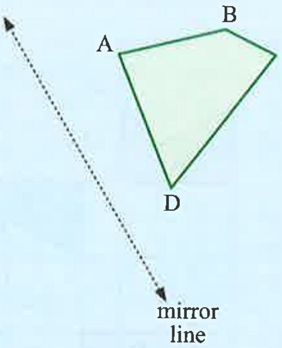


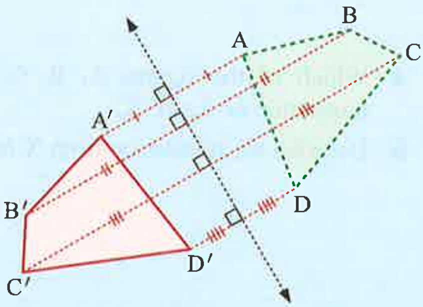
Courtesy of Pat Walsh - modified by permission

Example 1

Self Tutor

Reflect this quadrilateral in the mirror line.



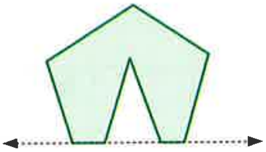


EXERCISE 19B.1

PRINTABLE DIAGRAMS

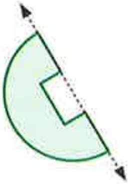
1 Draw the reflections of the following objects in the dashed mirror lines given:

a



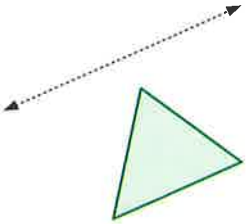
b

c

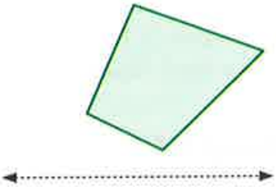


d

e

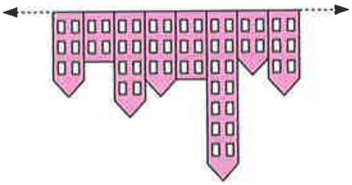


f

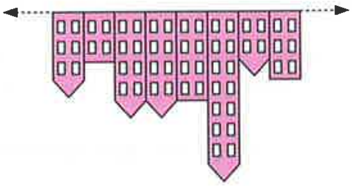


2 Which of the options below is the correct reflection of this row of buildings?

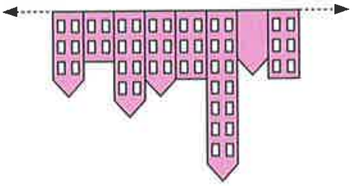
A



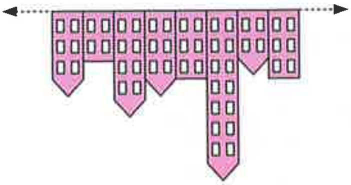
B



C

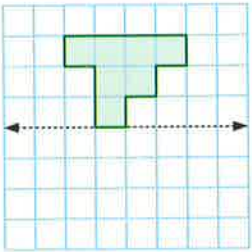


D

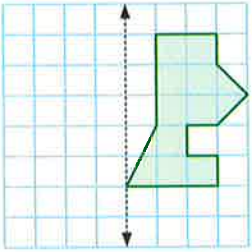


3 On grid paper, reflect these shapes in the given mirror lines:

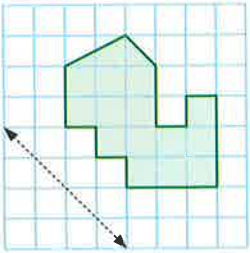
a



b

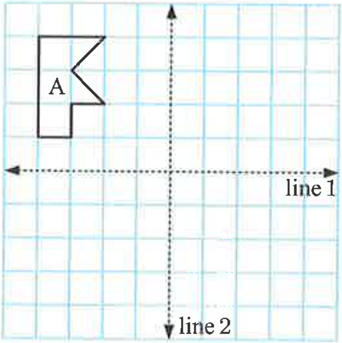


c



4 Find the image when figure A is reflected in:

a line 1 **b** line 2.

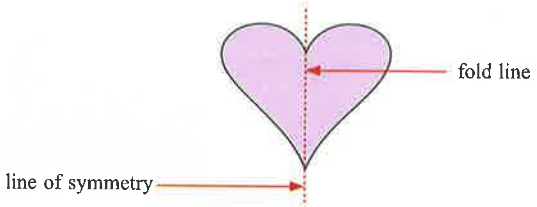


5 Suppose a figure X is reflected in a mirror line to give X′. What happens if X′ is reflected in the same mirror line? Illustrate your answer.

LINE SYMMETRY

A **line of symmetry** is a line along which a shape may be folded so the two parts of the shape will match.

For example:



DEMO

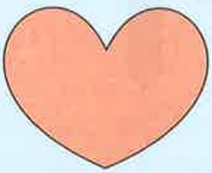
If a mirror is placed along the line of symmetry, the reflection in the mirror will be exactly the same as the half of the figure on the other side of the mirror line.

A shape has **line symmetry** if it has at least one line of symmetry.

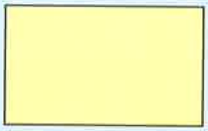
Example 2 Self Tutor

Draw all lines of symmetry of:

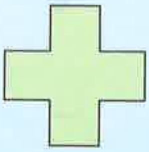
a



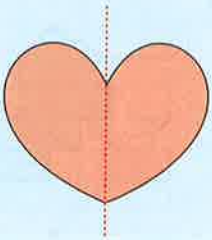
b



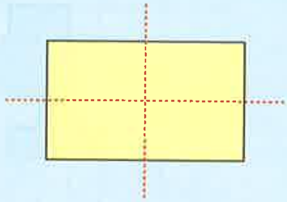
c



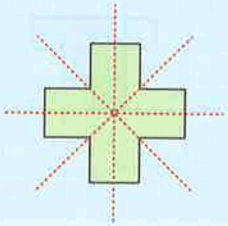
a



b



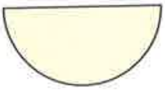
c



EXERCISE 19B.2

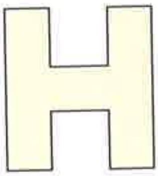
1 Copy these figures and draw their lines of symmetry:

a

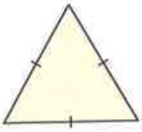


b

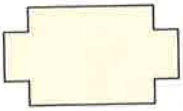
c



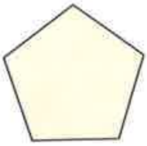
d



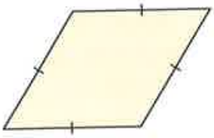
e



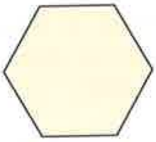
f



g



h

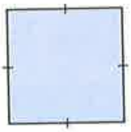


PRINTABLE DIAGRAMS

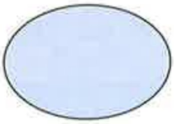
2 **a** Copy the following shapes and draw in all lines of symmetry.

i

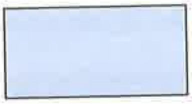
ii



iii



iv



b Which of these figures has the most lines of symmetry?

3 How many lines of symmetry do these patterns have?

a

b

4 **a** How many lines of symmetry can a triangle have? Draw all of the possible cases.

b How many lines of symmetry can a quadrilateral have? Draw all of the possible cases.

c How many lines of symmetry do you think a circle has?

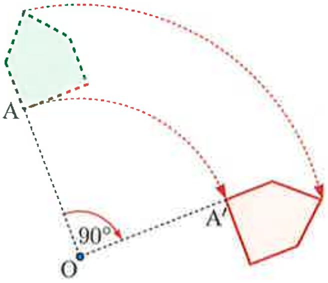
C ROTATIONS AND ROTATIONAL SYMMETRY

We are all familiar with objects which rotate, such as wheels, propellers, and the hands of a clock. We know that the Earth rotates on its axis once every day.

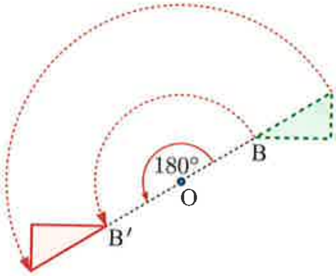
A **rotation** turns a shape or figure about a point and through a given angle.
The point about which a figure rotates is called the **centre of rotation**. We often label this point O.

DEMO

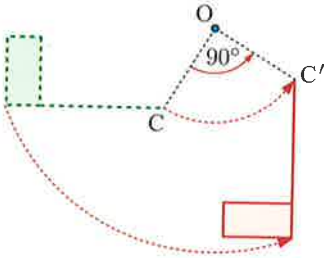
For example:



The figure is rotated clockwise about O through 90°.



The figure is rotated anticlockwise about O through 180°.



The figure is rotated anticlockwise about O through 90°.

Under a rotation, the distance of any point from the centre of rotation does not change.
So, OA = OA′, OB = OB′, and OC = OC′.

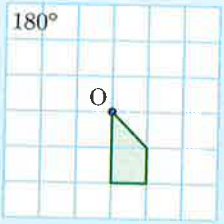
In mathematics we rotate in an anticlockwise direction unless we are told otherwise.

You should remember that 90° is a $\frac{1}{4}$-turn, 180° is a $\frac{1}{2}$-turn, 270° is a $\frac{3}{4}$-turn, and 360° is a full turn.

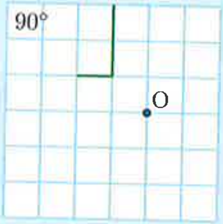
Example 3 **Self Tutor**

Rotate the given figures about O through the angle indicated:

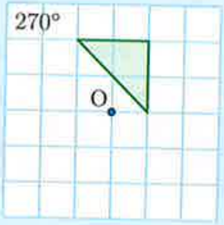
a



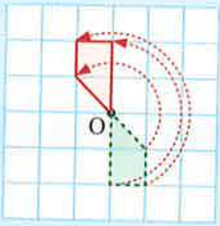
b



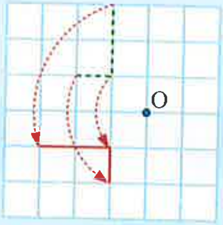
c



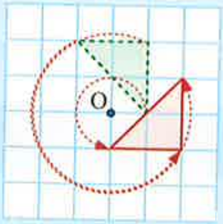
a

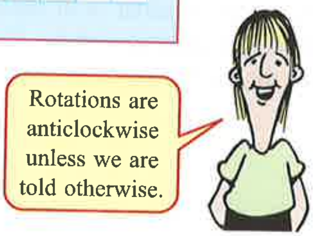


b



c





EXERCISE 19C.1

1 Consider the rotations of 

which follow:

A

B

C

D

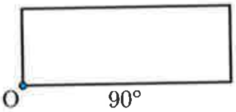
Which of **A**, **B**, **C**, or **D** is a rotation of the object through:

a 180° **b** 360° **c** 90° **d** 270°?

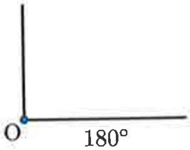
2 Copy and rotate each of the following shapes about the centre of rotation O, through the number of degrees shown. You could use tracing paper to help you.

PRINTABLE WORKSHEET

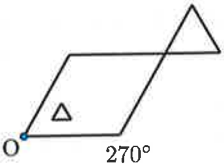
a



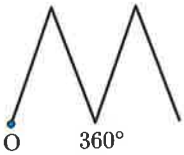
b



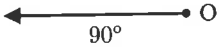
c



d

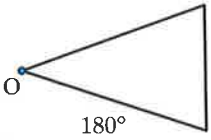


e

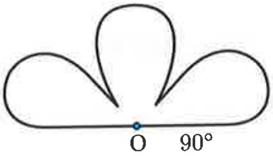


f

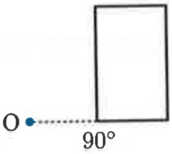
g



h



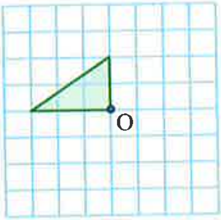
i



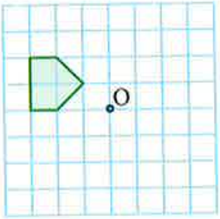
3 Rotate about O through the angle given:

DEMO

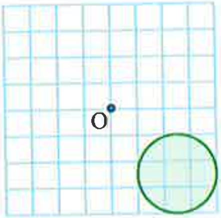
a 90°



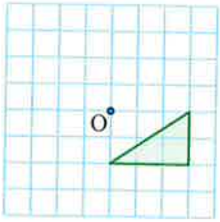
b 180°



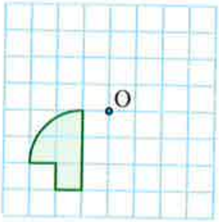
c 270°



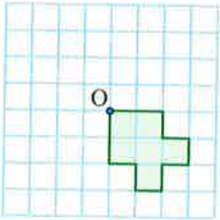
d 180°



e 90°



f 270°

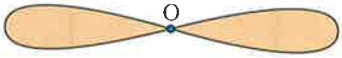


ROTATIONAL SYMMETRY

A shape has **rotational symmetry** if it can be rotated about a particular point through an angle **less than 360°** so that it maps onto itself.

The point through which the object rotates is called the **centre of rotational symmetry**.

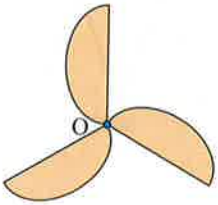
For example, this propeller shape has rotational symmetry. If it is rotated about O through 180° then it will look identical to how it did at the start. O is the centre of rotational symmetry.



Note that *every* shape will map onto itself under a 360° rotation, but this is not rotational symmetry.

If a figure has more than one line of symmetry then it will also have rotational symmetry. The centre of rotational symmetry will be the point where the lines of symmetry meet.

However, a figure which has rotational symmetry does not necessarily have line symmetry. For example, consider the figure alongside.

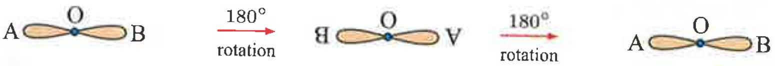


THE ORDER OF ROTATIONAL SYMMETRY

The **order of rotational symmetry** is the number of times a figure maps onto itself during one complete turn about the centre.

DEMO

For example,



has order 2

Click on the icon to see the order of rotational symmetry demonstrated for an equilateral triangle.

DEMO

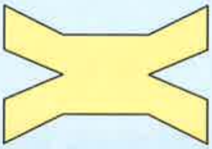
Example 4

Self Tutor

For each of the following figures:

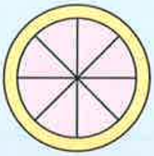
i mark the centre of rotational symmetry O
ii state the order of rotational symmetry.

a

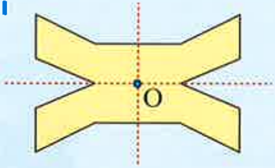


b

c

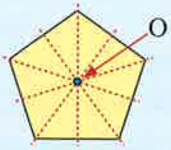


a **i**



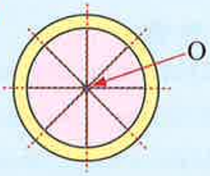
ii order = 2

b **i**



ii order = 5

c **i**



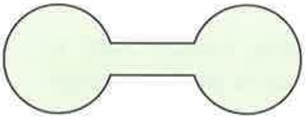
ii order = 8

EXERCISE 19C.2

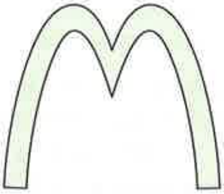
1 Which of the following shapes have rotational symmetry?

a

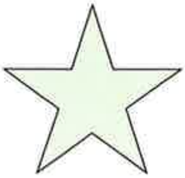
b



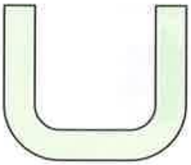
c



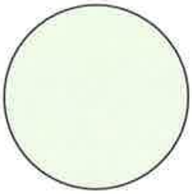
d



e



f

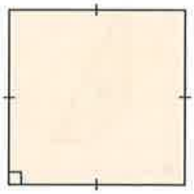


2 For each of the following figures:

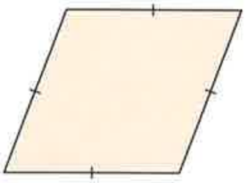
i mark the centre of rotational symmetry O
ii state the order of rotational symmetry.

PRINTABLE DIAGRAMS

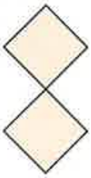
a



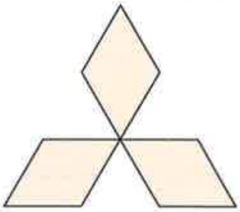
b



c

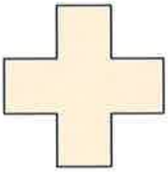


d

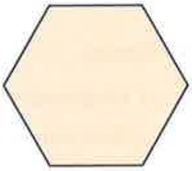


e

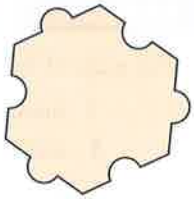
f



g



h



3 Draw a figure which has order of rotational symmetry: **a** 5 **b** 8.

ACTIVITY — USING TECHNOLOGY TO PERFORM ROTATIONS

In this Activity we use a computer package to construct a shape that has rotational symmetry.

What to do:

1. Click on the icon to load the software.
2. From the menu, choose the size of the sector angle.
3. Make a simple design with different shapes and colours in the sector.
4. Press Rotate to see your creation.

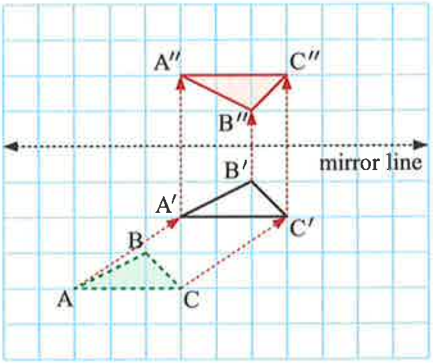
ROTATING FIGURES

D COMBINATIONS OF TRANSFORMATIONS

In this Section we perform several transformations on a figure, one after another.

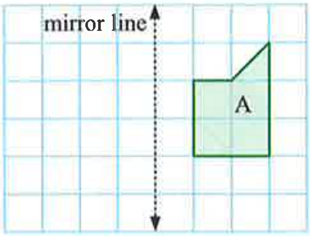
For example, in the figure opposite, triangle ABC is translated 3 units right and 2 units up to produce triangle $A'B'C'$. $A'B'C'$ is then reflected in the mirror line to produce triangle $A''B''C''$.

PRINTABLE DIAGRAMS

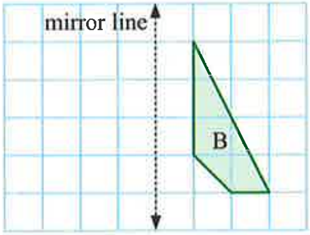


EXERCISE 19D

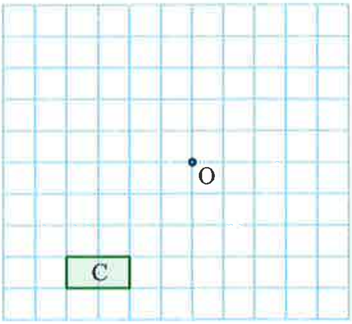
1 **a** Translate figure A 5 units down, then reflect the result in the mirror line.



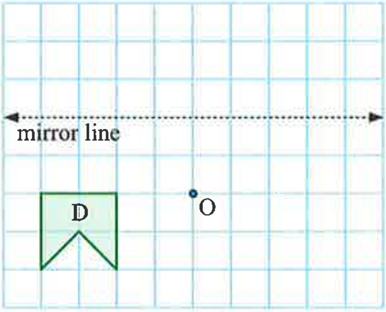
b Reflect figure B in the mirror line, then translate the result 4 units left.



c Translate figure C 4 units up, then rotate the result 90° clockwise about O.



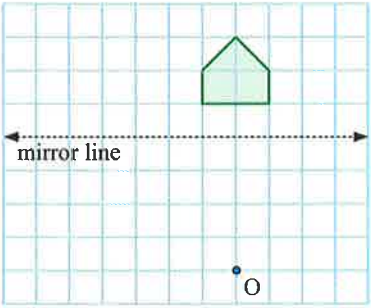
d Rotate figure D 180° anticlockwise about O, then reflect the result in the mirror line.



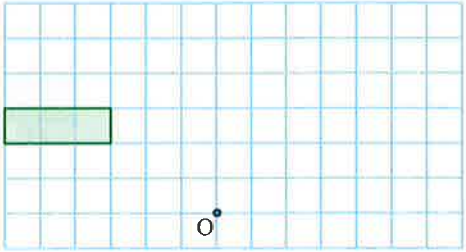
2 Find the image when the figure alongside is:

a translated 3 units left and 1 unit up, then reflected in the mirror line

b reflected in the mirror line, then rotated 90° anticlockwise about O.



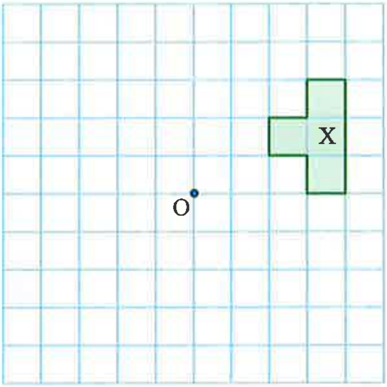
3



a Translate the rectangle alongside 4 units right, then rotate it 90° clockwise about O.

b Repeat **a**, but perform the transformations in the opposite order. Is the result the same?

4

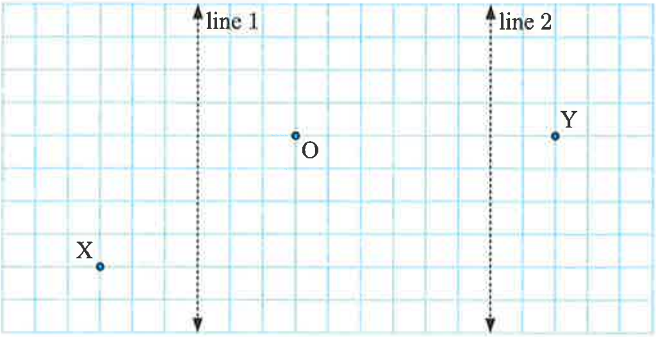


a Rotate figure X 90° anticlockwise about O to give X′, then rotate X′ 180° clockwise about O to give X″.

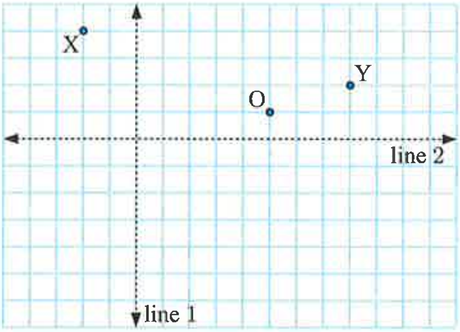
b What single transformation could be performed on X to give X″?

5 Describe how you could use reflections and rotations about O to transform X to Y.

a



b



GAME BATTLEGRID

Click on the icon to play a game where you must transform a point around a number plane.

BATTLEGRID

KEY WORDS USED IN THIS CHAPTER

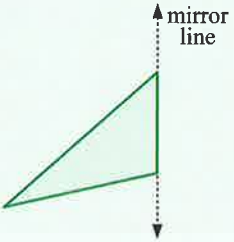
- centre of rotation
- mirror line
- rotation
- image
- object
- rotational symmetry
- line symmetry
- reflection
- translation

REVIEW SET 19A

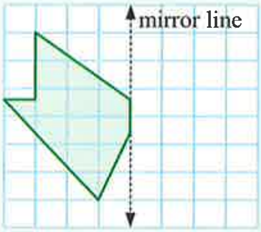
1 Draw the mirror image of:

PRINTABLE DIAGRAMS

a



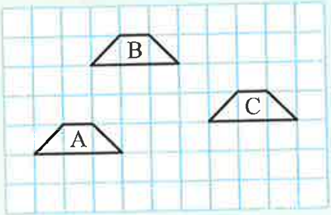
b



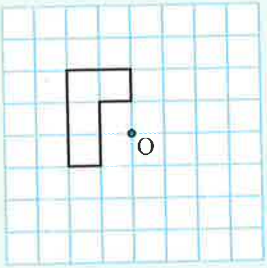
2 Describe the translation from:

a A to B **b** B to A

c B to C **d** C to A.



3 Rotate the given figure about O anticlockwise through 90°.



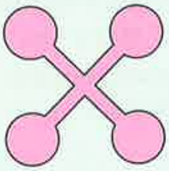
4 Does the boomerang alongside have:

a line symmetry

b rotational symmetry?

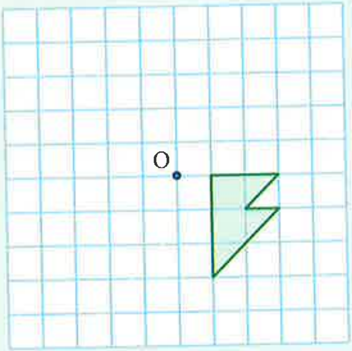
5 Find the order of rotational symmetry for the following shapes:

a



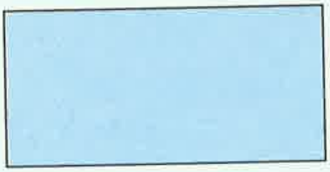
b

6 Find the image when the figure alongside is translated 5 units up, then rotated 180° about O.

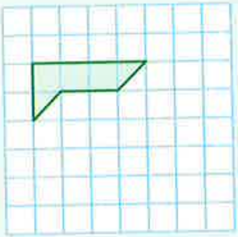


REVIEW SET 19B

1 Draw the lines of symmetry for this rectangle.



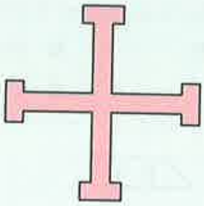
2 Translate the given figure 1 unit to the right and 3 units down.



PRINTABLE DIAGRAMS

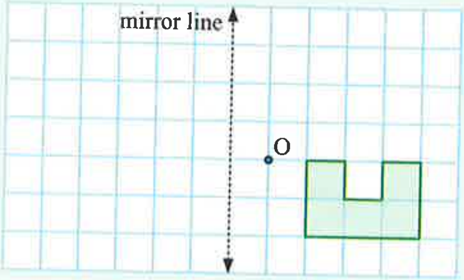
3 For the given figure:

- **a** locate the centre of rotational symmetry
- **b** find the order of rotational symmetry.

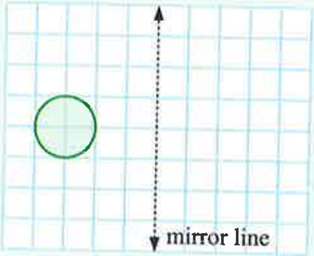


4 Find the image when the figure alongside is:

- **a** reflected in the mirror line
- **b** rotated 90° about O.

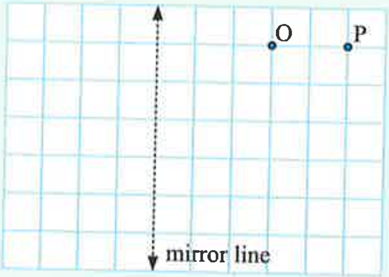


5



- **a** Find the image when the circle is translated 1 unit right, then reflected in the mirror line.
- **b** Find the image if the transformations in **a** are performed in the opposite order.

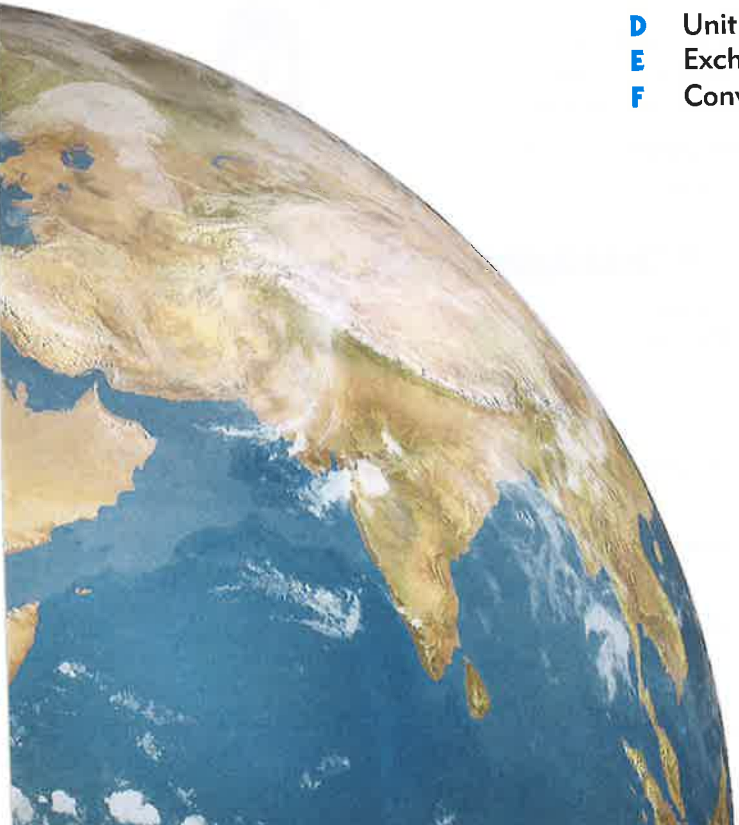
6 Show the result when point P is rotated 90° clockwise about O, then translated 3 units down, then reflected in the mirror line.



Chapter 20

Rates

Contents:

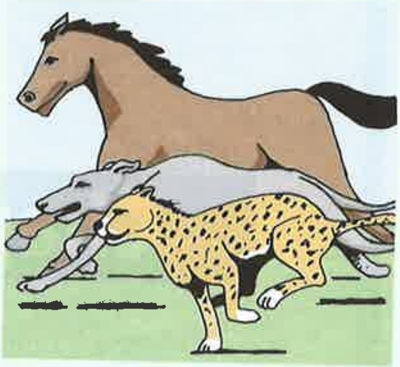


OPENING PROBLEM

A greyhound runs 515 m in 29.5 seconds.

A horse gallops 1650 m in 1 minute 45 seconds.

A cheetah sprints 380 m in 12.9 seconds.



Things to think about:

- **a** How can we compare the speeds of the three animals?
- **b** Which animal is fastest?

A RATES

We have seen that a **ratio** is an ordered comparison of quantities of the **same** kind. For example, we can have a ratio of lengths or a ratio of times.

A **rate** is an ordered comparison of quantities of **different** kinds.

For example, a person's *heart rate* is a comparison between the *number of heart beats* and the *time*.

When we write a rate, we do not use a ratio sign " : ", but instead we divide one quantity by another.

Since we are comparing quantities of *different* kinds, units are very important. We must always include units in our answer. We use the word *per* which means "for every", or a slash /, to separate the units.

For example, if a person's heart beats 65 times every minute, we write their heart rate as 65 beats per minute, or 65 beats/minute.

ACTIVITY 1 MEASURING YOUR HEART RATE

One way to determine your fitness level is to measure your heart rate. It is usually measured in beats per minute.

What to do:

1. Find your pulse on your wrist, or the side of your neck.
2. Count how many times you can feel your pulse in one minute.
3. Compare your heart rate with those of your classmates.
4. What happens to your heart rate when you exercise?

Other common examples of rates are:

	Examples of units
Rates of pay	dollars per hour
Petrol consumption	litres per 100 km or km per litre
Annual rainfall	mm per year
Unit cost	dollars per kg
Population density	people per square kilometre

Example 1 Self Tutor

A tap fills a 9 litre bucket in 3 minutes. Express this as a rate in simplest form.

$$\text{rate} = \frac{9\text{ L}}{3\text{ minutes}}$$

$$\therefore\ \text{rate} = \frac{9}{3}\text{ L per minute}$$

$$\therefore\ \text{rate} = 3\text{ L per minute}$$

Where necessary, round the rates to 2 decimal places.

EXERCISE 20A

1 Write down the meaning of each rate:

- **a** 5 km per h
- **b** 15 dollars per h
- **c** 7 L per s
- **d** 99 cents per L
- **e** 30 kg per h
- **f** 14 g per min
- **g** 96 dollars per day
- **h** 66 m per s
- **i** 21 mL per h

2 Suggest units which could be used to measure the following rates:

- **a** a person's rate of pay
- **b** an aeroplane's speed
- **c** the price of petrol
- **d** the typing speed of a secretary
- **e** the rate at which a car's temperature increases on a hot day.

3 Copy and complete:

- **a** A car uses 10 L of petrol every 160 km. The rate of petrol consumption is ...... km per litre.
- **b** A train travels 416 km over 8 hours. This is a rate of ...... km per hour.
- **c** 28 L of water drains from a tank in 8 seconds. This is a rate of ...... L per s.
- **d** A carton of milk costs €2.18 for 2 L. This is a rate of €...... per L.
- **e** A driver works for 3 hours and receives £51. His rate of pay is £...... per hour.

4 Jennifer's heart beats 375 times in 5 minutes. Express this as a rate in beats per minute.

5 The Peterson household used 1170 megajoules of gas during April. Express this rate of energy use in megajoules per day.

6 Annie travels 25 km by train to school. Her journey takes 45 minutes.
Victoria travels 20 km by car, a journey which takes 40 minutes.

- **a** Find the rate of travel for each girl in km per min.
- **b** Which mode of transport is more efficient?

7 Xinsong works 8 hours a week as a waiter, earning $168. Jay works 6 hours each week in the kitchen, earning $132.

a Find the rate of pay for each person in dollars per hour.

b Who is paid at a higher rate?

Example 2 **Self Tutor**

Henry eats 240 peanuts every 3 minutes.

a Find Henry's rate of eating peanuts.

b How many peanuts will Henry eat in 10 minutes?

a Henry's rate of eating peanuts

$$= \frac{240 \text{ peanuts}}{3 \text{ minutes}}$$

$= 80$ peanuts per minute

b In 10 minutes Henry will eat $80 \times 10 = 800$ peanuts.

8 A family of four uses 2800 litres of water each week.

a Find the rate of water usage in litres per day.

b How much water will the family use in 20 days?

9 Judy works part-time at a local café. She earned $50.40 for working 4 hours last week.

a Find Judy's rate of pay.

b This week Judy worked 19 hours. How much will she earn this week if she is paid the same hourly rate?

10 A milk truck takes 5 minutes to discharge 6750 litres of milk. At this rate, how much milk would the truck discharge in 18 minutes?

11 It costs $640 to buy a 32 m length of fibre optic cable.

a Find the cost of each metre of cable.

b Find the cost of a cable of length 27 m.

c Find the length of cable that could be bought for $4400.

12 To travel 518 km, a car uses 28 litres of petrol.

a Find the rate at which the petrol is used in:

i km per litre **ii** litres per 100 km.

b At this rate, how many litres of fuel would be needed to travel 1480 km?

c Fuel costs $1.35 per litre. How much would the journey in **b** cost?

B SPEED

The most common rate that we use is **speed**, which is a comparison between the *distance travelled* and the *time taken*.

The **instantaneous speed** of an object refers to how fast the object is travelling at a given point in time.

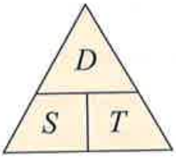
For example, when you are in a car, the speedometer might say that you are travelling at 50 km per hour.

However, when we go on a journey, we do not always travel at a constant speed. We need to slow down for other cars, and stop at traffic lights. For the whole journey, therefore, we calculate an **average speed** by comparing the total distance travelled with the total time taken.

$$\textbf{average speed} = \frac{\textbf{total distance travelled}}{\textbf{total time taken}}$$

This formula can be rearranged as: distance = speed × time

$$\text{or} \quad \text{time} = \frac{\text{distance}}{\text{speed}}$$



DEMO

You can use the triangle alongside to help you remember these.

Example 3

Self Tutor

Erica cycled 80 km in 2 hours.

- **a** Find her average speed.
- **b** Cycling at the same rate, how long would it take Erica to cycle 180 km?

a

$$\begin{aligned}&\text{average speed}\\ &= \frac{\text{distance travelled}}{\text{time taken}}\\ &= \frac{80 \text{ km}}{2 \text{ hours}}\\ &= \frac{80}{2} \text{ km/h}\\ &= 40 \text{ km/h}\end{aligned}$$

b

$$\begin{aligned}&\text{time}\\ &= \frac{\text{distance}}{\text{speed}}\\ &= \frac{180 \text{ km}}{40 \text{ km/h}}\\ &= \frac{180}{40} \text{ hours}\\ &= 4\frac{1}{2} \text{ hours}\end{aligned}$$

EXERCISE 20B.1

1 Find, in kilometres per hour, the average speed of:

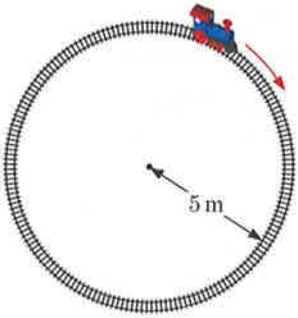
- **a** a cyclist who travels 100 km in 4 hours
- **b** a boat which travels 150 km in 5 hours
- **c** an athlete who runs 18 km in 1.5 hours
- **d** an aeroplane which takes 50 minutes to fly 750 km.

2 The speed limit on a freeway is 100 km/h. Jason drives 210 km along the freeway in 2 hours. Has he broken the law?

3 Bernadette drives her car at an average speed of 72 km/h.

- **a** If Bernadette drives for 3 hours, how far does she travel?
- **b** How long would it take Bernadette to travel 54 kilometres at this speed?

4 A model train travels around a circular track with radius 5 m as shown. The train takes 20 seconds to complete a lap of the track. Find the average speed of the train. Give your answer in metres per second, correct to 1 decimal place.



5 **a** How far does Liam travel if his aeroplane flies at 210 km/h for 1 hour and 40 minutes?

- **b** When Liam makes the return journey he is now flying against the wind, and his plane averages only 175 km/h. How long does the return flight take him?

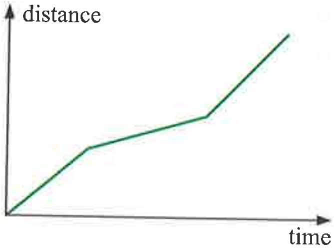
6 Yiren walks 60 metres in 22.5 seconds, while Sean walks 150 metres in 1 minute.

- **a** Find the average speed of each person.
- **b** Yiren and Sean each walk 2000 m at their normal speed. Who will finish first, and by how much?

TRAVEL GRAPHS

A **travel graph** for a journey shows the relationship between *distance travelled* and the *time taken*.

We will see how travel graphs can be used to calculate the speeds of travel at different stages of a journey.



INVESTIGATION — TRAVEL GRAPHS

Brian is riding his bicycle along a flat stretch of road. He travels 100 metres every 10 seconds.

What to do:

1 Copy and complete the table alongside, showing Brian's total distance travelled over 50 seconds.

Time (seconds)	*Distance* (metres)
0	0
10	100
20	200
30	
40	
50	

2 Plot these points on a graph, and join the points with a line. What feature of the graph indicates that Brian is travelling at a constant speed?

3 For Brian's journey so far, find:

- **a** the total distance travelled
- **b** the time taken
- **c** the average speed, in metres per second.

4 Brian then encounters a steep downhill section. He travels 200 metres in the next 10 seconds.

- **a** Extend your table to record the total distance travelled after 60 seconds.
- **b** Extend the graph to include this new point. What feature of the graph indicates that Brian's speed has changed?

5 For the period when Brian was travelling downhill, find:

- **a** the distance travelled
- **b** the time taken
- **c** the average speed, in metres per second.

6 For the *whole journey*, find:

- **a** the total distance travelled
- **b** the total time taken
- **c** the average speed, in metres per second.

From the **Investigation** you should have observed that when the speed is constant, the travel graph will be a straight line. The speed of travel is indicated by the **gradient** of the line.

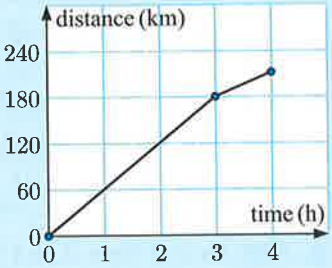
If the travel graph is not a straight line, we can still find the **average speed** between any two points on the graph.

Example 4

Self Tutor

The graph shows the progress of a train travelling between cities.

- **a** How far does the train travel in the first 3 hours?
- **b** Find the speed of the train for the first 3 hours.
- **c** Find the speed of the train during the final hour of the journey.
- **d** Find the average speed of the train for the entire journey.



a The train travels 180 km in the first 3 hours.

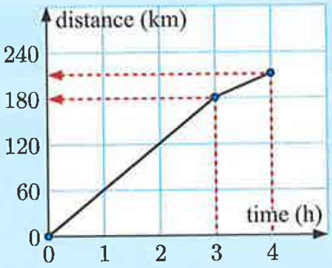
b Speed $= \dfrac{\text{distance travelled}}{\text{time taken}} = \dfrac{180 \text{ km}}{3 \text{ hours}} = 60$ km/h.

c The speed of the train was 30 km/h.

d Average speed $= \dfrac{\text{total distance travelled}}{\text{total time taken}}$

$$= \frac{210 \text{ km}}{4 \text{ hours}}$$

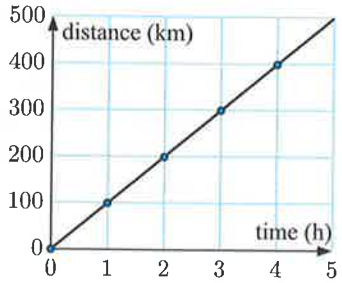
$$= 52.5 \text{ km/h}$$



EXERCISE 20B.2

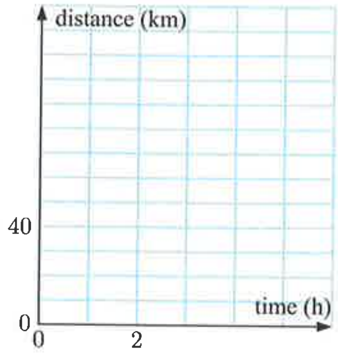
1 This travel graph shows the progress of a truck travelling between two cities.

- **a** Is the truck travelling at constant speed? Explain your answer.
- **b** How far does the truck travel in the first 2 hours?
- **c** Find the speed of the truck.



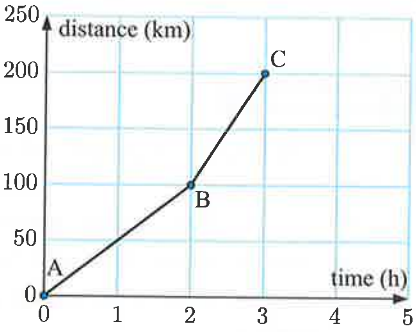
2 Every 2 hours, a cyclist travels 40 km.

- **a** Copy and complete the travel graph opposite.
- **b** How far will the cyclist travel in 3 hours?
- **c** How long will it take the cyclist to travel 100 km?
- **d** Find the speed of the cyclist.



3 This travel graph shows the progress of a car travelling from town A to B to C.

- **a** How far is it from A to B?
- **b** How long did the car take to get from A to B?
- **c** What was the speed of the car while travelling from A to B?
- **d** How far is it from B to C?
- **e** How long did the car take to get from B to C?
- **f** What was the speed of the car while travelling from B to C?
- **g** How far is it from A to C?
- **h** How long did the car take to get from A to C?
- **i** Find the *average speed* of the car from A to C.



C DENSITY

PUZZLE

Which is heavier, 1 tonne of lead, or 1 tonne of feathers?

The answer to this puzzle is that the objects are as heavy as each other, since both objects have mass 1 tonne. However, many people guess that the lead is heavier, since a *certain volume* of lead will be much heavier than the *same volume* of feathers. They have in fact compared the lead and feathers using a rate called **density**.

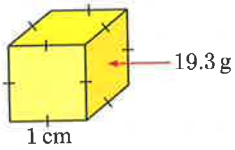
The **density** of an object is its mass per unit of volume.

Density can be found using the formula:

$$\textbf{density} = \frac{\textbf{mass}}{\textbf{volume}}$$

Density is usually measured in grams per cubic centimetre.

For example, the density of pure gold is 19.3 grams per cm^3. This means that every cubic centimetre of pure gold weighs 19.3 grams.



THE DENSITY OF WATER

In **Chapter 13**, we saw that 1 mL or 1 cm^3 of pure water at 4°C weighs 1 gram.

The density of pure water is 1 gram per cm^3.

If an object has density less than 1 gram per cm^3, then it will float on water.
If its density is greater than 1 gram per cm^3, then it will sink.

This table lists some common densities in g per cm^3.

Material	*Density*
carbon dioxide	0.002
petrol	0.70
ice	0.92
water	1.00
milk	1.03

Material	*Density*
aluminium	2.7
iron	7.8
lead	11.3
gold	19.3
platinum	21.4

Example 5 **Self Tutor**

Find the density of a piece of timber which is 60 cm by 10 cm by 3 cm and weighs 1.62 kg.

$$\begin{aligned}\text{The timber has mass} &= 1.62 \text{ kg}\\ &= 1620 \text{ g}\\ \text{and volume} &= 60 \times 10 \times 3 \text{ cm}^3\\ &= 1800 \text{ cm}^3\\ \therefore\ \text{density} &= \frac{\text{mass}}{\text{volume}}\\ &= \frac{1620 \text{ g}}{1800 \text{ cm}^3}\\ &= 0.9 \text{ g per cm}^3\end{aligned}$$

Example 6 **Self Tutor**

How many times more dense is gold than iron?

The ratio of densities $\dfrac{\text{density of gold}}{\text{density of iron}} = \dfrac{19.3}{7.8} \approx 2.47$

$\therefore$ gold is about 2.47 times more dense than iron.

EXERCISE 20C

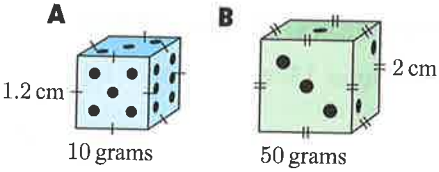
1 Find the density, in g per cm^3, of:

- **a** an object with mass 20 g and volume 5 cm^3
- **b** a metal disc which weighs 1.13 kg and has volume 50 cm^3
- **c** a block of ebony which is 1.1 m × 3 cm × 4 cm and weighs 1.4 kg.

2 How many times more dense is:

a lead than water **b** platinum than aluminium **c** milk than petrol?

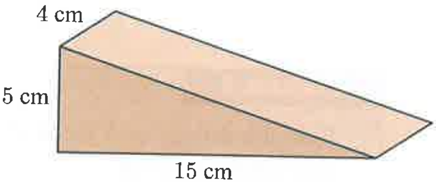
3 A pair of dice and their weights are shown alongside. Which die is made from the denser material?



4

Petrol and water do not mix. If the two liquids are poured into a container, they will separate into two layers. Which is the upper layer? Explain your answer.

5 The doorstop shown weighs 200 grams. If it was dropped into water, would it sink or float? Explain your answer.



ACTIVITY 2 WILL IT SINK OR FLOAT?

You will need: ruler, set of scales, container of water

What to do:

1 Gather several solid objects from around your classroom or your home. The objects should have a shape that you can calculate the volume of, such as a rectangular prism or a cylinder.

Before performing any calculations, predict whether each object will sink or float in water.

2 Measure the dimensions of each object. Use your measurements to find the volume of each object.

3 Use the scales to find the mass of each object.

4 Calculate the density of each object. If you wish, you may now change your predictions about whether each object will sink or float.

5 Place each object in the container of water. Were your predictions correct?

D UNIT COST

When shopping, it is important to get good value for money. However, it is often not obvious which item represents the best value for money, because the same item can come in several different sized packages.

To properly compare prices, we need to convert the cost of an item into a rate. This rate is called the **unit cost**. It could be the cost per gram, the cost per 100 grams, the cost per kilogram, or the cost per litre, for example. We then compare the unit cost for packages of different sizes.

ACTIVITY 3 UNIT PRICING

Most supermarkets include unit pricing on the price tags of their items.

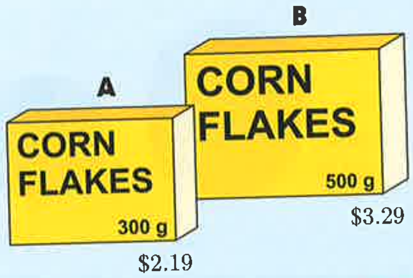
Next time you are at a supermarket, find out how the unit prices for the following items are measured:

- milk
- batteries
- flour
- dishwashing liquid
- steak
- paper towels

Example 7

Self Tutor

By comparing the cost per 100 g, decide which box of cereal is better value for money.



A 300 g = 3 lots of 100 g

$\therefore$ cost per 100 g $= \frac{\$2.19}{3}$

$= \$0.73$ per 100 g

B 500 g = 5 lots of 100 g

$\therefore$ cost per 100 g $= \frac{\$3.29}{5}$

$= \$0.658$ per 100 g

So, **B** is better value for money.

EXERCISE 20D

1 Use your calculator to find the unit cost for each of the following items. Express your answer in the units in brackets.

a	packet of 3 tennis balls for $11.40	($ per ball)
b	5 kg potatoes for $8.45	($ per kg)
c	250 g packet of chips for $2.40	(cents per g)
d	1.25 L soft drink for €0.99	(cents per L)
e	4.2 m of ribbon for $8.40	($ per m)
f	35 L of petrol for £43.40	(pence per L)

2 Consider the following grocery items and decide which is the better value for money:

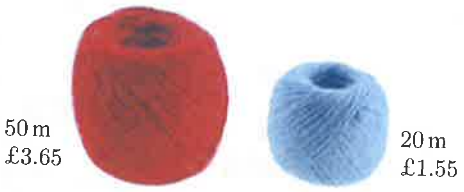
a compare cost per 100 g

b compare cost per 100 mL

c

d

e compare cost per 10 m



f compare cost per 10 g

3 A supermarket sells 110 g tubes of toothpaste for $3.19, and 160 g tubes of toothpaste for $3.99.

a Calculate the price per 10 g for each size of toothpaste.

b Which size is better value for money?

c The supermarket offers a '3 for 2' deal where if you buy two 110 g tubes of toothpaste, you receive a third one free. Does this represent better value for money than buying the 160 g tubes?

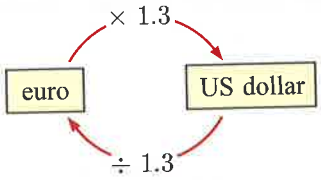
E EXCHANGE RATES

If you have travelled to other countries, you may have noticed that in different places people use different types of money. We call these **currencies**. For example, the United States of America uses the US dollar, most European countries use the euro, and Japan uses the yen.

If you visit a place which uses a different currency, you will need to sell some of your money and buy some of theirs in **exchange**. An **exchange rate** is used to work out how much your money is worth in the other currency.

For example, suppose that the exchange rate between the euro and the US dollar is 1 euro = 1.3 US dollars. This means that 1 euro can be exchanged for 1.3 US dollars.

- To convert euros into US dollars, we **multiply** by 1.3.
- To convert US dollars into euros, we **divide** by 1.3.



Exchange rates change constantly. The current exchange rate is the exchange rate at this point in time.

Example 8 **Self Tutor**

Suppose the exchange rate between the euro and the US dollar is 1 euro = 1.3 US dollars.

a Convert 200 euros into US dollars.
b Convert 650 US dollars into euros.

a 200 euros = (200 × 1.3) US dollars
= 260 US dollars

b 650 US dollars = (650 ÷ 1.3) euros
= 500 euros

EXERCISE 20E

1 Suppose the exchange rate between the US dollar and the Australian dollar is 1 US dollar = 1.1 Australian dollars. Convert:

a 100 US dollars into Australian dollars
b 250 US dollars into Australian dollars
c 55 Australian dollars into US dollars
d 1100 Australian dollars into US dollars.

2 Suppose the exchange rate between the euro and the Swiss franc is 1 euro = 1.2 Swiss francs. Convert:

a 40 euros into Swiss francs
b 270 euros into Swiss francs
c 90 Swiss francs into euros
d 1500 Swiss francs into euros.

3 Suresh has travelled from India to Japan. He wants to convert 2000 Indian rupees into Japanese yen. The current exchange rate is 1 rupee = 1.7 yen. How many yen will Suresh receive?

4 Estelle lives in Paris, and has 700 euros to spend on accommodation for a 5 night trip to London. She looks online, and sees a hotel which costs 120 British pounds per night. The current exchange rate is 1 euro = 0.8 British pounds. Will Estelle be able to afford the hotel?

5 Steve is travelling from Canada to New Zealand for a holiday. The current exchange rate is
1 Canadian dollar = 1.15 New Zealand dollars.

  a When Steve arrives in New Zealand, he converts 2000 Canadian dollars into New Zealand dollars. How many New Zealand dollars does he receive?

  b During his holiday, Steve spends 1380 New Zealand dollars. How many New Zealand dollars does he have left at the end of his holiday?

  c When he returns to Canada, Steve exchanges his New Zealand dollars back into Canadian dollars. How many Canadian dollars does he receive?

F CONVERTING RATES

It is often useful to convert a rate into different units so it is easier to understand for the situation we are dealing with.

DISCUSSION

2 metres per second is the same rate as 7.2 kilometres per hour. Which rate makes it easier to understand the situation if you are:

- walking 300 m to the bus stop
- hiking for 5 hours?

Example 9 **Self Tutor**

A petrol bowser pumps petrol at the rate of 600 L per hour. Write this rate in L per minute.

In 1 hour, the bowser pumps 600 L.

There are 60 minutes in 1 hour, so in 1 minute the bowser pumps $\frac{600}{60} = 10$ L.

This is a rate of 10 L per minute.

EXERCISE 20F.1

1 A fire hose discharges water at the rate of 180 litres per minute. Write this rate in L per hour.

2 Kelly's heart rate is 60 beats per minute. Write her heart rate in:

  a beats per second **b** beats per hour **c** beats per day.

3 A shower head has a flow rate of 7.5 L per minute. Write this rate in:

  a mL per second **b** L per hour.

4 A bamboo plant grows 18 m in 60 days. Write this growth rate in:

  a m per day **b** m per hour **c** mm per hour.

5 Reg eats 175 g of potato chips per day. Write this rate in:

a grams per week **b** kilograms per week.

6 The density of a material is 6.8 g per cm^3. Write this density in kg per m^3.

SPEED CONVERSIONS

Roger rides his bicycle at 36 km/h. To write his speed in m/s, consider this conversion:

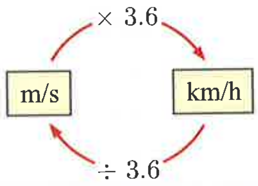
$$\begin{aligned} 36 \text{ km/h} &= \frac{36 \text{ km}}{1 \text{ hour}} \\ &= \frac{36\,000 \text{ metres}}{3600 \text{ seconds}} \qquad \{1 \text{ h} = 60 \text{ min} = 60 \times 60 \text{ s}\} \\ &= 10 \text{ m/s} \end{aligned}$$

So, travelling at 10 m/s is the same as travelling at 36 km/h. We say that these are **equivalent** rates.

Notice that travelling at 1 m/s is the same as travelling at 3.6 km/h.

Speed conversions:

- To convert m/s into km/h we **multiply by 3.6**.
- To convert km/h into m/s we **divide by 3.6**.



EXERCISE 20F.2

1 Using 10 m/s = 36 km/h, convert to km/h:

a 30 m/s **b** 70 m/s **c** 5 m/s

2 Mentally convert to m/s:

a 72 km/h **b** 144 km/h **c** 9 km/h

Example 10 **Self Tutor**

a A sprinter runs at 11 m/s. Convert this to km/h.

b An aeroplane travels at 900 km/h. Convert this to m/s.

a
11 m/s
= (11 × 3.6) km/h
= 39.6 km/h

b
900 km/h
= (900 ÷ 3.6) m/s
= 250 m/s

3 Convert to km/h:

a 200 m/s **b** 45 m/s **c** 27 m/s **d** 800 m/s

4 Convert to m/s:

a 50 km/h **b** 110 km/h **c** 21 km/h **d** 540 km/h

Example 11

Self Tutor

A 400 m sprinter finishes a race in 45 seconds.

Find his speed in km/h.

$$\begin{aligned}\text{speed} &= \frac{400\text{ m}}{45\text{ sec}}\\ &= 8.8888\text{.... m/s}\\ &= (8.8888\text{....} \times 3.6)\text{ km/h}\\ &= 32\text{ km/h}\end{aligned}$$

5 In 2009, Usain Bolt achieved a world record time of 19.19 seconds for the 200 metre sprint. Find his average speed, correct to 2 decimal places, in:

a m/s **b** km/h.

6 Find the following speeds in km/h:

a A sprinter runs 100 m in 9.7 seconds.

b A greyhound races 500 m in 29 seconds.

c A horse gallops 2000 m in 2 min 10 seconds.

d A swimmer travels 1500 m in 15 minutes.

Global context

click here

Population density

Statement of inquiry:	Performing calculations allows us to compare the characteristics of different countries.
Global context:	Globalisation and sustainability
Key concept:	Relationships
Related concepts:	Quantity, Measurement
Objectives:	Communicating, Applying mathematics in real-life contexts
Approaches to learning:	Thinking, Communication

KEY WORDS USED IN THIS CHAPTER

- average speed
- exchange rate
- travel graph
- conversion
- rate
- unit cost
- density
- speed

REVIEW SET 20A

1 A petrol pump delivers 42 litres of petrol into a car in 3 minutes. Write this rate in litres per minute.

2 Convert:

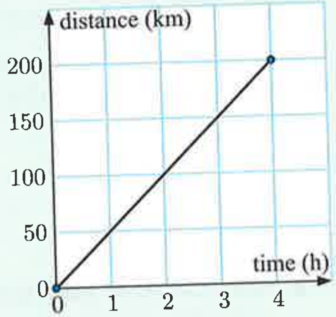
a 150 beats per minute into beats per second **b** 54 m/s into km/h.

3 A freight train travels 770 km in 8 hours, while a truck on the highway travels 120 km in 85 minutes. Which mode of transport travels at the faster rate?

4 Water from a tap will fill a 9 L watering can in 45 seconds. How long will it take to fill a 120 L pond?

5 The graph shows the progress of a car as it travels between cities.

- **a** How far does the car travel in 3 hours?
- **b** How long does it take for the car to travel 100 km?
- **c** Find the speed of the car.



6 A runner travels 32.5 km in 2 hours and 30 minutes. Find his speed in:

- **a** km/h
- **b** m/s.

7 Find the density of a 420 g paperweight with a volume of 150 cm^3.

8 Suppose the exchange rate between the Singapore dollar and the Chinese yuan is
1 Singapore dollar = 5 Chinese yuan.

- **a** Convert 60 Singapore dollars into Chinese yuan.
- **b** Convert 2000 Chinese yuan into Singapore dollars.

9 Find the density of a 5 cm by 30 cm by 60 cm piece of packing foam weighing 126 g.

10 Which of the chocolate bars is the better value for money?

REVIEW SET 20B

1 Margot receives €220 for an eight hour nursing shift. What is her hourly rate of pay?

2 Convert:

- **a** 80 km/h to m/s
- **b** 15 cm per year to mm per month.

3 At a local market it costs $5.10 to buy 0.6 kg of rhubarb.

- **a** Find the price per kilogram of the rhubarb.
- **b** How much would it cost to buy 2.5 kg of rhubarb?

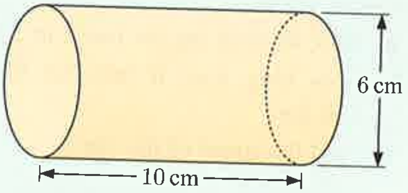
4 Trent rides his motorcycle for 3 hours. In this time he covers a distance of 198 km, and uses 11 litres of fuel. Find:

  a Trent's average speed in km/h

  b the petrol consumption of the motorcycle in km/L.

5 A pack of 12 fruit bars costs $4.92. Find the unit cost in cents per bar.

6 This cylindrical object weighs 200 g. If it was dropped into water, would it sink or float?



7 **a** Which of these packets of cereal is better value for money?

  b The 750 g packet is put on special, and now costs only $3.49. Which packet is better value for money now?

8 The travel graph shows Sylvie's progress when driving her car from home to the beach.

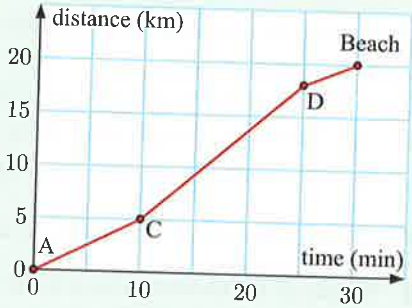
  a How far is the beach from Sylvie's home?

  b How long did it take Sylvie to get to the beach?

  c Find Sylvie's average speed for the whole journey.

  d What was Sylvie's speed between C and D?

  e At which points did the car change its speed?



9 Lucy has travelled from the United States to Mexico. She wants to convert 500 US dollars into Mexican pesos. The current exchange rate is 1 US dollar = 13.1 Mexican pesos. How many pesos will Lucy receive?

10 Alex drove 200 km in 4 hours.

  a Find his average speed.

  b Driving at this speed, how long would it take Alex to drive 325 km?

  c Write Alex's average speed in metres per second.

ANSWERS

EXERCISE 1A

1 **a** 17 **b** 64 **c** 328 **d** 810 **e** 2901

f 5 402 390

2 **a** thirty six dollars

b four hundred and five dollars

c six thousand, five hundred and one pounds

d eleven thousand and eighty five dollars

e fifty four thousand, seven hundred and sixty dollars

f two hundred and eighty five thousand, four hundred euros

3 **a** seven or 7 **b** seventy or 70

c seven or 7 **d** seventy or 70

e seven hundred or 700 **f** seven thousand or 7000

g seven or 7 **h** seventy thousand or 70 000

i seventy thousand or 70 000 **j** seven million or 7 000 000

k seven million or 7 000 000 **l** seventy or 70

4 **a** 12 **b** 10 **c** 101 **d** 2998 **e** 99

5 **a** one hundred and three dollars, $113, $130

b Xiao 109 cm, Kylie 116 cm, Wendy 118 cm, Sarah 126 cm

c giraffe 674 kg, hippopotamus 1872 kg, rhinoceros 2156 kg, elephant 3058 kg

d Milan 107 m, Salisbury 123 m, Rome 138 m, Cologne 157 m

e fourteen pounds, four thousand pounds, £4100, fourteen thousand pounds, forty thousand pounds

6 **a** 65 **b** 721 **c** 430 **d** 9084 **e** 50 690

f 7 002 063

7 **a** 700 + 30 + 4 **b** 3000 + 900 + 20 + 8

c 20 000 + 1000 + 80 **d** 600 000 + 30 000 + 400

8 **a** 245 **b** 865 320

EXERCISE 1B.1

1 **a** 40 **b** 57 **c** 54 **d** 359 **e** 115

f 69 **g** 124 **h** 542

2 **a** 107 **b** 149 **c** 104 **d** 1061 **e** 295

f 1703 **g** 3311 **h** 234

3 **a** 39 **b** 51 **c** 51 **d** 61 **e** 66

f 91 **g** 117 **h** 122

4 **a** 24 **b** 59 **c** 28 **d** 47 **e** 79

f 88 **g** 72 **h** 108

EXERCISE 1B.2

1 **a** 280 **b** 480 **c** 440 **d** 6300 **e** 7200

f 10 000 **g** 20 000 **h** 630 000

2 **a** 130 **b** 900 **c** 3100 **d** 600 **e** 840

f 600 **g** 1400 **h** 9000

3 **a** 64 **b** 144 **c** 54 **d** 110 **e** 72

f 192 **g** 84 **h** 240

4 **a** 144 **b** 341 **c** 224 **d** 954 **e** 693

f 824 **g** 182 **h** 7400 **i** 11 976 **j** 12 060

k 32 048 **l** 74 925

EXERCISE 1B.3

1 **a** 5 **b** 6 **c** 5 **d** 2 **e** 3 **f** 26

g 18 **h** 17

2 **a** $3\frac{1}{5}$ **b** $7\frac{1}{4}$ **c** $15\frac{1}{3}$ **d** $4\frac{4}{6} = 4\frac{2}{3}$

e 41 **f** 62 **g** 49 **h** 31

EXERCISE 1C

1 **a** 60 **b** 40 **c** 70 **d** 130 **e** 100

f 230 **g** 310 **h** 10 000

2 **a** 400 **b** 300 **c** 100 **d** 900 **e** 900

f 2000 **g** 18 700 **h** 25 900

3 **a** 6000 **b** 2000 **c** 7000 **d** 1000 **e** 14 000

f 10 000 **g** 26 000 **h** 254 000

4 **a** 50 **b** 200 **c** 400 **d** 500 **e** 900

f 1000 **g** 9000 **h** 50 000

5 **a** 680 **b** 210 **c** 590 **d** 170 **e** 2000

f 3900 **g** 9000 **h** 17 000

6 **a** $3200 **b** 300 g **c** 4600 m **d** 68 000

e $27 000 **f** 700 flights

7 **a** 178 380 people **b** 180 000 people

c 178 000 people **d** 178 400 people

EXERCISE 1D

1 **a** 8000 **b** 14 000 **c** 180 000 **d** 40 000

e 2 000 000 **f** 800 000 **g** 32 000 000 **h** 4 000 000

i 20 000 000

2 **a** 20 **b** 4 **c** 5 **d** 30 **e** 200 **f** 350

g 100 **h** 50 **i** 75

3 1200 tests **4** 1000 biscuits **5** 800 spaces

6 6000 avocados **7** 6 000 000 people **8** 15 eggs

9 **a** **i** 5 600 000

ii 5 600 000 is close enough to 5 699 386 ∴ it is reasonable.

b **i** 720 000

ii 720 000 is approximately 10 times larger than the computed answer of 69 950 ∴ it is not reasonable.

c **i** 500

ii 500 is close enough to 590 ∴ it is reasonable.

d **i** 22 500

ii 22 500 is approximately 10 times larger than the computed answer of 2177 ∴ it is not reasonable.

EXERCISE 1E.1

1 **a** 5 **b** 4 **c** 45 **d** 100 **e** 150 **f** 14

2 **a** 163 **b** 341 **c** 1122 **d** 818 **e** 1098 **f** 995

3 **a** 150 **b** 518 **4** **a** 22 **b** 31 **c** 83

5 21 **6** 319 m **7** 775 **8** 703

9 **a** €1050 **b** €1775 **10** 62 kg **11** €1249

EXERCISE 1E.2

1 **a** 45 **b** 24 **c** 6 **d** 48 **e** 0 **f** 42

g 0 **h** 64 **i** 44 **j** 44 **k** 54 **l** 54

m 70 **n** 70 **o** 70 **p** 36

2 **a** 28 **b** 280 **c** 2800 **d** 40 **e** 400

f 40 000 **g** 84 **h** 8400 **i** 840 000

3 **a** 385 **b** 153 **c** 1313 **d** 2079 **e** 384 **f** 485

4 **a** 300 **b** 700 **c** 1100 **d** 14 000

e 8000 **f** 370 000 **g** 1 000 000 **h** 1 000 000

5 **a** 555 **b** 840 **c** 1080 **d** 2717

e 5628 **f** 12 960 **g** 11 016 **h** 9071

6 **a** 437 **b** 432 **c** 120 **7** $60

8 91 **9** 1365 people **10** $924 **11** 144 socks

EXERCISE 1E.3

1 **a** 3 **b** 9 **c** 9 **d** 29 **e** 0 **f** 11 **g** 0
h 8 **i** 0 **j** 11 **k** undefined **l** 12

2 **a** 3 **b** 30 **c** 300 **d** 8 **e** 8 **f** 800
g 9 **h** 9 **i** 9000

3 **a** 25 **b** 35 **c** 15 **d** 45 **e** 14 **f** 25
g 21 **h** 103 **i** 63 **j** 22 **k** 54 **l** 49

4 **a** 13 **b** 23 **c** 28 **d** 18 **e** 19 **f** 38

5 17 buses **6** 22 kg **7** 32 bags **8** $9

EXERCISE 1F

1 **a** **D** **b** **C** **c** **A** **d** **E** **e** **B**

2 **a** 2×3^2 **b** $2^2 \times 3 \times 5$ **c** 2×5^3 **d** $3^2 \times 5^3$
e $2^3 \times 5 \times 7$ **f** $3^3 \times 7^2$ **g** $3^4 \times 5^2$ **h** $7^5 \times 11^3$

3 **a** 8 **b** 18 **c** 36 **d** 280 **e** 4851 **f** 4116

4 **a** 11 664 **b** 42 017 500 **c** 104 544
d 178 200 **e** 5 282 739 **f** 54 925 000

5 **a** 2^1 **b** 2^2 **c** 2^4 **d** 2^6

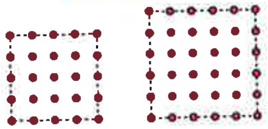
6 **a** 3^1 **b** 3^3 **c** 3^4 **d** 3^6

7 **a** 10^2 **b** 10^3 **c** 10^5 **d** 10^6

8 **a** 5^2 **b** 6^2 **c** 5^3 **d** 7^3

EXERCISE 1G

1 **a**



b $5^2 = 25$, $6^2 = 36$

2 **a** $7^2 = 49$, $8^2 = 64$, $9^2 = 81$, $10^2 = 100$
b $15^2 = 225$, $25^2 = 625$, $40^2 = 1600$

3 **a** 25, 49 **b** 64, 100

4 **a**
$$\begin{aligned} 1^2 &= 1 \\ 11^2 &= 121 \\ 111^2 &= 12\,321 \\ 1111^2 &= 1\,234\,321 \end{aligned}$$
b **i** 123 454 321
ii 12 345 654 321

5 **a**
$$\begin{aligned} 1 &= 1 = 1^2 \\ 1+3 &= 4 = 2^2 \\ 1+3+5 &= 9 = 3^2 \\ 1+3+5+7 &= 16 = 4^2 \\ 1+3+5+7+9 &= 25 = 5^2 \end{aligned}$$
b **i** $6^2 = 36$
ii $10^2 = 100$

6 $5^3 = 125$

7 $6^3 = 216$, $7^3 = 343$, $10^3 = 1000$, $13^3 = 2197$

8 $21^3 = 9261$ and $22^3 = 10\,648$
$\therefore$ 21 cubic numbers are less than 10 000.

9 8, 9

10 **a**
$$\begin{aligned} 1^3 &= 1 &&= 1 &&= 1^2 \\ 1^3+2^3 &= 1+8 &&= 9 &&= (1+2)^2 \\ 1^3+2^3+3^3 &= 1+8+27 &&= 36 &&= (1+2+3)^2 \\ 1^3+2^3+3^3+4^3 &= 1+8+27+64 &&= 100 &&= (1+2+3+4)^2 \end{aligned}$$
b **i** $(1+2+3+4+5)^2 = 225$
ii $(1+2+3+4+5+6+7+8+9+10)^2 = 3025$

EXERCISE 1H

1 **a** 5 **b** 9 **c** 7 **d** 1 **e** 9 **f** 20
g 13 **h** 5 **i** 39 **j** 7 **k** 15 **l** 16

2 **a** 3 **b** 6 **c** 88 **d** 14 **e** 27 **f** 7
g 21 **h** 22 **i** 44

3 **a** 11 **b** 4 **c** 17 **d** 0 **e** 9 **f** 69

4 **a** 60 **b** 39 **c** 23 **d** 4 **e** 8 **f** 13

5 **a** 50 **b** 68 **c** 9 **d** 17 **e** 1 **f** 225
g 9 **h** 19 **i** 17

6 **a** $5 + 9 \div 3 = 8$ **b** $7 \times 11 - 21 = 56$
c $18 - 16 \div 2 = 10$ **d** $17 - 3^2 = 8$
e $13 - 4 \times 2 = 5$ **f** $4 \times 13 - 6 \times 7 = 10$

7 **a** $3 \times (4+2) \times 5 = 90$ **b** $(3 \times 4 - 5) \times 4 = 28$
c $4 \times (16-1) - 6 = 54$ **d** $(6 + 7 \times 2) \div 5 = 4$
e $4 + 4 \div (2+2) = 5$ **f** $(3 + 11 - 5) \div 3 = 3$

REVIEW SET 1A

1 nine thousand, six hundred and two

2 seven thousand or 7000

3 **a** 0 **b** 11 **c** undefined **4** 2103

5 **a** 49 550 **b** 49 600 **c** 50 000

6 276 **7** 965 210 **8** 3 600 000 **9** 11 buses

10 **a** 7 **b** 26 **c** 90 **11** $240

12 $17^3 = 4913$ **13** four; 49, 64, 81, 100

14 **a** $2 + 12 \div (4-2) = 8$ **b** $30 \div (5+1) + 4 = 9$

15 **a** 8 even numbered bookcases **b** 54 shelves
c 92 books **d** 1978 books **e** $29 670

REVIEW SET 1B

1 forty thousand, seven hundred and one

2 30 000 + 500 + 2

3 **a** 153 **b** 86 **c** 6 **d** 38 **e** 540 000 **f** $5\frac{1}{7}$

4 **a** 140 **b** 1700 **c** 9000 **5** $396

6 **a** 570 **b** 37 000 **c** 4200 **d** 120 000

7 **a** 128 **b** 3125 **c** 3969

8 300 books **9** €3467 **10** $(44-8) \div (4+2) = 6$

11 **a** 416 **b** 12 648 **c** 27 **d** 47

12 $8^2 + 7^3 = 407$ **13** **a** 56 **b** 9

14 **a** largest 9852, smallest 2589 **b** 7263

15 **a** 1000 minutes **b** 1176 minutes **c** 176 minutes
d 30 minutes

EXERCISE 2A

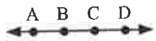
1 **a**

b

c

d

e



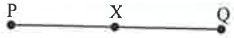
f

2 **a** (AB) or (BA) **b** (XY), (YX), (XZ), (ZX), (YZ), or (ZY)

3 **a** [PQ], [PR], [QR] **b** [QP], [PR]

4 **a** B **b** C **c** C **d** B

5 **a**



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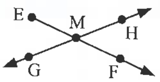

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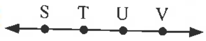

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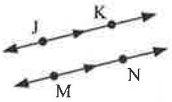
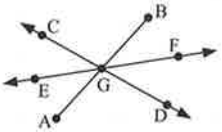


e

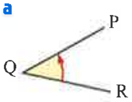


6 **a** Any three of (AC), (AE), (BA), (BC), (BE), (CA), (CB), (CE), (EA), (EB), (EC).
b 3 lines
c **i** intersect at F **ii** are collinear **iii** are parallel

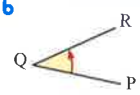
EXERCISE 2B

1 **a** **C** **b** **A** **c** **D** **d** **B**

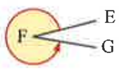
2 **a**



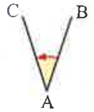
b



c



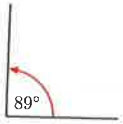
d



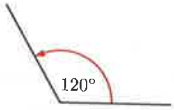
3 **a** **i** 25° **ii** 90° **iii** 45° **iv** 115°
b **i** acute **ii** right **iii** acute **iv** obtuse

4 **a**

b

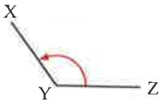


c

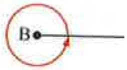


5 **a** **i** b **ii** g **iii** d
b **i** reflex **ii** obtuse **iii** acute

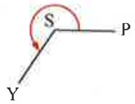
6 **a**



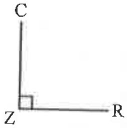
b



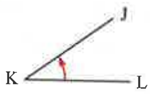
c



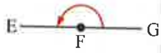
d



e



f



7 **a** **i** 68° **ii** 117° **iii** 112°
b **i** 70° **ii** 80° **iii** 65°

8 ≈ 28.5° **9** ≈ 110°

10 **a** **i** 8 right angles **ii** 8 acute angles
b **i** 8 right angles **ii** 14 acute angles
c **i** 14 right angles **ii** 68 acute angles

EXERCISE 2C

1 **a** supplementary **b** neither **c** complementary
d neither **e** neither **f** supplementary
2 **a** 75° **b** 3° **c** 47°
3 **a** 51° **b** 123° **c** 90°
4 **a** supplementary **b** neither **c** complementary
d neither
5 **a** $(90 - x)°$ **b** $(180 - y)°$

6 **a** $p = 125$ **b** $q = 38$ **c** $k = 94$ **d** $b = 85$
e $q = 26$ **f** $t = 45$ **g** $s = 21$ **h** $a = 90$
i $g = 30$
7 **a** $r = 266$ **b** $z = 120$ **c** $m = 236$
8 **a** $s = 50$ **b** $b = 115$ **c** $m = 31$ **d** $s = 75$
e $j = 161$

EXERCISE 2D

1 **a** a and c, b and d **b** p and q, r and s **2** **B and C**
3 **a** c **b** d **c** d **d** b **4** **A, C, and D**
5 **a** s **b** s **c** q **d** q **6** **B and D**
7 **a** z **b** z **c** x **d** y
8 **a** corresponding **b** alternate **c** co-interior
d corresponding **e** corresponding
f vertically opposite **g** vertically opposite
h co-interior **i** alternate

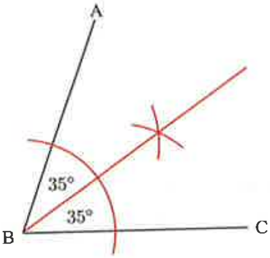
EXERCISE 2E

1 **a** $x = 124$ {equal corresponding angles}
b $b = 82$ {supplementary co-interior angles}
c $q = 42$ {equal alternate angles}
d $y = 57$ {equal corresponding angles}
e $k = 62$ {equal alternate angles}
f $a = 135$ {equal corresponding angles}
g $x = 147$ {equal alternate angles}
h $y = 73$ {supplementary co-interior angles}
i $d = 15$ {equal corresponding angles}
2 **a** $a = 76$ {vertically opposite angles}
$b = 104$ {supplementary co-interior angles}
b $a = 117$ {equal corresponding angles}
$b = 117$ {vertically opposite angles}
c $a = 38$ {vertically opposite angles}
$b = 38$ {equal alternate angles}
d $a = 145$ {angles at a point}
$b = 35$ {supplementary co-interior angles}
e $m = 96$ {supplementary co-interior angles}
$n = 84$ {supplementary co-interior angles}
f $a = 36$ {equal corresponding angles}
$b = 36$ {equal alternate angles}
3 **a** $x = y$ {equal alternate angles}
b $a + b = 180$ {supplementary co-interior angles}
c $p = q$ {equal corresponding angles}
d $a + b = c$ {equal alternate angles}
4 **a** parallel {equal alternate angles}
b not parallel {co-interior angles do not sum to 180°}
c not parallel {alternate angles are not equal}
d parallel {equal corresponding angles}
e parallel {angles on a straight line, equal corresponding angles}
f parallel {angles on a straight line, equal corresponding angles}
5 **a** The figure contains a pair of parallel lines.
{co-interior angles sum to 180°}
$\therefore$ $a = 120$ {supplementary co-interior angles}
b The figure contains a pair of parallel lines.
{angles on a straight line, equal corresponding angles}
$\therefore$ $a = 115$ {equal corresponding angles, vertically opposite angles}
6 $a = 70$

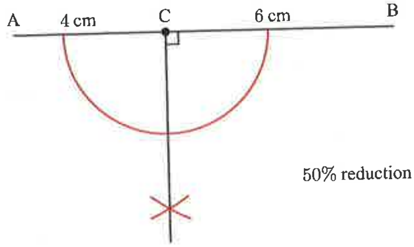
7 $a = 40$ {equal alternate angles}
$b = 40$ {vertically opposite angles with a}
$c = 40$ {equal alternate angles with a}
$d = 90$ {equal corresponding angles}
$e = 90$ {vertically opposite angles with d}
$f = 50$ {angles on a line sum to 180° with a and d}
$g = 50$ {equal corresponding angles with f}

EXERCISE 2F

1 **a**, **b**

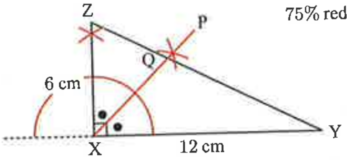


2 **a**, **b**



50% reduction

3 **a**, **b**, **c**, **e**

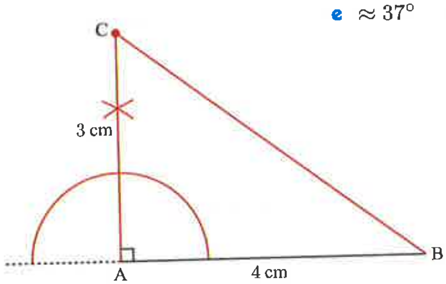


75% reduction

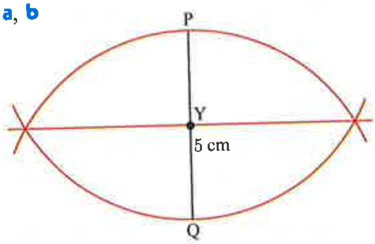
d ≈ 13.4 cm **f** **i** ≈ 8.9 cm **ii** ≈ 108°

4 **a**, **b**, **c**

d BC = 5 cm
e ≈ 37°



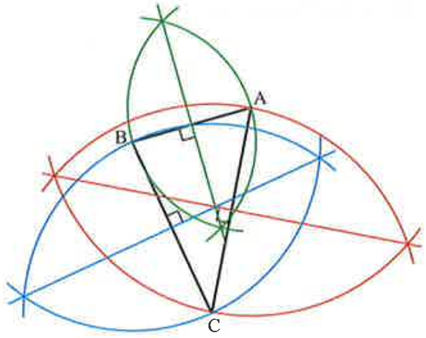
5 **a**, **b**



50% reduction

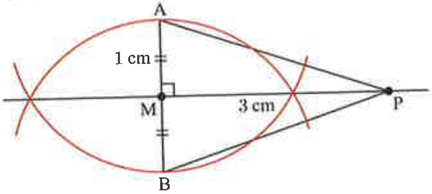
c PY = QY = 2.5 cm

6 **a**



c "The three perpendicular bisectors of the sides of a triangle are concurrent (meet at the same point)."

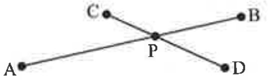
7 **a**, **b**, **c**



d AP = BP ≈ 3.2 cm
e Both angles are equal and are ≈ 18°.

REVIEW SET 2A

1 **a** 37° **b** 50°
2 **a** f **b** a **c** d **d** d
3 **a** $a = 16$ **b** $b = 27$ **c** $c = 120$ **4** two points
5



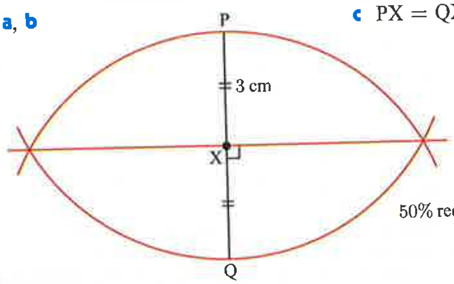
6 **a** $x = 62$ {vertically opposite angles, supplementary co-interior angles}
b $x = 61$ {equal alternate angles}
c $x = 88$ {angles on a line (twice)}

7 **a** $m = 116$ {equal alternate angles}
b $m = 81$ {equal corresponding angles}
c $m = 141$ {supplementary co-interior angles}

8 **a** $x = y$ {vertically opposite angles, equal corresponding angles}
b $a + b = 180$ {supplementary co-interior angles}

9 **a** parallel {vertically opposite angles, equal corresponding angles}
b parallel {angles at a point, supplementary co-interior angles}

10 **a**, **b** **c** PX = QX = 3 cm



50% reduction

REVIEW SET 2B

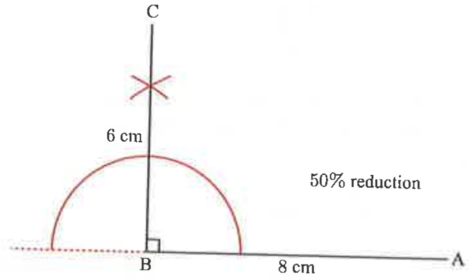
1 **a** Any two of (AC), (BA), (BC), (CA), (CB)
b **i** they are collinear **ii** they intersect at D

2 **a** 25° **b** 92°

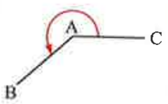
3 **a** $x = 110$ {equal corresponding angles}
b $c = 126$ {angles at a point}

4 **a** $a = 35$ {complementary angles}
b $b = 45$ {angles on a line}

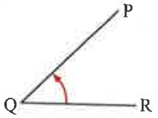
5



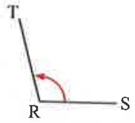
6 **a**



b



c



7 **a** **i** f **ii** d **iii** b
b **i** reflex **ii** obtuse **iii** acute

8 **a** 338° **b** 22° {angles at a point}

9 **a** supplementary **b** neither
c complementary **d** neither

10 parallel {vertically opposite angles, supplementary co-interior angles, equal alternate angles}

EXERCISE 3A

1

	Statement	*Number*	*Opposite of statement*	*Number*
a	winning by 5 goals	+5	losing by 5 goals	−5
b	25 m east of a building	+25	25 m west of a building	−25
c	a clock is 3 min slow	−3	a clock is 3 min fast	+3
d	a gain of 4 kg	+4	a loss of 4 kg	−4
e	a loss of \$1250	−1250	a gain of \$1250	+1250
f	20 km south of the city	−20	20 km north of the city	+20
g	200 m above sea level	+200	200 m below sea level	−200
h	11°C below zero	−11	11°C above zero	+11
i	a decrease of \$100	−100	an increase of \$100	+100
j	one floor above ground level	+1	one floor below ground level	−1

2 **a** +1 **b** −3 **c** −2 **d** −5

3 A: +2, B: −3, C: +6, D: −5, E: −1

4 **a** −4 **b** +21 **c** +13 **d** −17 **e** +32

5 **a** −3 **b** +15 **c** −250 **d** −2000 **e** +57

6 **a** a deposit of \$30 **b** a fall of 3°C
c a 1 km trip east **d** no change
e going down 1 floor **f** a loss of 2 kg

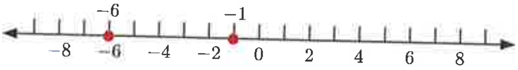
7 **a** Day 1: +56 g, Day 2: −16 g, Day 3: −28 g, Day 4: +73 g, Day 5: −19 g
b 3336 grams

8 **a** Tarfia: 15°C, Palermo: 10°C, Marseille: 5°C, Berlin: 0°C, Reykjavik: −5°C
b **i** 5°C **ii** 15°C **iii** 20°C
c **i** 5°C **ii** 10°C **iii** 15°C
d **i** 10°C **ii** 10°C

EXERCISE 3B

1 **a** 3 **b** −15 **c** 10 **d** 9 **e** −38 **f** 6
g −7 **h** 0

2



Since −1 is further to the right, −1 is greater than −6. We could also say −6 is less than −1.

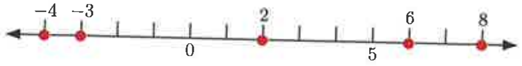
3 2 is greater than negative 5. This statement is true.

4 **a** true **b** true **c** false **d** true **e** false **f** true

5 **a** $8 > 6$ **b** $18 > 7$ **c** $-9 < -4$
d $-3 < 15$ **e** $20 > -15$ **f** $-6 < -2$

6 **a**

b



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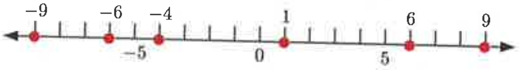
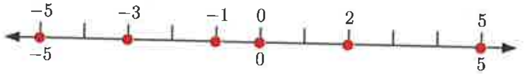
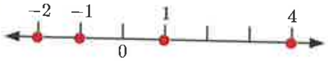


d

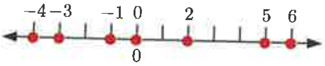


7



−2, −1, 1, 4

8



−4, −3, −1, 0, 2, 5, 6

9 **a** 8, 6, 0, −2, −5, −7 **b** −10, −7, −2, 0, 7, 8

10 Moscow −6°C, Oslo −4°C, Tokyo 1°C, Ulaanbaatar 3°C, Melbourne 19°C, Singapore 33°C

11 **a** −4 **b** −1 **c** −11 **d** −1 **e** 0 **f** −7

12 **a** 10 **b** 2 **c** −4 **d** −4 **e** −6 **f** −4
g −8 **h** −12

EXERCISE 3C

1 **a** 1 **b** 7 **c** −7 **d** −1 **e** 1 **f** 7
g −7 **h** −1

2 **a** −4 **b** 8 **c** −8 **d** 4 **e** −4 **f** 8
g −8 **h** 4

3 **a** −1 **b** −8 **c** −2 **d** 4 **e** −10 **f** 9
g 3 **h** −17
4 2nd floor above ground level
5 **a** −9 **b** −2 **c** −9 **d** −21 **e** −12 **f** 3
g −10 **h** −1 **i** 6
6 **a** 1 **b** 6 **c** 3 **d** 2 **e** −7 **f** 17
g −17 **h** −27 **i** 16
7 **a** 9 **b** 3 **c** 9 **d** 2 **e** 16 **f** 15
8 −1°C

EXERCISE 3D

1 **a** 24 **b** −24 **c** −24 **d** 24 **e** −24 **f** −24
g 24 **h** 24
2 **a** −6 **b** −30 **c** 14 **d** −50 **e** −48 **f** −45
g −88 **h** −33 **i** −81 **j** 24 **k** −55 **l** 42
3 **a** □ = 1 **b** □ = −2 **c** □ = −11 **d** □ = −4
e □ = −6 **f** □ = −2 **g** □ = 4 **h** □ = −1
i □ = 6 **j** □ = −3 **k** □ = −3 **l** □ = −10
4 **a** 280 m **b** $120
5 **a** −200 **b** 63 **c** 20 **d** −27 **e** 200 **f** 4
g −7 **h** 90 **i** −125 **j** −24 **k** 48 **l** 80
6 **a** 1 **b** 1 **c** −1 **d** −1 **e** 1
For an even power, the answer is positive.
For an odd power, the answer is negative.

EXERCISE 3E

1 **a** 5 **b** −5 **c** −5 **d** 5 **e** 5 **f** 5
g −5 **h** −5 **i** 1 **j** −1 **k** −1 **l** 1
m 11 **n** −11 **o** 11 **p** −11
2 **a** □ = −4 **b** □ = −6 **c** □ = −4 **d** □ = −25
e □ = −9 **f** □ = −12 **g** □ = −8 **h** □ = −5
i □ = −40 **j** □ = −9 **k** □ = 3 **l** □ = −28
m □ = −8 **n** □ = −120 **o** □ = 144 **p** □ = 12
3 **a** debt share is $50 000 **b** −9°C (drops 9°C per hour)

EXERCISE 3F

1 **a** 1 **b** −7 **c** 1 **d** 2 **e** 4 **f** −15
g 16 **h** 8 **i** 4 **j** −7 **k** −5 **l** −9
2 No, $-3^2 = -9$ and $(-3)^2 = 9$. **3** $480 000 profit
4 **a** $918 profit **b** $153 average profit **5** −13

EXERCISE 3G

1 **a** −21 **b** 53 **c** −51 **d** −54 **e** −950
f −4 **g** −24 **h** 140
2 2 m above **3** $84 **4** €5200
5 **a** −$2029 **b** $376 **c** −$705

REVIEW SET 3A

1 **a** −3 **b** 3 **c** −6 **d** 12
2 **a** −12 **b** negative ÷ positive = negative **c** $-5 < 3$
3 **a** borrowing 3 books **b** withdrawing $10
4 **a** 1 **b** 64 **c** −25
5 **a** −6, −4, −3, 0, 2, 3, 7 **b** 13
6 **a** −7 **b** $3 > -8$ **c** −12 **7** $52
8 **a** −9°C **b** +28 m **c** −36 points **9** **A**
10 **a** Ying **b** Cathy
c **i** 19 minutes **ii** 21 minutes **iii** 8 minutes

11 **a** Amy scored 48 points, Sean scored −8 points
b 56 points
12 **a** **i** 20 m **ii** 4 m **iii** −12 m **iv** −28 m
b 32 m **c** 32 m **d** 48 m
13 **a** **i** −2 **ii** +6 **iii** −5 **b** **i** −12 **ii** −6

REVIEW SET 3B

1 A: 3, B: −1, C: 7, D: 0, E: −4
2 **a** negative × negative = positive **b** 77
3 **a** −7 **b** −21 **4** **a** −1 **b** 10 **c** −24
5 **a** 5, 3, 1, 0, −1, −4, −6 **b** 11 **c** −2
6 **a** 4 kg loss **b** 8 kg **7** **a** 2 **b** −129 **c** 4
8 −1°C **9** 21 pianos
10 **a** 8 floors **b** floor 4 **c** floor 4
d **i** $(3+2)-(3-2)$ **ii** no
11 **a** −25 **b** −14
c **i** −12 **ii** 2 knots above the top of the plant
12 **a** +6 **b** −8 **c** +10 **d** 9 jumps

EXERCISE 4A.1

1 **a** divisible **b** not divisible **c** not divisible
d divisible **e** divisible **f** not divisible
2 **a** even **b** odd **c** odd **d** even **e** even
f odd **g** even **h** odd
3 96
4 **a** 24, 30, 36 **b** 33 **c** 22, 26, 30, 34, 38
d 23, 25, 29, 31, 35, 37 **e** 36 **f** 27
5 **Note:** Other answers are possible.
a $18 + 42$ **b** $11 + 49$ **c** 2×30 **d** 4×15
6 **a** even **b** odd **c** even **d** even

EXERCISE 4A.2

1 **a** true **b** false **c** false **d** true **e** true **f** true
g false **h** false **i** true **j** false **k** true **l** true
2 **a** divisible **b** not divisible **c** not divisible
d divisible **e** not divisible **f** divisible
3 **a** not divisible **b** divisible **c** divisible
d not divisible
4 **a** divisible **b** divisible **c** not divisible
d divisible
5 **a** not divisible **b** divisible **c** not divisible
d divisible
6 **a** divisible by 2 and 5 **b** divisible by 3 and 9
c divisible by 3 and 5 **d** not divisible by any of them
7 $a + b = 3$, 6, 9, 12, 15, or 18
8 **a** 2, 5, or 8 **b** 2 or 6 **c** 0 or 5 **d** 2 or 8
e 2 **f** 1
9 **a** **Note:** Other answers are possible.
i 1485 **ii** 1548
b $1 + 4 + 5 + 8 = 18$, which is divisible by 9.
Therefore, any number containing these digits, regardless of their order, is divisible by 9.
10 When the digits of a number are reversed, the difference between the sum of the even digits and the sum of the odd digits will stay the same.
So, if a given number is divisible by 11, then if we reverse its digits, the result will also be divisible by 11.

EXERCISE 4B.1

1 **a** yes **b** no **c** no **d** yes

2 **a** 1, 2, 4, 5, 10, 20 **b** $20 = 4 \times 5$
c $20 = 1 \times 20$, $20 = 2 \times 10$

3 **a** 1, 2, 4, 8 **b** 1, 2, 4, 8, 16
c 1, 2, 3, 5, 6, 10, 15, 30 **d** 1, 2, 3, 4, 6, 9, 12, 18, 36
e 1, 2, 4, 11, 22, 44 **f** 1, 2, 4, 7, 8, 14, 28, 56
g 1, 2, 5, 10, 25, 50
h 1, 2, 3, 4, 6, 7, 12, 14, 21, 28, 42, 84
i 1, 7, 11, 77 **j** 1, 7, 49 **k** 1, 5, 13, 65
l 1, 7, 13, 91

4 **a** $24 = 6 \times 4$ **b** $25 = 5 \times 5$ **c** $28 = 4 \times 7$
d $100 = 5 \times 20$ **e** $88 = 11 \times 8$ **f** $88 = 2 \times 44$
g $36 = 2 \times 18$ **h** $36 = 3 \times 12$ **i** $36 = 9 \times 4$
j $49 = 7 \times 7$ **k** $121 = 11 \times 11$ **l** $72 = 6 \times 12$
m $60 = 12 \times 5$ **n** $48 = 12 \times 4$ **o** $96 = 8 \times 12$

5 **a** 7 **b** 9 **c** 28 **d** 22 **e** 25 **f** 45

6 **a** 30 **b** 105 **c** 210 **7** 63

8 **a** 50 lockers **b** 33 lockers
c 16 lockers {6, 12, 18, 24, 30, 36, 42, 48, 54, 60, 66, 72, 78, 84, 90, 96}
d 49 lockers

9 **a** **i** 3 factors **ii** 3 factors **iii** 3 factors
iv 9 factors
b square numbers

10 **a** Every locker with an odd number of factors remains open.
b 1, 4, 9, 16, 25, 36, 49, 64, 81, 100

EXERCISE 4B.2

1 **a** 3 **b** 4 **c** 7 **d** 3 **e** 8 **f** 7
g 4 **h** 9 **i** 2 **j** 22 **k** 27 **l** 45

2 6 metres **3** 40 nails

EXERCISE 4C.1

1 **a** 4, 8, 12, 16, 20, 24 **b** 9, 18, 27, 36, 45, 54
c 10, 20, 30, 40, 50, 60 **d** 15, 30, 45, 60, 75, 90
e 22, 44, 66, 88, 110, 132 **f** 35, 70, 105, 140, 175, 210

2 **a** 42 **b** 99 **c** 165 **d** 9900

3 **a** 504 **b** 996

4 **a, b**

1	2	③	$\boxed{4}$	5	⑥	7	$\boxed{8}$	⑨	10
11	$\boxed{⑫}$	13	14	⑮	$\boxed{16}$	17	⑱	19	$\boxed{20}$
㉑	22	23	$\boxed{㉔}$	25	26	㉗	$\boxed{28}$	29	㉚

c 12, 24

5 **a** 63, 99 **b** 36 **c** 35 **6** 30, 60, 90

EXERCISE 4C.2

1 **a** 20 **b** 15 **c** 24 **d** 60 **e** 30 **f** 28
g 72 **h** 42 **i** 66 **j** 65 **k** 75 **l** 108

2 56 minutes **3** 156 buns

EXERCISE 4D

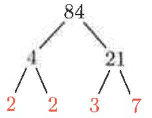
1 **a** 2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47
b No, a prime has exactly two factors, one and itself.
c yes, 2

2 **a** 3 **b** 15 **c** 5 **3** 11

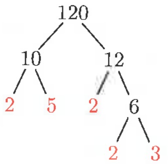
4 **a** $6485 = 5 \times 1297$ **b** $9320 = 2 \times 4660$
c $2222 = 2 \times 1111$ **d** $4279 = 11 \times 389$

5 **Note:** Other factor trees may be possible.

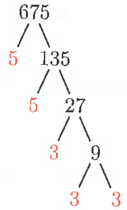
a



b



c



6 **a** $24 = 2^3 \times 3$ **b** $70 = 2 \times 5 \times 7$ **c** $63 = 3^2 \times 7$
d $72 = 2^3 \times 3^2$ **e** $225 = 3^2 \times 5^2$ **f** $88 = 2^3 \times 11$
g $480 = 2^5 \times 3 \times 5$ **h** $1024 = 2^{10}$

7 **a** $28 = 2^2 \times 7$ **b** $27 = 3^3$
c $84 = 2^2 \times 3 \times 7$ **d** $160 = 2^5 \times 5$
e $216 = 2^3 \times 3^3$ **f** $528 = 2^4 \times 3 \times 11$
g $784 = 2^4 \times 7^2$ **h** $138 = 2 \times 3 \times 23$
i $250 = 2 \times 5^3$ **j** $189 = 3^3 \times 7$
k $726 = 2 \times 3 \times 11^2$ **l** $9625 = 5^3 \times 7 \times 11$

8 4^2 is not a product of prime factors, as 4 is not a prime number.

EXERCISE 4E

1 **a** 4 **b** 7 **c** 9 **d** 11 **e** 16 **f** 0
g 32 **h** 34 **i** 65 **j** 99 **k** 100 **l** 120

2 **a** 1 and 2 **b** 2 and 3 **c** 5 and 6 **d** 8 and 9

3 **a** 1 **b** 4 **c** 5 **d** 7 **e** -1 **f** -3
g -4 **h** -10

REVIEW SET 4A

1 **Note:** Other answers are possible.
a $10 + 26$ **b** $9 + 29$ **c** 2×18 **d** 4×9

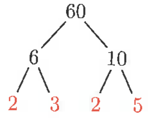
2 **a** 0, 4, or 8 **b** 2, 5, or 8 **c** 0 or 9

3 **a** 1, 2, 3, 4, 6, 8, 9, 12, 18, 24, 36, 72
b 1, 2, 3, 5, 6, 9, 10, 15, 18, 30, 45, 90
c 1, 2, 3, 6, 7, 9, 14, 18, 21, 42, 63, 126

4 42, 49, 56

5 **a** $2950 = 2 \times 1475$ **b** $1863 = 3 \times 621$

6



7 48

8 **a** $44 = 2^2 \times 11$
b $504 = 2^3 \times 3^2 \times 7$
c $693 = 3^2 \times 7 \times 11$

9 **a** 6 cm × 6 cm
b 117 pieces

10 **a** 14 **b** -5

REVIEW SET 4B

1 **a** divisible **b** not divisible **2** 53, 59, 61, 67

3 **a** divisible **b** divisible **c** not divisible **d** divisible

4 21 **5** 66 **6** **a** 12 **b** -7 **7** **a** 7 **b** 6

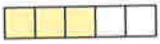
8 **a** $48 = 2^4 \times 3$ **b** $495 = 3^2 \times 5 \times 11$
c $900 = 2^2 \times 3^2 \times 5^2$

9 70 days **10** 13 and 14

EXERCISE 5A

1 **a** $\frac{1}{4}$ **b** $\frac{5}{6}$ **c** $\frac{5}{8}$ **d** $\frac{7}{15}$

2 **a**



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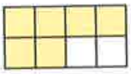

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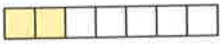

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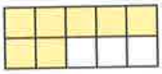
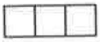
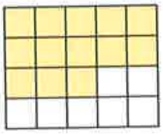


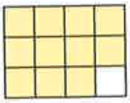
e



f



g



h

3 **a** D **b** C **c** B **d** A **e** B **f** A **g** D **h** C

4 **a** C **b** A **c** D **d** B

5 **a** $\frac{9}{23}$ **b** $\frac{5}{23}$ **c** $\frac{7}{23}$ **d** $\frac{16}{23}$

EXERCISE 5B

1 **a** $\frac{1}{2}$ **b** $\frac{1}{5}$ **c** $\frac{4}{7}$ **d** $\frac{8}{9}$ **e** $\frac{2}{3}$ **f** $\frac{9}{10}$

g $\frac{9}{6}$ **h** $\frac{20}{4}$

2 **a** $3 \div 5$ **b** $2 \div 7$ **c** $6 \div 10$ **d** $5 \div 8$

e $1 \div 11$ **f** $6 \div 7$ **g** $11 \div 12$ **h** $13 \div 3$

3 **a** $8 \div 2 = 4$ **b** $15 \div 5 = 3$ **c** $24 \div 8 = 3$

d $10 \div 10 = 1$ **e** $16 \div 4 = 4$ **f** $42 \div 6 = 7$

4 **a** $\frac{-2}{3}$ **b** $\frac{4}{-5}$ **c** $\frac{-6}{-7}$ **d** $\frac{10}{-12}$ **e** $\frac{-11}{22}$ **f** $\frac{-23}{-5}$

g $\frac{16}{-8}$ **h** $\frac{-18}{2}$

5 **a** $-1 \div 8$ **b** $-4 \div -6$ **c** $3 \div -9$

d $-10 \div -2$ **e** $-24 \div 6$

6 **a** $25 \div 5 = 5$ **b** $25 \div -5 = -5$ **c** $-25 \div 5 = -5$

d $-25 \div -5 = 5$ **e** $27 \div 9 = 3$ **f** $-27 \div 9 = -3$

g $27 \div -9 = -3$ **h** $-27 \div -9 = 3$

7 **a** $15 \div -3 = -5$ **b** $-15 \div 3 = -5$

c $-63 \div -7 = 9$ **d** $63 \div -7 = -9$

e $40 \div -10 = -4$ **f** $-40 \div 10 = -4$

g $-96 \div 12 = -8$ **h** $96 \div -12 = -8$

8 **a** $12 \div 6 = 2$ **b** $18 \div 6 = 3$

c $48 \div 12 = 4$ **d** $30 \div -3 = -10$

e $14 \div 7 = 2$ **f** $10 \div 2 = 5$

g $-44 \div 11 = -4$ **h** $-28 \div -4 = 7$

EXERCISE 5C

1 **a** $1\frac{4}{5}$ **b** $1\frac{1}{4}$ **c** $2\frac{1}{10}$ **d** $10\frac{1}{3}$ **e** $3\frac{5}{6}$ **f** $3\frac{4}{7}$

g $7\frac{1}{9}$ **h** $10\frac{3}{10}$

2 **a** $\frac{5}{2}$ **b** $\frac{5}{4}$ **c** $\frac{14}{5}$ **d** $\frac{11}{8}$ **e** $\frac{16}{7}$ **f** $\frac{13}{10}$

g $\frac{31}{16}$ **h** $\frac{14}{3}$

3 **a** $8\frac{3}{8}$ **b** 8 teams **4** $3\frac{3}{4}$ m

EXERCISE 5D

1 **a** $A = \frac{5}{3}$ or $1\frac{2}{3}$, $B = \frac{1}{3}$, $C = \frac{4}{3}$ or $1\frac{1}{3}$, $D = \frac{2}{3}$

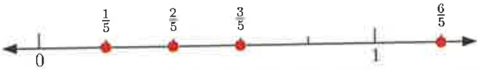
b $A = \frac{10}{6}$ or $1\frac{4}{6}$, $B = \frac{3}{6}$, $C = \frac{7}{6}$ or $1\frac{1}{6}$, $D = \frac{1}{6}$

c $A = \frac{6}{4}$ or $1\frac{2}{4}$, $B = \frac{9}{4}$ or $2\frac{1}{4}$, $C = \frac{13}{4}$ or $3\frac{1}{4}$,

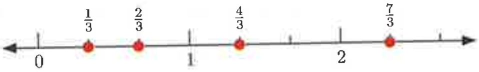
$D = \frac{11}{4}$ or $2\frac{3}{4}$

d $A = \frac{2}{7}$, $B = \frac{10}{7}$ or $1\frac{3}{7}$, $C = \frac{6}{7}$, $D = \frac{8}{7}$ or $1\frac{1}{7}$

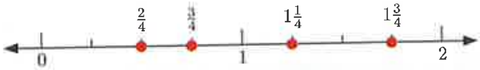
2 **a**



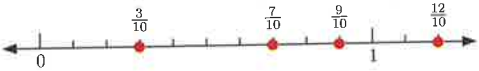
b



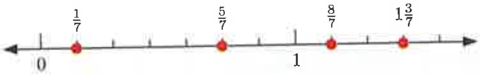
c



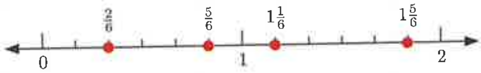
d



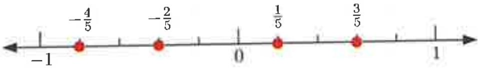
e



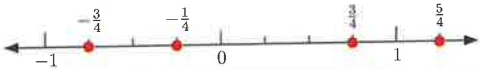
f



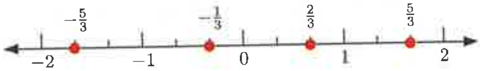
3 **a**



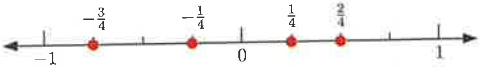
b



c

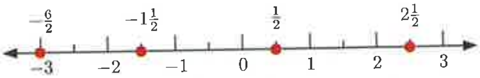


4 **a**



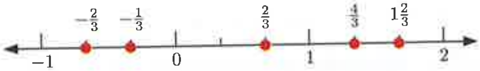
$-\frac{3}{4}, -\frac{1}{4}, \frac{1}{4}, \frac{2}{4}$

b



$-\frac{6}{2}, -1\frac{1}{2}, \frac{1}{2}, 2\frac{1}{2}$

c



$-\frac{2}{3}, -\frac{1}{3}, \frac{2}{3}, \frac{4}{3}, 1\frac{2}{3}$

EXERCISE 5E.1

1 **a** equal **b** not equal **c** equal **d** not equal

2 C **3** **a** $\frac{2}{8}, \frac{3}{12}$ **b** $\frac{6}{8}, \frac{9}{12}$ **c** Any two of $\frac{4}{6}, \frac{6}{9}, \frac{8}{12}$

4 **a** $\frac{3}{12}$ **b** $\frac{8}{12}$ **c** $\frac{10}{12}$ **d** $\frac{54}{12}$ **e** $\frac{10}{12}$

5 **a** $\frac{4}{20}$ **b** $\frac{15}{20}$ **c** $\frac{26}{20}$ **d** $\frac{13}{20}$ **e** $\frac{9}{20}$

EXERCISE 5E.2

1 **a** $\frac{1}{3}$ **b** $\frac{1}{4}$ **c** $\frac{4}{5}$ **d** $\frac{2}{3}$ **e** $\frac{1}{2}$ **f** $\frac{1}{3}$

2 **a** $\frac{3}{4}$ **b** $\frac{4}{5}$ **c** $\frac{6}{7}$ **d** $\frac{4}{5}$ **e** $\frac{1}{3}$ **f** $\frac{7}{10}$
g $\frac{2}{5}$ **h** $\frac{5}{9}$ **i** $\frac{16}{25}$ **j** $\frac{3}{8}$

3 **a** $\frac{3}{2}$ **b** $\frac{4}{3}$ **c** $\frac{8}{5}$ **d** $\frac{7}{4}$ **e** $\frac{5}{3}$ **f** $\frac{7}{6}$
g $\frac{10}{9}$ **h** $\frac{12}{11}$

4 **a** $\frac{2}{3}$ **b** $\frac{2}{3}$ **c** $\frac{1}{2}$ **d** $\frac{5}{3}$

EXERCISE 5E.3

1 **a** $\frac{1}{5}$ **b** $\frac{1}{5}$ **c** $\frac{3}{5}$ **d** $\frac{1}{8}$ **e** $\frac{3}{16}$ **f** $\frac{1}{4}$

2 **a** $\frac{1}{5}$ **b** $\frac{4}{7}$ **c** $\frac{3}{8}$ **d** $\frac{3}{5}$ **e** $\frac{1}{5}$ **f** $\frac{1}{9}$

3 **a** $\frac{1}{3}$ **b** $\frac{1}{4}$ **c** $\frac{2}{5}$

4 **a** $\frac{1}{8}$ **b** $\frac{1}{6}$ **c** $\frac{2}{3}$ **5** $\frac{7}{10}$ **6** $\frac{2}{5}$ **7** $\frac{9}{16}$

EXERCISE 5F

1 **a** $\frac{3}{8}$ **b** $\frac{3}{5}$ **c** $\frac{2}{11}$ **d** $\frac{19}{25}$ **e** $\frac{3}{4}$ **f** $\frac{7}{12}$

2 **a** $\frac{2}{3} < \frac{3}{4}$ **b** $\frac{3}{5} > \frac{5}{9}$ **c** $\frac{5}{6} > \frac{13}{18}$ **d** $\frac{3}{11} < \frac{2}{7}$
e $\frac{11}{25} > \frac{2}{5}$ **f** $\frac{11}{16} < \frac{7}{10}$

3 $\frac{5}{8}, \frac{2}{3}, \frac{11}{15}, \frac{7}{9}$

EXERCISE 5G

1 **a** 1 **b** $\frac{2}{5}$ **c** $\frac{9}{7}$ **d** 4 **e** 1 **f** 1
g $\frac{3}{2}$ **h** $\frac{4}{5}$

2 **a** $\frac{3}{8}$ **b** $\frac{3}{10}$ **c** $\frac{1}{6}$ **d** $\frac{27}{20}$ **e** $\frac{1}{21}$ **f** $\frac{7}{12}$
g $\frac{1}{2}$ **h** $\frac{13}{24}$ **i** $\frac{1}{14}$ **j** $\frac{11}{20}$ **k** $\frac{29}{18}$ **l** $\frac{7}{40}$

3 **a** $\frac{3}{4}$ **b** $2\frac{1}{3}$ **c** $2\frac{1}{6}$ **d** $2\frac{13}{14}$ **e** $\frac{9}{10}$ **f** $3\frac{17}{18}$
g $\frac{5}{6}$ **h** $6\frac{5}{8}$

4 **a** $\frac{53}{35}$ **b** $1\frac{7}{8}$ **c** $\frac{2}{9}$ **d** $2\frac{2}{5}$

EXERCISE 5H

1 **a** $\frac{1}{6}$ **b** $\frac{3}{10}$ **c** $\frac{4}{9}$ **d** $\frac{4}{15}$ **e** $\frac{4}{27}$ **f** $\frac{5}{12}$
g $1\frac{2}{3}$ **h** $\frac{10}{9}$ **i** $\frac{3}{8}$ **j** $\frac{8}{15}$ **k** $1\frac{7}{8}$ **l** $9\frac{5}{8}$

2 **a** $\frac{1}{4}$ **b** $\frac{1}{3}$ **c** $\frac{4}{7}$ **d** $\frac{5}{9}$ **e** 18 **f** $\frac{3}{2}$
g $\frac{1}{4}$ **h** $\frac{1}{12}$ **i** $\frac{4}{3}$ **j** 1 **k** $\frac{3}{4}$ **l** $1\frac{1}{2}$

3 **a** $\frac{1}{8}$ **b** $1\frac{1}{2}$ **c** 20

4 **a** $\frac{1}{6}$ **b** $\frac{1}{6}$ **c** $\frac{1}{14}$ **d** $\frac{1}{10}$ **e** $\frac{1}{5}$ **f** $\frac{6}{5}$

5 **a** 25 **b** 20 **c** $2\frac{1}{4}$

EXERCISE 5I

1 **a** $\frac{4}{3}$ **b** $\frac{3}{2}$ **c** $\frac{6}{5}$ **d** $\frac{7}{4}$ **e** $\frac{3}{8}$ **f** $\frac{5}{18}$

2 **a** $\frac{2}{3}$ **b** $\frac{3}{8}$ **c** $\frac{5}{11}$ **d** $\frac{4}{19}$ **e** $\frac{8}{15}$ **f** $\frac{6}{31}$

3 **a** $-\frac{4}{3}$ **b** -3 **c** $-1\frac{1}{5}$ **d** $-\frac{5}{12}$ **e** $-\frac{8}{9}$ **f** $-\frac{5}{14}$

EXERCISE 5J

1 **a** 3 **b** 2 **c** $\frac{5}{3}$ **d** 4

2 **a** $\frac{4}{3}$ **b** $\frac{12}{5}$ **c** $\frac{9}{10}$ **d** $\frac{2}{15}$

3 **a** $\frac{5}{16}$ **b** $2\frac{2}{3}$ **c** $\frac{1}{5}$ **d** $\frac{5}{18}$ **e** $\frac{1}{10}$ **f** $1\frac{3}{7}$
g $1\frac{4}{5}$ **h** $4\frac{1}{8}$

4 **a** $\frac{9}{16}$ **b** $3\frac{1}{2}$

EXERCISE 5K

1 **a** 10 **b** 3 **c** $8\frac{1}{2}$ **d** $\frac{1}{2}$ **e** $\frac{1}{2}$ **f** 23
g $9\frac{1}{2}$ **h** $\frac{1}{2}$

2 **a** $\frac{1}{2}$ **b** 3 **c** 5

EXERCISE 5L

1 **a** $42 **b** 15 kg **2** €81 **3** 45 questions

4 $50 **5** $\frac{5}{7}$ **6** $\frac{1}{40}$ **7** $\frac{23}{90}$ **8** $\frac{17}{60}$

9 80 packets **10** 2880 bottles **11** $18 000 **12** $\frac{1}{6}$

REVIEW SET 5A

1 **a** $\frac{4}{15}$ **b** $\frac{7}{8}$

2 **a** $36 \div 12 = 3$ **b** $50 \div 2 = 25$ **c** $-21 \div 7 = -3$

3 **a** $\frac{1}{6}$ **b** $\frac{6}{11}$ **c** $\frac{9}{5}$

4

5 **a** $9\frac{2}{3}$ **b** $7\frac{3}{5}$ **c** $7\frac{4}{7}$ **6** $\frac{5}{12}$

7 **a** $\square = 12$ **b** $\square = 3$ **c** $\square = 6$ **8** $\frac{15}{22}$

9 **a** $\frac{16}{15}$ **b** $\frac{5}{24}$ **c** $\frac{12}{55}$ **d** $3\frac{1}{2}$

10 **a** $\frac{1}{2}$ **b** 4 **11** 9 students **12** $\frac{27}{40}$

REVIEW SET 5B

1 **a** $\frac{7}{20}$ **b** $\frac{1}{5}$ **c** $\frac{1}{2}$ **2** $-28 \div 7 = -4$

3 **a** $\frac{19}{10}$ **b** $\frac{36}{5}$ **4** $2\frac{1}{14}$

5 $\mathrm{A} = \frac{13}{10}$ or $2\frac{3}{5}$, $\mathrm{B} = -\frac{2}{5}$, $\mathrm{C} = \frac{6}{5}$ or $1\frac{1}{5}$

6 **a** $\frac{7}{5}$ **b** $\frac{8}{7}$

7 **a** $\frac{24}{35}$ **b** $\frac{5}{6}$ **c** $\frac{19}{20}$ **d** $\frac{9}{16}$ **e** $5\frac{5}{6}$ **f** $1\frac{11}{21}$
g $3\frac{7}{18}$ **h** $\frac{18}{25}$

8 $\frac{8}{25}$ **9** $\frac{2}{7}$ **10** 250 pots **11** $\frac{11}{20}$ **12** $\frac{5}{2}$

EXERCISE 6A

1 **a** $4 + \frac{2}{10}$ **b** $7 + \frac{5}{10} + \frac{3}{100}$ **c** $9 + \frac{1}{10} + \frac{8}{100}$
d $3 + \frac{3}{100}$ **e** $\frac{2}{10} + \frac{3}{100} + \frac{4}{1000}$ **f** $1 + \frac{5}{100} + \frac{9}{1000}$
g $5 + \frac{6}{1000} + \frac{1}{10\,000}$ **h** $\frac{7}{10\,000} + \frac{1}{100\,000}$
i $2 + \frac{5}{10} + \frac{1}{1000}$ **j** $\frac{7}{100} + \frac{7}{1000} + \frac{1}{10\,000}$
k $10 + 1 + \frac{9}{10} + \frac{1}{100} + \frac{2}{1000}$ **l** $\frac{1}{100} + \frac{1}{10\,000}$

2 **a** 0.7 **b** 0.15 **c** 0.549 **d** 0.03 **e** 0.105
f 0.067 **g** 0.084 **h** 0.0039 **i** 0.6155

3 **a** 0.71 **b** 0.13 **c** 0.54 **d** 0.267
e 0.506 **f** 0.097 **g** 0.803 **h** 0.022

4 **a** 700 **b** $\frac{7}{10}$ **c** $\frac{7}{100}$ **d** $\frac{7}{1000}$
e $\frac{7}{100}$ **f** 70 000 **g** $\frac{7}{10}$ **h** $\frac{7}{10}$

5 **a** 7.6 **b** 3.67 **c** 12.17 **d** 2.59
e 1.461 **f** 6.039 **g** 2.001 **h** 3.0007
i 5.39 **j** 7.0203 **k** 7.21 **l** 3.723

EXERCISE 6B

1 **a** $\frac{7}{10}$ **b** $\frac{2}{5}$ **c** $1\frac{1}{10}$ **d** $2\frac{3}{5}$ **e** $\frac{19}{100}$ **f** $\frac{29}{100}$
g $\frac{1}{4}$ **h** $\frac{4}{25}$ **i** $\frac{17}{20}$ **j** $\frac{24}{25}$ **k** $\frac{3}{20}$ **l** $\frac{1}{20}$
m $\frac{7}{100}$ **n** $3\frac{13}{100}$ **o** $5\frac{2}{25}$ **p** $7\frac{11}{20}$

2 **a** $\frac{101}{1000}$ **b** $\frac{23}{500}$ **c** $\frac{1}{125}$ **d** $\frac{41}{200}$
e $\frac{1}{8}$ **f** $\frac{1}{2500}$ **g** $\frac{73}{500}$ **h** $\frac{7}{8}$
i $\frac{1}{2000}$ **j** $\frac{3}{400}$ **k** $1\frac{3}{8}$ **l** $4\frac{19}{250}$

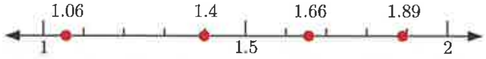
3 **a** 0.5 **b** 0.6 **c** 0.65 **d** 0.25
e 0.34 **f** 0.45 **g** 0.92 **h** 0.02
i 0.124 **j** 0.515 **k** 0.56 **l** 0.414
m 0.95 **n** 0.016 **o** 0.625 **p** 0.175

EXERCISE 6C

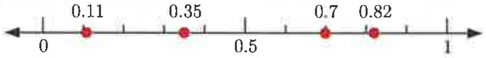
1 **a** 5 **b** 4.8 **c** 4.77 **d** 4.769
2 **a** 23 **b** 23.1 **c** 23.06 **d** 23.060
3 **a** 8 **b** 8.0 **c** 8.04 **d** 8.0424
4 42.72 **5** $128 million **6** 5.2 goals per game
7 $57.29
8 **a** 0.35 **b** 1.43 **c** 0.44 **d** 19.16
e 1.73 **f** 6.56 **g** 8.60 **h** 10.30

EXERCISE 6D

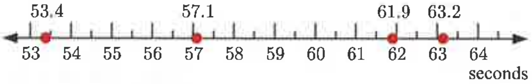
1 **a** 28.3 cm **b** 37.8°C **c** 74.6 kg **d** 10.5 seconds
2 **a** 6.7 **b** 47.8 **c** 3.77 **d** 1.953 **e** 0.042 **f** 0.404
3 **a**



b



4



EXERCISE 6E

1 **a** $0.339 < 0.393$ **b** $5.05 > 0.55$ **c** $0.6 = 0.60$
d $2.62 > 2.6$ **e** $0.39 < 0.4$ **f** $12.121 < 21.121$
g $0.123 < 0.132$ **h** $\frac{150}{1000} = 0.15$ **i** $2.4 = 2.400$
j $0.902 > 0.209$ **k** $0.008\,76 < 0.0876$
l $3.20 < 3.201$

2 **a** 1.036, 1.3, 1.36 **b** 8.6, 8.67, 8.76
c 0.052, 0.495, 0.5 **d** 32.7, 32.71, 33.17
e 7.999, 8.066, 8.1 **f** 6.043, 6.304, 6.34, 6.403
g 9.009, 9.09, 9.1, 9.2 **h** 0.09, 0.099, 0.9, 0.99

3 16.91 seconds, 16.98 seconds, 17.1 seconds, 17.19 seconds

4 **a** Wednesday 1.3602, Friday 1.3578, Tuesday 1.3571, Monday 1.3537, Thursday 1.3519
b **i** Wednesday **ii** Thursday

EXERCISE 6F

1 **a** 0.9 **b** 0.93 **c** 1.69 **d** 2.53
e 1.13 **f** 19.633 **g** 13.81 **h** 0.548
i 0.6638 **j** 13.962 **k** 4.01 **l** 5.24

2 **a** 2.3 **b** 2.13 **c** 3.0028 **d** 0.7
e 0.7 **f** 0.01 **g** 1.29 **h** 1.0074
i 6.55 **j** 0.0739 **k** 6.46 **l** 0.9766

3 **a** 160.06 **b** 823.67 **c** 311.349 **d** 130.29
4 **a** 11.907 **b** 5.44 **c** 5.882 **d** 49.065
5 0.125 kg **6** 4.27 m **7** $3.75 **8** £655.54
9 **a** 53.54 s **b** **i** 1.36 s **ii** 0.73 s

EXERCISE 6G.1

1 **a** **i** 32.71 **ii** 327.1 **iii** 3271 **iv** 32 710 000
b **i** 76 **ii** 7600 **iii** 76 000 **iv** 7 600 000
2 **a** 270 **b** 400 **c** 22 **d** 16 400 **e** 2 **f** 79
g 810 **h** 50 **i** 16 700 **j** 36 **k** 76.1 **l** 33 800

EXERCISE 6G.2

1 **a** **i** 8.46 **ii** 0.846 **iii** 0.0846 **iv** 0.000 846
b **i** 0.07 **ii** 0.0007 **iii** 0.000 07 **iv** 0.000 0007

2 **a** 0.6 **b** 9.2 **c** 52.9 **d** 5.29
e 0.529 **f** 0.0529 **g** 0.03 **h** 0.0003
i 0.0097 **j** 0.006 **k** 0.0006 **l** 0.000 022
m 0.000 77 **n** 0.002 963 **o** 0.000 035 **p** 0.0516

EXERCISE 6H

1 **a** 2.8 **b** 7.2 **c** 3 **d** 0.08
e 0.042 **f** 0.015 **g** 0.000 12 **h** 0.45
i 0.4 **j** 0.024 **k** 18 **l** 21
m 1800 **n** 2500 **o** 0.04 **p** 0.105

2 **a** 1036.2 **b** 10.362 **c** 103.62 **d** 10.362
e 0.103 62 **f** 1.0362 **g** 1.0362 **h** 103.62
i 0.010 362

3 **a** **i** 4 **ii** 3.6 **b** **i** 6 **ii** 5.89
c **i** 36 **ii** 37.38 **d** **i** 63 **ii** 67.16
e **i** 280 **ii** 274.06 **f** **i** 30 **ii** 30.527

4 **a** 14.84 **b** 0.01 **c** 0.004 14 **5** $23.40
6 €87.50 **7** £16.79 (rounded) **8** $62.05
9 67.5 kg **10** **a** 10 750 kg **b** 6 truck loads

EXERCISE 6I

1 **a** 3.6 **b** 10.9 **c** 20.1 **d** 0.09
e 0.025 **f** 0.26 **g** 7.7 **h** 0.83
i 0.059 **j** 0.0137 **k** 0.48 **l** 0.057

2 **a** 2 **b** 7 **c** 30 **d** 0.5 **e** 2 **f** 5
g 300 **h** 800 **i** 8 **j** 64 **k** 0.4 **l** 5400

3 **a** 1.5 **b** 11 **c** 0.2 **d** 20 **e** 44
f 0.029 **g** 0.022 **h** 0.7

4 26 minutes
5 **a** 140 **b** 1.4 **c** 1.4 **d** 140 **e** 0.014 **f** 0.14
6 **a** 250 lengths **b** $226.35
c **i** 6.3201 carats **ii** 1.264 02 carats
d 28 nut bars **e** 300 lengths **f** 23 tins

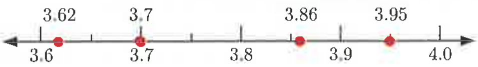
REVIEW SET 6A

1 **a** $\frac{5}{10}$ **b** $\frac{6}{1000}$
2 **a** $2 + \frac{1}{10} + \frac{2}{1000} + \frac{3}{10\,000}$ **b** $\frac{1}{250}$
3 **a** 0.8 **b** 0.36 **c** 0.065
4 **a** 28.55 **b** 0.5 **c** 46 **d** 0.539
5 **a** 5.4 **b** 2.38
6 **a** 1.006 **b** 6.23 **c** 2.9 **d** 0.48 **e** 23.13 **f** 90
7 **a** $3.03 < 3.303$ **b** $0.514 < 0.541$ **c** $2.404 > 2.044$
8 **a** **i** 859 **ii** 8590 **iii** 859 000
b **i** 6.74 **ii** 0.0674 **iii** 0.006 74
9 60 laps **10** 10.17 tonnes

REVIEW SET 6B

1 **a** 0.43 **b** 0.701 **c** 0.0208
2 **a** $\frac{43}{50}$ **b** $2\frac{6}{25}$ **c** $\frac{9}{200}$ **3** 3.3 steals per game
4 **a** 0.004 76 **b** 0.0476

5



6 **a** 62.5 **b** 2.78 **c** 3.2 **d** 6.87 **e** 0.013 **f** 16

7 **a** 14.74 **b** 14

8 8 cups **9** 176.8 cm **10** 3.023, 3.204, 3.23, 3.234

EXERCISE 7A.1

1 $p-7$ passengers **2** **a** $g+3$ goals **b** $2g$ goals

3 **a** $n-12$ years **b** $n+6$ years **c** $2n$ years

4 xy apartments **5** **a** x tails **b** $2x$ eyes **c** $4x$ legs

6 **a** $3\times 8+7$ blueberries **b** $3\times 12+7$ blueberries
c $3\times b+7$ blueberries

7 **a** $6\times 2+5$ horses **b** $6\times 4+5$ horses
c $6\times h+5$ horses

8 **a** $S=8\times n+4$ where S is the number of strawberries
b $S=m\times n+p$

EXERCISE 7A.2

1 **D** **2** **C**

3 **a** $3a$ **b** $3a$ **c** $5x$ **d** $5x$ **e** $6n$ **f** cd
g km **h** bn **i** $9xy$ **j** xyz **k** bhk **l** $2st$

4 **a** $xy+z$ **b** $3p+4q$ **c** $pq-r$ **d** $p-qr$
e $u-7w$ **f** $4c+9d$ **g** $ef-gh$ **h** $9-2mn$
i $3(d-3)$ **j** $4(g+1)$ **k** $7(x-5)$ **l** $2(x-y)$

5 **a** $x\times x$ **b** $y\times y\times y$ **c** $3\times x\times x$
d $4\times m\times m\times m$ **e** $8\times x\times x\times x\times y$
f $5\times p\times q\times q$ **g** $c\times c+4\times d\times d\times d$
h $3\times v\times v-5\times w\times w$

6 **a** x^2 **b** p^4 **c** $4a^2$ **d** $5b^3$
e $3ab^2$ **f** f^2g^3h **g** f^2+f **h** w^3+7
i e^3-2e^2 **j** $5a^3+b^2$ **k** $4xy^2+z^2$ **l** $5a+a^2$

EXERCISE 7B

1 **a** false, $\dfrac{x+y}{2}$ is an expression **b** true **c** true
d true **e** false, the constant term is -2
f false, $\dfrac{3}{q}=6$ is an equation

2 **a** 5 **b** 4 **c** 8 **d** 4 **e** 1 **f** -2
g 1 **h** -1 **i** -7

3 **a** 1 term **b** 2 terms **c** 3 terms **d** 3 terms
e 3 terms **f** 3 terms **g** 2 terms **h** 2 terms
i 2 terms

4 **a** 4 terms **b** -7 **c** -2 **d** $5y$ and $-2y$

5 **a** $2x$ and $5x$, 3 and 5 **b** x and $5x$, y and $-y$
c $2x$ and $3x$ **d** q^2 and $4q^2$, 3 and 7
e no like terms **f** ab and $3ab$

EXERCISE 7C.1

1 **a** **i** $(2p+1)+(2p+1)$ **ii** $(p+1)+(3p+1)$
iii $(2p+2)+2p$

b The total number of strawberries is the same in each case, and is equal to $4p+2$.

So, $(2p+1)+(2p+1)=(p+1)+(3p+1)$
$=(2p+2)+2p$
$=4p+2$

2 **Note:** Other answers are possible.
a $(2b+1)+(2b+1)$, $4b+2$
b $(2b+2)+(2b+2)$, $4b+4$
c $(b+3)+b$, $2b+3$ **d** $(2b+4)+(b+5)$, $3b+9$

3 **a** true **b** false, $4(b+2)=4b+8$
c true **d** false, $(b+2)+(2b+2)=3b+4$

4 **a** $4p$ **b** $6p$ **c** $2p+3$ **d** $4p+5$ **e** $4p+12$
f $2p+7$ **g** $2p+4$ **h** $3p+12$ **i** $5p+5$

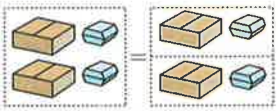
5 **a** $2p$ **b** $3p$ **c** 0 **d** $2p+3$ **e** $2p+4$
f $2p+2$ **g** $2p$ **h** $p+1$ **i** 1

6 **a** **E** **b** **A** **c** **D** **d** **B** **e** **F** **f** **C**

EXERCISE 7C.2

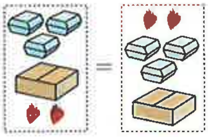
1 **a** $c+1$ **b** $c+2p$ **c** $p+3$ **d** $c+2p+3$
e $3c+2$ **f** $c+p+3$ **g** $2c+p+2$

2 **a**



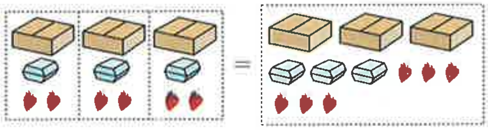
$2c+2p=2(c+p)$

b



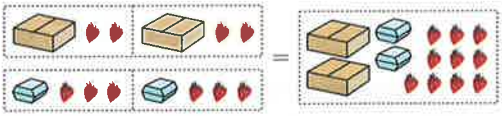
$3p+c+2=2+3p+c$

c



$3(c+p+2)=3c+3p+6$

d



$2(c+2)+2(p+3)=2c+2p+10$

EXERCISE 7D

1 **a** $2a$ **b** $3b$ **c** $2a+3b$ **d** $3+2x$
e $3f+3$ **f** 5 **g** $3p-2q$ **h** $5-3g$
i $5x-2$ **j** $1-2r$ **k** $5+3z$ **l** $2m+3n+5$

2 **a** $4a$ **b** $4y$ **c** cannot be simplified **d** $3x+y$
e $12b$ **f** $3r$ **g** cannot be simplified **h** $6n$
i $2p$ **j** cannot be simplified **k** 0 **l** $3d+3$
m $2+2y$ **n** $2q$ **o** cannot be simplified **p** $5w$
q cannot be simplified **r** $4p$ **s** $4h$ **t** $11x$
u $x+3y$

3 **a** $-2z$ **b** $-4b$ **c** cannot be simplified **d** $-2m$
e $-2x$ **f** $-4f$ **g** $-6y$ **h** $6s-6$
i cannot be simplified **j** $-4k$ **k** $-4k+4$
l cannot be simplified **m** $-20r$ **n** $-4t$ **o** $-v$
p $-w-5$ **q** $-x+y$ **r** $-y-x+1$

4 **a** $8x+6y$ **b** $3p+8q$ **c** $7a+6b$ **d** $2d$ **e** $-v-4$
f $4x+2z$ **g** $5h$ **h** $4r+t+5$ **i** $12x-13y$
j $-a^2b$ **k** $2xy+3x^2y$ **l** $2ab-2a^2b-ab^2$

5 **a** $-5\neq 7$, so Pat is wrong
b $4-(p+p+p)=4-3p$, $4-p+p+p=4+p$

EXERCISE 7E

1 **a** $8c$ **b** $30x$ **c** $21y$ **d** $24t$ **e** $45pq$
f $28mn$ **g** $48ct$ **h** $99yz$

2 **a** $3x^2$ **b** $5y^2$ **c** $14a^2$ **d** $32m^2$ **e** $54z^2$ **f** $4x^3$
g $49x^2$ **h** $5x^3$ **i** $12n^3$ **j** $42y^3$ **k** $80k^3$
l $4x^3$ **m** $5x^2y$ **n** $3a^2b$ **o** $40x^2y$ **p** $63x^2y^2$

EXERCISE 7F.1

1 **a** 9 **b** 11 **c** 9 **d** 20 **e** 10 **f** 4
g 21 **h** 25
2 **a** $3p + 10$ potatoes
b **i** 25 potatoes **ii** 46 potatoes **iii** 85 potatoes
3 **a** 6 **b** 15 **c** 17 **d** 28 **e** 2 **f** 11
g -3 **h** 22 **i** 0 **j** 1 **k** -42 **l** -3
4 **a** €$(40x + 15y)$ **b** **i** €110 **ii** €180 **iii** €265
5 **a** 7 **b** 40 **c** 16 **d** 8 **e** 3 **f** 2
g -2 **h** 12 **i** 30 **j** 0 **k** 18 **l** 18
6 **a** 14 **b** 23 **c** -4 **d** -4 **e** 9 **f** 36
g -6 **h** 46 **i** 0 **j** -72 **k** 0 **l** -42

EXERCISE 7F.2

1 **a** 3 **b** -32 **c** -3 **d** -15 **e** 8 **f** -7
g -8 **h** -10 **i** 12 **j** 6 **k** -4 **l** 14
2 **a** -14 **b** -20 **c** 9 **d** -5 **e** 52 **f** 6
g 93 **h** -88

REVIEW SET 7A

1 $p - 8$ peaches **2** **E** **3** **a** 4 terms **b** -7
4 **a** $2 \times 4 + 5$ strawberries **b** $2 \times 7 + 5$ strawberries
c $2 \times s + 5$ strawberries
5 **a** $2x^2$ and $3x^2$, $-4x$ and $-6x$
b $5a$ and a, $-3b$ and $-2b$ **c** $3c$ and $3c$
d $2e$ and $-4e$, ef and $2ef$
6 **a** $12x + 5$ **b** $5p - 2$ **c** $8x$ **d** $5d^2 + cd - 3c$
7 **a** k^3 **b** $5m^2n$ **c** $7a^2 + ab^2$
8 **a** 27 **b** 28 **9** **a** $35p$ **b** $8f^2$ **c** $36s^3$
10 **a** 30 **b** -45 **c** -63

REVIEW SET 7B

1 **a** 11 birds **b** $3b + 2$ birds **c** $3b + f$ birds
2 **a** $m \times m \times m$ **b** $7 \times t \times t$
c $6 \times x \times x \times y \times y \times y$ **d** $4 \times p \times p - q \times q$
3 -3 **4** **a** 1 **b** $6x$ and $2x$, $-6y$ and y
5 **a** $7x$ **b** $2(c - d)$ **c** $8pq$
6 **a** $6c$ **b** $5a + 5$ **c** $4q - 8$ **d** $5x - 2$
7 **a** $24ab$ **b** $9y^3$ **c** $44mn^3$
8 **a** $3t + 2$ teddy bears
b **i** 11 teddy bears **ii** 17 teddy bears **iii** 38 teddy bears
9 **a** false, $(b + 4) + (b + 5) = 2b + 9$ **b** true
10 **a** 25 **b** 4 **c** -5

EXERCISE 8A

1 **a** 20% **b** 35% **c** 53% **d** 96%
2 **a** $\frac{13}{100}$ **b** $\frac{37}{100}$ **c** $\frac{6}{100}$ **d** $\frac{92}{100}$ **e** $\frac{79}{100}$
3 **a** 17% **b** 38% **c** 90% **d** 125% **e** 1%
4 **a** **B** **b** **C** **c** **D** **d** **A**
5 **a** Patrick's **b** Toby's **c** Katie's **d** Lily's

EXERCISE 8B.1

1 **a** 70% **b** 36% **c** 55% **d** 50% **e** 40%
f 75% **g** 66% **h** 20.5% **i** 34.1% **j** 70.9%
2 **a** 22.5% **b** 37.5% **c** 8.75% **d** 8.4% **e** $33\frac{1}{3}\%$
3 **a** 16.7% **b** 71.4% **c** 44.4% **d** 76.9% **e** 48.6%
4 **a** $\frac{9}{20}$ **b** 25% **c** 70%
5 **a** $\frac{5}{8}$ **b** 62.5% **c** 37.5%

EXERCISE 8B.2

1 **a** $\frac{3}{4}$ **b** $\frac{3}{50}$ **c** $\frac{3}{20}$ **d** $\frac{1}{5}$ **e** 1 **f** $\frac{11}{20}$
g $\frac{3}{2}$ **h** $\frac{2}{25}$ **i** $\frac{22}{25}$ **j** 7 **k** $\frac{31}{50}$ **l** $\frac{49}{20}$
2 **a** $\frac{21}{25}$ right-handed **b** $\frac{4}{25}$ left-handed

EXERCISE 8B.3

1 **a** 38% **b** 93% **c** 15% **d** 31.7%
e 54.6% **f** 80.2% **g** 7% **h** 158%
2 **a** 90% **b** 0.4% **c** 5.9% **d** 40.73%
e 160% **f** 420% **g** 300% **h** 0.26%

EXERCISE 8B.4

1 **a** 0.89 **b** 0.6 **c** 0.18 **d** 0.08
e 0.495 **f** 1.25 **g** 2 **h** 0.3801
i 0.375 **j** 0.000 02 **k** 1.298 **l** 0.777

2

Percentage	*Fraction*	*Decimal*
100%	1	1
75%	$\frac{3}{4}$	0.75
50%	$\frac{1}{2}$	0.5
25%	$\frac{1}{4}$	0.25
20%	$\frac{1}{5}$	0.2
10%	$\frac{1}{10}$	0.1
5%	$\frac{1}{20}$	0.05
1%	$\frac{1}{100}$	0.01
$33\frac{1}{3}\%$	$\frac{1}{3}$	0.333 ....
$66\frac{2}{3}\%$	$\frac{2}{3}$	0.666 ....

EXERCISE 8C

1 **a** 40% **b** 65% **c** 10% **d** 50% **e** 40%
f 75% **g** 90% **h** 80% **i** 20% **j** 40%
k 45% **l** 40% **m** 25% **n** 50% **o** 20%
p 20% **q** $16\frac{2}{3}\%$ **r** $66\frac{2}{3}\%$
2 **a** 86% **b** 47% **c** 40% **d** 62.5% **e** 75%
3 **a** 70% **b** 22% **c** 6% **d** 42% **e** 55% **f** 75%
4 **a** 24% **b** 20% **c** 44%
5 yes (average is $55\frac{5}{9}\%$) **6** 86%
7 **a**

	Forest area (km^2)	*Land area* (km^2)	*Forest as % of land area*
Bangladesh	14 394	130 170	11.1%
Colombia	603 980	1 109 500	54.4%
Finland	221 570	303 890	72.9%
Indonesia	937 470	1 811 570	51.7%
Madagascar	124 960	581 540	21.5%
Niger	11 916	1 266 700	0.9%
Philippines	77 198	298 170	25.9%
Spain	183 493	498 800	36.8%

b Finland

EXERCISE 8D

1 **a** 9 **b** 7 **c** 9 **d** £1512 **e** $750 **f** 4.75 tonnes
g 3.8 m **h** 5600 mL **i** 9 minutes **j** 290 kg
2 7 sweets **3** 132 seeds **4** 54 laps
5 **a** 14 students **b** 13 students **c** 17 students
d 19 students
6 3.15 kg **7** **a** 180 mL **b** 1.2 L
8 **a** 12 tonnes **b** 108 tonnes **c** $214 620

EXERCISE 8E.1

1 **a** 176 kg **b** 9 km **c** $29 925 **d** 390 mL
e 205 L **f** £369
2 **a** 13 students **b** 78 students

EXERCISE 8E.2

1 **a** 1.05 **b** 0.94 **c** 1.12 **d** 0.75 **e** 0.51 **f** 1.34
2 **a** €840 **b** 133.4 kg **c** 672 m **d** 1320 L
e 139.52 km **f** $578 000
3 6160 loaves **4** £265 per week **5** $6.05
6 Multipliers are 1.1 for 10% increase and 0.9 for 10% decrease.
$x \times 1.1 \times 0.9 = x \times 0.99$. This represents a 1% decrease.
7 **a** 1080 tonnes **b** **i** 2011 **ii** 2012 **8** ≈ 38.6%

EXERCISE 8F

1 **a** a 36 cm increase **b** a 19 kg decrease
c a £55 increase **d** a 100 mL decrease
e a 3 minute decrease **f** a $7 increase
2 **a** 20% increase **b** 60% increase **c** 16% decrease
d 25% decrease **e** 25% decrease **f** 50% increase
3 30% decrease **4** 8% decrease **5** ≈ 55.9% increase

EXERCISE 8G.1

1

	Profit or loss?	How much profit or loss?
a	profit	$30
b	loss	£65
c	profit	€150
d	profit	¥6500

2

	Cost price	Selling price	Profit or loss?
a	€56	€76	€20 profit
b	$420	$345	$75 loss
c	£385	£580	£195 profit
d	€265	€200	€65 loss

3 **a** **i** profit of $20 **ii** ≈ 44.4% profit
b **i** loss of £1150 **ii** ≈ 16.4% loss
c **i** loss of €23 **ii** ≈ 60.5% loss
d **i** profit of $60 **ii** ≈ 31.6% profit
e **i** profit of £90 **ii** 37.5% profit
4 ≈ 26.9% **5** ≈ 64.7% **6** **a** £4500 **b** $33\frac{1}{3}\%$
7 €525 profit, ≈ 63.6%
8 **a** loss of $1.50 **b** ≈ 0.469% loss
9 **a** $780 **b** €360 **c** $2975 **d** ¥28 000
10 $2592 **11** £140

EXERCISE 8G.2

1 **a** $34 **b** £249 **2** $2254 **3** €158.40
4 **a** £8 **b** 16% **5** **a** $171 **b** $296 **c** $801

6

	Marked price	Discount	Selling price	Discount as a % of marked price
a	$160	$40	$120	25%
b	£500	£170	£330	34%
c	$2.40	36 cents	$2.04	15%
d	$4.15	75 cents	$3.40	≈ 18.1%
e	€252	€89	€163	≈ 35.3%

REVIEW SET 8A

1 D **2** **a** $\frac{29}{100}$ **b** $\frac{37}{50}$ **c** $\frac{9}{20}$ **d** $\frac{19}{10}$
3 **a** 56% **b** 23.9% **c** 260% **d** 0.71%
4 **a** 54% **b** 25% **c** 40.9% **5** 65%
6 **a** 7% **b** 536 students **7** 12 150 spectators
8 **a** 1.45 **b** 0.25 **c** 1.09
9 **a** €162 loss **b** 27% loss **10** $78

REVIEW SET 8B

1 **a** B **b** C **c** A
2 **a** 68% **b** 95% **c** 27.5% **d** 12.5%
3 **a** 0.47 **b** 0.06 **c** 0.927 **d** 1.65
4 **a** 98 m **b** 115.6 kg **5** No, only 62.5% are present.
6 **a** 5% **b** **i** 1.75 L **ii** 1.25 L **c** 700%
7 **a** $625 **b** 36 km
8 **a** 48% **b** **i** 16% increase **ii** ≈ 24.1%
9 18% **10** $60

EXERCISE 9A

1 **a** true **b** false **c** true **d** false **e** false **f** true
2 **a** true **b** false **c** false **d** false **e** true **f** true
3 **a** false **b** true **c** false

EXERCISE 9B

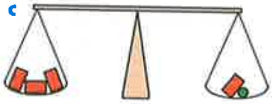
1 **a** $x = 6$ **b** $x = 12$ **c** $x = 6$ **d** $x = 9$
e $x = 5$ **f** $x = 7$
2 **a** $x = 7$ **b** $x = 9$ **c** $x = 11$ **d** $x = 7$
e $x = 7$ **f** $x = 8$
3 **a** $x = 3$ is too low **b** $x = 12$ is too high **c** $x = 9$

EXERCISE 9C.1

1 **a**

b

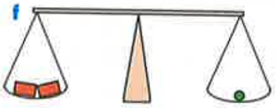
c



d

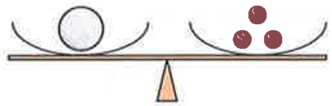
e

f



2 **a** 3 strawberries must be taken from the right side.
b 8 strawberries
c 1 banana is equal to 4 strawberries in mass.
3 **a** 6 marbles must be taken from the right side.
b 1 golf ball must be removed from the left side.

c



d 1 golf ball is equal to 3 marbles in mass.

4 **a** 3○ = □ **b** ◠ = ○ **c** 4⬠ = ☆
d ▱ = 2○ **e** ▯ = ▱ + ○ **f** △ = ○
g ◔ = ⬭ **h** △ = 3○ + 3□ **i** ◔ = ☆

5 **a** ■ = 6 **b** ✚ = 15 **c** ✦ = 2 **d** ▲ = 24

EXERCISE 9C.2

1 **a** $x+3=7$ **b** $x+5=9$ **c** $x=11$
d $2x=14$ **e** $7-x=8$ **f** $8-2x=6$

2 **a** $x+1=9$ **b** $x=3$ **c** $-x=9$
d $2x=-2$ **e** $5x=-2$ **f** $2x-4=2$

3 **a** $2x=4$ **b** $8x=12$ **c** $x=18$
d $2(x+1)=6$ **e** $x-1=6$ **f** $2-x=-6$

4 **a** $x=2$ **b** $x+1=4$ **c** $x+3=2$
d $x-1=2$ **e** $x=\frac{8}{5}$ **f** $x-2=6$
g $x=-7$ **h** $1-x=6$

EXERCISE 9D

1 **a** -4 **b** $\div 2$ **c** $\times 3$ **d** $+1$ **e** $+\frac{1}{3}$ **f** $\times \frac{1}{2}$
g $-\frac{3}{4}$ **h** $\div 7$ **i** $+11$

2 **a** x **b** x **c** x **d** x **e** x **f** x
g x **h** x **i** $3x$

3 **a** $x=7$ **b** $x=9$ **c** $x=11$ **d** $x=-5$
e $x=-3$ **f** $x=-10$ **g** $x=8$ **h** $x=9$
i $x=-4$ **j** $x=-17$ **k** $x=0.2$ **l** $x=\frac{2}{3}$

4 **a** $y=9$ **b** $y=25$ **c** $y=7$ **d** $y=10$
e $y=-7$ **f** $y=0$ **g** $y=33$ **h** $y=3.9$
i $y=4\frac{1}{2}$

5 **a** $a=5$ **b** $a=4$ **c** $a=8$ **d** $a=4$
e $a=-4$ **f** $a=5$ **g** $a=-7$ **h** $a=8$
i $a=\frac{7}{10}$

6 **a** $x=6$ **b** $x=12$ **c** $x=28$ **d** $x=-5$
e $x=54$ **f** $x=-22$ **g** $x=12$ **h** $x=-10$
i $x=18$

7 **a** $a=8$ **b** $b=13$ **c** $c=6$ **d** $d=44$
e $e=-7$ **f** $f=-5$ **g** $g=41$ **h** $h=21$
i $i=-8$ **j** $j=-7$ **k** $k=1\frac{1}{2}$ **l** $l=-2$
m $m=-4$ **n** $n=8$ **o** $\square=-5$ **p** $p=0.5$
q $q=-50$ **r** $r=5$ **s** $s=\frac{2}{5}$ **t** $t=-28$
u $u=\frac{5}{9}$

EXERCISE 9E

1 **a** $\boxed{x} \xrightarrow{\times 2} \boxed{2x} \xrightarrow{-3} \boxed{2x-3}$

b $\boxed{x} \xrightarrow{+1} \boxed{x+1} \xrightarrow{\times 4} \boxed{4(x+1)}$

c $\boxed{x} \xrightarrow{-2} \boxed{x-2} \xrightarrow{\div 3} \boxed{\frac{x-2}{3}}$

d $\boxed{x} \xrightarrow{\div 2} \boxed{\frac{x}{2}} \xrightarrow{+5} \boxed{\frac{x}{2}+5}$

2 **a** $\boxed{x} \xrightarrow{\times 3} \boxed{3x} \xrightarrow{+2} \boxed{3x+2}$

b $\boxed{x} \xrightarrow{\div 3} \boxed{\frac{x}{3}} \xrightarrow{-1} \boxed{\frac{x}{3}-1}$

c $\boxed{x} \xrightarrow{-7} \boxed{x-7} \xrightarrow{\div 2} \boxed{\frac{x-7}{2}}$

d $\boxed{x} \xrightarrow{+4} \boxed{x+4} \xrightarrow{\times 2} \boxed{2(x+4)}$

3 **a** Build up: $\boxed{x} \xrightarrow{\times 2} \boxed{2x} \xrightarrow{+4} \boxed{2x+4}$
Undoing: $\boxed{2x+4} \xrightarrow{-4} \boxed{2x} \xrightarrow{\div 2} \boxed{x}$

b Build up: $\boxed{x} \xrightarrow{\times 3} \boxed{3x} \xrightarrow{-1} \boxed{3x-1}$
Undoing: $\boxed{3x-1} \xrightarrow{+1} \boxed{3x} \xrightarrow{\div 3} \boxed{x}$

c Build up: $\boxed{x} \xrightarrow{\times 4} \boxed{4x} \xrightarrow{+3} \boxed{4x+3}$
Undoing: $\boxed{4x+3} \xrightarrow{-3} \boxed{4x} \xrightarrow{\div 4} \boxed{x}$

d Build up: $\boxed{x} \xrightarrow{\times 5} \boxed{5x} \xrightarrow{-12} \boxed{5x-12}$
Undoing: $\boxed{5x-12} \xrightarrow{+12} \boxed{5x} \xrightarrow{\div 5} \boxed{x}$

4 **a** Build up: $\boxed{x} \xrightarrow{\div 3} \boxed{\frac{x}{3}} \xrightarrow{-1} \boxed{\frac{x}{3}-1}$
Undoing: $\boxed{\frac{x}{3}-1} \xrightarrow{+1} \boxed{\frac{x}{3}} \xrightarrow{\times 3} \boxed{x}$

b Build up: $\boxed{x} \xrightarrow{-1} \boxed{x-1} \xrightarrow{\div 3} \boxed{\frac{x-1}{3}}$
Undoing: $\boxed{\frac{x-1}{3}} \xrightarrow{\times 3} \boxed{x-1} \xrightarrow{+1} \boxed{x}$

c Build up: $\boxed{x} \xrightarrow{+5} \boxed{x+5} \xrightarrow{\div 3} \boxed{\frac{x+5}{3}}$
Undoing: $\boxed{\frac{x+5}{3}} \xrightarrow{\times 3} \boxed{x+5} \xrightarrow{-5} \boxed{x}$

d Build up: $\boxed{x} \xrightarrow{\div 3} \boxed{\frac{x}{3}} \xrightarrow{+5} \boxed{\frac{x}{3}+5}$
Undoing: $\boxed{\frac{x}{3}+5} \xrightarrow{-5} \boxed{\frac{x}{3}} \xrightarrow{\times 3} \boxed{x}$

e Build up: $\boxed{x} \xrightarrow{\times 3} \boxed{3x} \xrightarrow{+8} \boxed{3x+8}$
Undoing: $\boxed{3x+8} \xrightarrow{-8} \boxed{3x} \xrightarrow{\div 3} \boxed{x}$

f Build up: $\boxed{x} \xrightarrow{+8} \boxed{x+8} \xrightarrow{\times 3} \boxed{3(x+8)}$
Undoing: $\boxed{3(x+8)} \xrightarrow{\div 3} \boxed{x+8} \xrightarrow{-8} \boxed{x}$

g Build up: $\boxed{x} \xrightarrow{-6} \boxed{x-6} \xrightarrow{\times 2} \boxed{2(x-6)}$
Undoing: $\boxed{2(x-6)} \xrightarrow{\div 2} \boxed{x-6} \xrightarrow{+6} \boxed{x}$

h Build up: $\boxed{x} \xrightarrow{\times 2} \boxed{2x} \xrightarrow{-6} \boxed{2x-6}$

Undoing: $\boxed{2x-6} \xrightarrow{+6} \boxed{2x} \xrightarrow{\div 2} \boxed{x}$

EXERCISE 9F

1 **a** $x = 5$ **b** $x = 1$ **c** $x = 3$ **d** $x = 4$
e $x = -3$ **f** $x = \frac{1}{3}$ **g** $x = 2$ **h** $x = \frac{5}{6}$
i $x = 0$ **j** $x = 4$ **k** $x = \frac{1}{7}$ **l** $x = \frac{1}{2}$

2 **a** $x = 4$ **b** $x = 21$ **c** $x = -30$ **d** $x = -20$
e $x = 0$ **f** $x = -80$

3 **a** $x = 12$ **b** $x = 3$ **c** $x = 1$ **d** $x = 32$
e $x = 2$ **f** $x = -6$ **g** $x = 1$ **h** $x = 0$

4 **a** $x = 5$ **b** $x = 4$ **c** $x = 8$ **d** $x = -3$
e $x = 1$ **f** $x = 4$ **g** $x = 5$ **h** $x = -5$
i $x = 2$ **j** $x = 4$ **k** $x = -2$ **l** $x = 2\frac{2}{3}$

5 **a** $x = 3$ **b** $x = 10$ **c** $x = 5$ **d** $x = 23$
e $x = -8$ **f** $x = 6$ **g** $x = -9$ **h** $x = -4$
i $x = -7$ **j** $x = 5$ **k** $x = -14$ **l** $x = 0$
m $x = -4$ **n** $x = -5$ **o** $x = -5$ **p** $x = -5$
q $x = -10$ **r** $x = 40$

EXERCISE 9G

1 **a** $x = 6$ **b** $x = 12$ **c** $x = -9$ **d** $x = 8$
e $x = \frac{1}{2}$ **f** $x = 3$

2 **a** $x = 70$ **b** $x = 80$ **c** $x = 40$ **d** $x = 50$

EXERCISE 9H.1

1 **a** $5n = 30$ **b** $n + 10 = 23$ **c** $\dfrac{n}{4} + 6 = 8$
d $\dfrac{11 - n}{3} = 2$

2 **a** $x + 12 = 27$ **b** $x - 150 = 80$ **c** $\dfrac{x}{3} = 12$
d $2x + 10 = 31$

EXERCISE 9H.2

1 The number is 6. **2** The number is 5. **3** The number is 10.
4 24 chocolates **5** 9 singers **6** \$5 **7** 32 cars
8 5 balloons **9** 4 boxes **10** £1200

REVIEW SET 9A

1 **a** true **b** false **c** true

2 $x = 7$ **3** **a** ▲ = 2◎ **b** □ = 2▲ + ◎

4 **a** $+\,7$ **b** $\times\,8$ **c** $-\,13$ **d** $\div\,\frac{1}{2}$

5 **a** $x = 13$ **b** $x = 10$ **c** $x = -5$ **d** $x = 24$

6 **a** $\boxed{x} \xrightarrow{\times 2} \boxed{2x} \xrightarrow{-9} \boxed{2x-9}$

b $\boxed{x} \xrightarrow{\div 3} \boxed{\dfrac{x}{3}} \xrightarrow{+4} \boxed{\dfrac{x}{3}+4}$

7 **a** $\boxed{x} \xrightarrow{-4} \boxed{x-4} \xrightarrow{\times 3} \boxed{3(x-4)}$

b $\boxed{x} \xrightarrow{+2} \boxed{x+2} \xrightarrow{\div 3} \boxed{\dfrac{x+2}{3}}$

8 **a** $x = 2$ **b** $x = -13$ **c** $x = 27$ **d** $x = -10$
e $x = 4$ **f** $x = 22$

9 €6 **10** 36 lollies

REVIEW SET 9B

1 **a** true **b** false

2 **a** $4x = 16$ **b** $x - 2 = 5$ **3** 🟩 = 18

4 **a** $x = 4$ **b** $x = 15$ **c** $x = -6$ **d** $x = -12$

5 The number is 36.

6 **a** $\boxed{x} \xrightarrow{\times 4} \boxed{4x} \xrightarrow{-7} \boxed{4x-7}$

b $\boxed{x} \xrightarrow{+9} \boxed{x+9} \xrightarrow{\times 2} \boxed{2(x+9)}$

7 **a** $\boxed{\dfrac{x-9}{4}} \xrightarrow{\times 4} \boxed{x-9} \xrightarrow{+9} \boxed{x}$

b $\boxed{2(x+5)} \xrightarrow{\div 2} \boxed{x+5} \xrightarrow{-5} \boxed{x}$

8 **a** $x = 6$ **b** $x = 2$ **c** $x = 12$ **d** $x = -11$
e $x = -18$ **f** $x = -10$

9 10 friends **10** **a** $x = 9$ **b** $x = 25$

EXERCISE 10A

1 **a** polygon
b not a polygon as a side is curved, crossed, and not closed
c polygon **d** polygon
e not a polygon as sides are not straight **f** polygon
g not a polygon as sides cross over **h** polygon
i not a polygon as sides are not straight **j** polygon
k polygon **l** not a polygon as figure is not closed

2 **a** triangle **b** quadrilateral **c** hexagon
d heptagon **e** octagon **f** nonagon

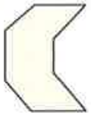
3 **a** convex quadrilateral **b** convex triangle
c non-convex decagon **d** convex pentagon
e non-convex quadrilateral **f** non-convex decagon
g non-convex heptagon **h** convex nonagon

4 **a**

b

c

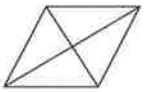
d



Note: There may be other answers.

5 **a** all angles not equal **b** all angles not equal
c all sides not equal in length **d** all sides not equal in length

6 **a**



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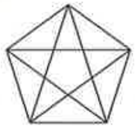
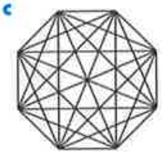


c



EXERCISE 10B

1 **a** scalene **b** isosceles **c** equilateral **d** isosceles

2 **a** obtuse **b** right angled **c** acute **d** acute

3 **a** right angled, scalene **b** acute, isosceles
c acute, equilateral **d** right angled, isosceles

EXERCISE 10C

1 **a** $x = 79$ **b** $x = 21$ **c** $x = 54$ **d** $x = 73$
e $x = 38$ **f** $x = 41$

2 **a** $a = 137$ **b** $a = 48$ **c** $a = 61$ **d** $a = 99$
e $a = 120$ **f** $a = 30$

3 **a** $x = 40$ {angle sum of triangle}
b $c = 65$ {exterior angle of triangle}
c $t = 50$ {angle sum of triangle}
d $x = 30$ {angle sum of triangle}
e $a = 28$ {exterior angle of triangle}
f $n = 60$ {angle sum of triangle}

4 **a** The sum of two obtuse angles is greater than 180°.
b The sum of a right angle and an obtuse angle is greater than 180°.
c The sum of three angles less than 60° is less than 180°.

5 **a** $x = 30$ {angle sum of triangle}
b $x = 36$ {angle sum of triangle}
c $x = 55$ {angle sum of triangle}
d $x = 50$ {angle sum of triangle}
e $x = 65$ {angle sum of triangle}
f $x = 63$ {angle sum of triangle}

6 $Q\widehat{A}C = c^\circ$ {equal alternate angles}
$P\widehat{A}B = b^\circ$ {equal alternate angles}
But $P\widehat{A}B + B\widehat{A}C + Q\widehat{A}C = 180^\circ$ {angles on a line}
So, $a + b + c = 180$

7 **a** $a = 18$ {angle sum of triangle}
$b = 108$ {angles on a line}
b $a = 36$ {angle sum of triangle}
$b = 36$ {vertically opposite angles}
$c = 64$ {angle sum of triangle}
c $a = 65$ {exterior angle of triangle}
$b = 45$ {angle sum of triangle}

EXERCISE 10D

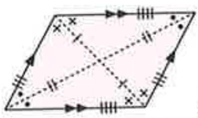
1 **a** $x = 75$ **b** $x = 64$ **c** $x = 75.5$ **d** $p = 90$
e $x = 5$ **f** $x = 80$, $y = 6$ **g** $q = 108$
h $r = 48$ **i** $x = 135$ **j** $x = 70$ **k** $x = 25$
l $y = 30$

2 **a** $x = 37$, $\triangle ABC$ is isosceles with $AC = BC$
b $x = 45$, $\triangle KLM$ is isosceles with $KL = LM$
c $x = 20$, $\triangle PQR$ is isosceles with $PR = QR$

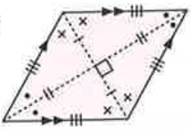
3 **a** **F** **b** **D** **c** **A** **d** **E** **e** **B** **f** **G** **g** **C**

EXERCISE 10E

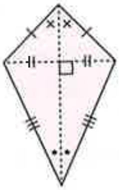
1 **a**



b



c

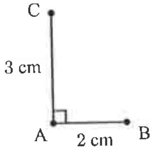


2 **a** rectangle **b** kite **c** rhombus
d square **e** trapezium **f** parallelogram

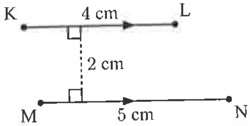
3 **a** true **b** true **c** true **d** true
e true **f** true **g** true **h** true

4 **a** $[AC] \perp [BD]$ **b** $[PQ] \perp [QR]$ **c** $[AB] \parallel [DC]$
d $[HI] \parallel [KJ]$, $[HI] \perp [IJ]$, $[KJ] \perp [JI]$
e $[KM] \perp [LN]$
f $[WX] \parallel [ZY]$, $[WZ] \parallel [XY]$, $[WZ] \perp [ZY]$, $[ZY] \perp [YX]$, $[YX] \perp [XW]$, $[XW] \perp [WZ]$

5 **a**



b



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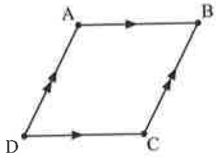
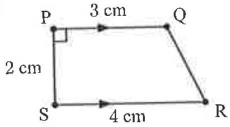


d



6 **a** $x = 5$ {equal sides}
$y = 90$ {opposite angles of a rhombus}
b $x = 130$ {opposite angles of a kite}
$y = 4$ {equal adjacent sides of a kite}
c $x = 2$ {diagonals of a rhombus bisect each other}
$y = 90$ {diagonals of a rhombus meet at 90°}
d $x = 6$ {diagonals of a rectangle are equal in length}
e $x = 30$ {diagonals of a rhombus bisect angles}
$y = 60$ {diagonals meet at 90°, angle sum of triangle}
f $x = 60$ {co-interior angles supplementary}
g $x = 10$ {opposite sides of a rectangle}
$y = 7$ {diagonals of a rectangle bisect each other}
h $x = 5$ {equal adjacent sides of a kite}
$y = 90$ {diagonals of a kite intersect at right angles}
i $x = 80$ {opposite angles of parallelogram}
$y = 100$ {opposite angles of parallelogram}

EXERCISE 10F

1 **a** $x = 122$ {angle sum of quadrilateral}
b $a = 100$ {angle sum of quadrilateral}
c $a = 70$ {angles on a line}
$b = 100$ {angle sum of quadrilateral}
d $a = 105$ {angle sum of quadrilateral}
$b = 75$ {angles on a line}
e $a = 98$ {angle sum of quadrilateral}
f $a = 76$ {angle sum of quadrilateral}
$b = 104$ {angles on a line}

2 **a** $a = 82$ {angle sum of quadrilateral}
b $a = 97$ {angle sum of quadrilateral}
$b = 83$ {angles on a line}
$c = 121$ {angles on a line}
c $a = 118$ {angles on a line}
$b = 92$ {angles on a line}
$c = 94$ {angle sum of quadrilateral}
$d = 86$ {angles on a line}

REVIEW SET 10A

1 **a** non-convex quadrilateral **b** convex hexagon
c non-convex octagon

2 **a** right angled **b** obtuse

3 **a** $x = 58$ {angle sum of triangle}
b $x = 55$ {exterior angle of triangle}
c $x = 137$ {exterior angle of triangle}
d $x = 68$ {base angles of isosceles triangle}
e $x = 69$ {exterior angle of triangle}
f $x = 115$ {opposite angles of a parallelogram}

4 **a** polygon **b** not a polygon as a side is not straight

c not a polygon as figure is not closed

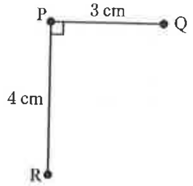
d not a polygon as sides cross over

5 **a** $x = 60$ {exterior angle of triangle}

b $x = 40$ {angle sum of triangle}

c $x = 70$ {vertically opposite angles, angle sum of triangle}

6



7 **a** true **b** false

8 **a** right angled triangle {missing angle equals 90°, angle sum of triangle}

b parallelogram {opposite angles are equal}

c square {diagonals bisect each other at 90° and are equal in length}

9 **a** $a = 90$ **b** $b = 34$ **c** $c = 34$

10 **a** $x = 125$ {angle sum of quadrilateral}

b $x = 70$ {angles on a line}

$y = 65$ {angle sum of quadrilateral}

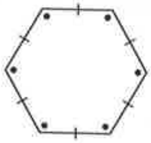
c $a = 80$ {angles on a line}

$b = 55$ {angles on a line}

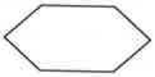
$c = 120$ {angle sum of quadrilateral}

REVIEW SET 10B

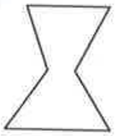
1 **a**



b



c



2 **a** isosceles **b** scalene

3 **a** $x = 75$ {angle sum of triangle}

b $b = 58$ {angle sum of triangle}

4 **a** $x = 30$ {angles of isosceles triangle}

b $x = 140$ {exterior angle of triangle}

5 $a = 60$ {angles on a line}

$b = 30$ {angle sum of triangle}

6 parallelogram {diagonals bisect each other}

7 $a = 65$ {angles on a line}

$b = 70$ {angle sum of triangle}

$c = 45$ {vertically opposite angles, angle sum of triangle}

8 $x = 90$ {diagonals of a kite intersect at right angles}

$y = 64$ {angle sum of triangle}

$z = 30$ {equal opposite angles of kite}

9 **a** parallelogram

b $x = 88$ {opposite angles of a parallelogram}

$y = 92$ {co-interior angles supplementary}

10 The sum of four acute angles is less than 360°.

EXERCISE 11A

1 **a** km **b** mm **c** m **d** cm **e** m **f** mm

2 **a** B **b** C **c** C

3 **a** 25 mm **b** 26 mm **c** 8 mm **d** 59 mm

e 5 mm **f** 18 mm

4 **a** 400 cm **b** 2 cm **c** 290 cm **d** 300 000 cm

5 **a** 8 m **b** 7000 m **c** 120 000 m **d** 32 m

6 **a** 90 mm **b** 3000 mm **c** 1200 mm **d** 450 000 mm

7 **a** 15 km **b** 7.5 km **c** 0.6 km **d** 2.5 km

8 **a** 60 mm **b** 7 m **c** 3 km **d** 8 cm

e 1100 cm **f** 4000 m **g** 32 mm **h** 2.4 m

i 3.8 km **j** 1.7 cm **k** 780 cm **l** 600 m

9 **a** 459.5 cm **b** 512.7 cm **c** 3432.2 m **d** 8926.5 m

10 40 000 pipes **11** 19.6 cm **12** 200 lengths **13** 10.8 km

EXERCISE 11B

1 **a** 10 units **b** 14 units **c** 18 units **d** 12 units

e 16 units **f** 26 units

2 **a** 112 mm **b** 125 mm **c** 89 mm

3 **a** 15 cm **b** 14 cm **c** 16 cm **d** 36 cm

e 56 m **f** 24.6 m

4 **a** 16 m **b** 44 cm **c** 12 m **d** 15 m

e 6.4 cm **f** 11.6 m

5 **a** 29 mm or 2.9 cm **b** 114 mm or 11.4 cm

c 1020 cm or 10.2 m **d** 4.4 m **e** 11.8 km

f 5.2 km

6 **a** 480 cm **b** 544 cm **7** \$13 770 **8** 540 cm

9 34.58 km **10** **a** **i** 26 cm **ii** 35 cm **b** 8.2 m

EXERCISE 11C.1

1 **a** 20 units^2 **b** 6 units^2 **c** 33 units^2 **d** 35 units^2

2 **a** m^2 **b** cm^2 **c** km^2 **d** mm^2 **e** ha

EXERCISE 11C.2

1 **a** 500 mm^2 **b** 25 cm^2 **c** 70 000 m^2

d 36 000 cm^2 **e** 40 ha **f** 8300 mm^2

g 800 000 m^2 **h** 1.56 m^2 **i** 12 km^2

j 9 cm^2 **k** 7.6 ha **l** 280 mm^2

m 25 ha **n** 124 800 cm^2 **o** 9200 mm^2

2 15 000 mm^2 **3** **a** Bruno **b** Carlos

4 **a** 57 000 m^2 **b** 3420 kg **c** €2719.45

EXERCISE 11D.1

1 **a** 40 cm^2 **b** 36 cm^2 **c** 225 m^2 **d** 220 m^2

2 **b** 21 cm × 29.7 cm ≈ 624 cm^2

3 **a** 2400 m^2 **b** 80 minutes

4 **a** 720 pavers **b** £3960

5 **a** 1440 ha **b** \$1 728 000

6 6 cm **7** 12 m **8** 4.8 m

EXERCISE 11D.2

1 **a** 18 cm^2 **b** 66 m^2 **c** 24 m^2 **d** 35 cm^2

e 54 m^2 **f** 7.5 cm^2 **g** 13 m^2 **h** 45 cm^2

2 36 m^2 **3** 42 m^2 **4** 6 cm

EXERCISE 11D.3

1 **a** 48 m^2 **b** 66 cm^2 **c** 80 cm^2 **d** 20 cm^2

e 24 cm^2 **f** 84 m^2

2 **a** 16 cm^2 **b** 63 m^2 **c** 25 cm^2 **d** 42 cm^2

e 13 m^2 **f** 42 cm^2

3 600 cm^2 **4** 22.62 cm^2

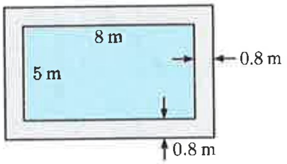
5 **a** 7040 cm^2 **b** £11.26 **6** 5 cm

EXERCISE 11E

1 **a** 90 cm^2 **b** 34 m^2 **c** 33 m^2 **d** 30 km^2
e 24 cm^2 **f** 50 m^2

2 **a** 41 cm^2 **b** 45 m^2

3 **a**



b 23.36 m^2
c \$747.52

4 **a** 400 cm^2 **b** 128 cm^2 **c** 144 cm^2

5 **a** 5600 cm^2 **b** 7200 cm^2 **c** 900 cm^2
d 76 300 cm^2 or 7.63 m^2

6 **a** 4 cm^2 **b** 0.5 cm^2 **c** 2 cm^2 **d** $\sqrt{2}$ cm

REVIEW SET 11A

1 **a** m^2 **b** km^2 **c** cm^2

2 **a** 129 mm **b** 3950 m **c** 243 cm
d 0.1459 m^2 **e** 94 000 m^2 **f** 1280 mm^2

3 **a** 28 m **b** 38 cm **c** 310 cm or 3.1 m

4 **a** 5.29 m^2 **b** 60 m^2 **c** 85.5 cm^2

5 2511.9 m **6** **a** 184 ha **b** 3680 trees

7 800 triangles **8** **a** 80 m^2 **b** 28 cm^2

9 **a** 72 m **b** 36 m^2 **c** 13 m^2

10 **a** **i** 350 cm^2 **ii** 175 cm^2
b 19×350 cm^2 $+$ 2×175 cm^2 $= 350$ cm $\times$ 20 cm ✓

REVIEW SET 11B

1 **a** m **b** cm **2** **a** 4.9 cm **b** 2.99 cm^2 **c** 684 ha

3 **a** 30 m **b** 21 cm **c** 44 cm

4 **a** 30 m^2 **b** 30 cm^2 **5** 14.6 m **6** \$570

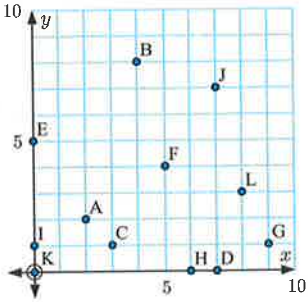
7 43.73 m^2 **8** **a** 128 cm^2 **b** 8 m^2

9 **a** 1500 cm^2 **b** **i** 495 cm^2 **ii** 66.9 cm^2
c 376.2 cm^2

10 **a** 6 km **b** 300 km **c** 1.2 km^2

EXERCISE 12A

1



2 **a** The y-coordinate of a point on the x-axis is 0.
b The x-coordinate of a point on the y-axis is 0.

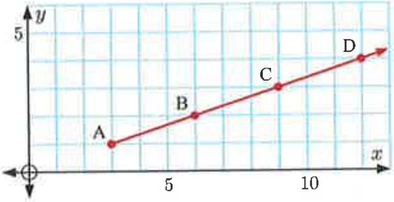
3 **a** **i** 6 **ii** 2 **iii** 9 **iv** 6
b **i** 2 **ii** 1 **iii** 5 **iv** 6
c A(2, 8), B(6, 4), C(9, 9), D(2, 3), E(8, 2), F(5, 7), G(6, 6), H(4, 1), I(9, 5), J(2, 5), K(0, 6)
d O(0, 0)

4 **a** C and G. They lie on the same vertical line.
b E and F. They lie on the same horizontal line. **c** E(7, 7)

5 D(2, 3)

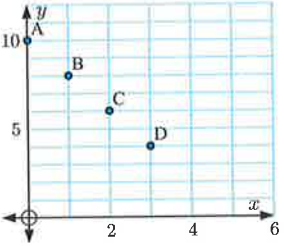
6 **a** **i** (7, 5) **ii** (9, 2) **iii** (7, 7) **iv** (2, 1) and (6, 6)
b **i** Treasure Trove **ii** Lion's Den **iii** Mt. Ogre
iv Oasis

7 **a, b**



The points lie in a straight line.
c E(15, 5)

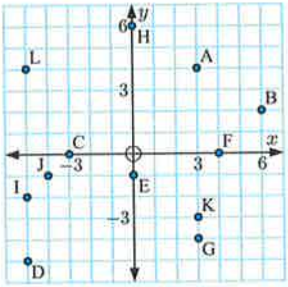
8 **a**



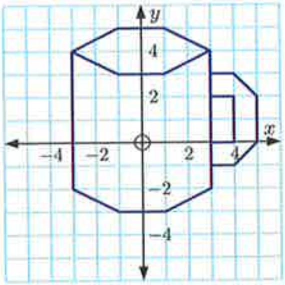
b E(4, 2) and F(5, 0)

EXERCISE 12B

1



2



3 **a** **i** 0 **ii** 3 **iii** −5 **iv** 6
b **i** 1 **ii** −5 **iii** 0 **iv** 4
c A(−1, 1), B(3, 1), C(−2, −5), D(0, 4), E(1, −4), F(−3, 0), G(6, −2), H(−5, −3), I(4, 4), J(−5, 2)
d **i** B, I **ii** A, J **iii** C, H **iv** E, G **v** F **vi** D

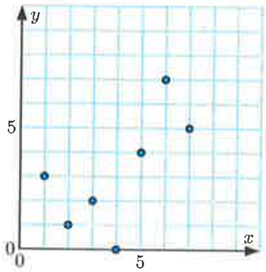
4 **a** **i** (2, 1) **ii** (−3, −2) **iii** (−1, 4) **iv** (4, −3)
v (1, 5) **vi** (−4, 2) **vii** (5, −1) **viii** (−2, −5)
b **i** dog, house **ii** flower garden, carrot patch
iii tree, toolshed **iv** car, letterbox

5 **a** first quadrant **b** third quadrant
c second quadrant **d** fourth quadrant

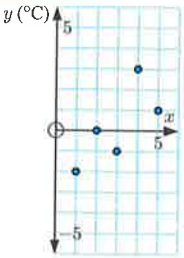
6 **a** first **b** fourth **c** third **d** second **e** fourth **f** fourth **g** third **h** second

EXERCISE 12C

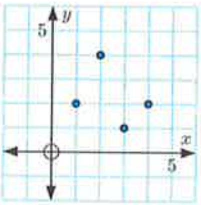
1



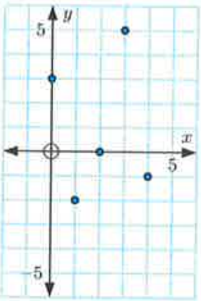
2



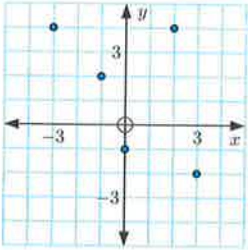
3 **a**



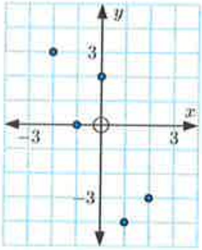
b



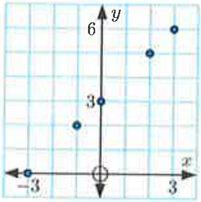
c



d



4 **a**



b The points lie in a straight line.

c Yes, each y-value is 3 more than its corresponding x-value.

EXERCISE 12D.1

1 **a** $y = x + 5$ **b** $y = x - 7$ **c** $y = 3x$ **d** $y = \frac{1}{2}x$

2 **Note:** Other answers are possible.

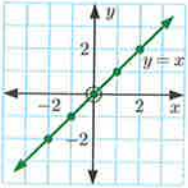
a (1, 6) **b** (10, 3) **c** (1, 3) **d** (2, 1)

3 10 **4** −15 **5** P(−2, −3), Q(3, 7)

EXERCISE 12D.2

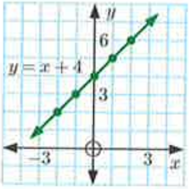
1 **a**

x	−2	−1	0	1	2
y	−2	−1	0	1	2



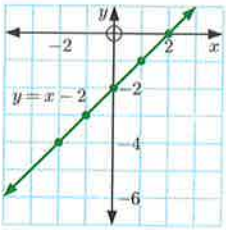
b

x	−2	−1	0	1	2
y	2	3	4	5	6

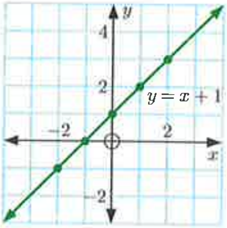


c

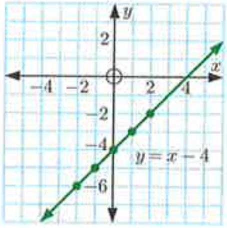
x	−2	−1	0	1	2
y	−4	−3	−2	−1	0



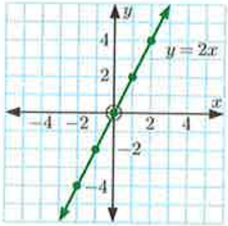
2 **a**



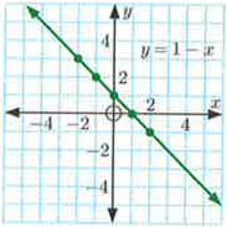
b



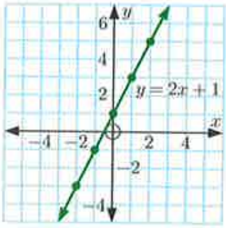
c



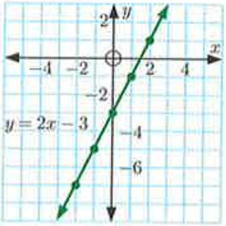
d



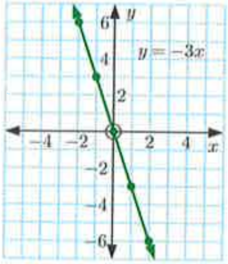
e



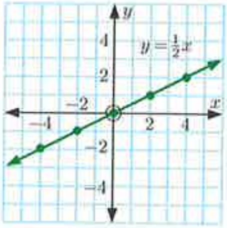
f



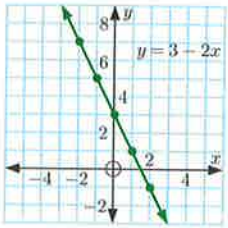
g



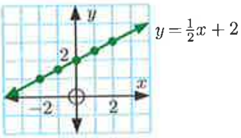
h



i



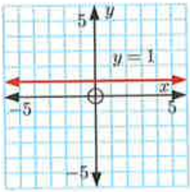
3 **a**



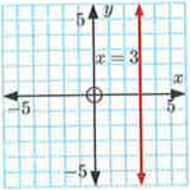
b **i** (0, 2) **ii** (−4, 0)

EXERCISE 12E

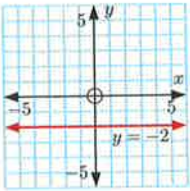
1 **a**



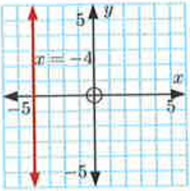
b



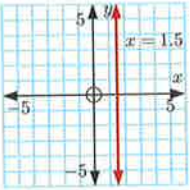
c



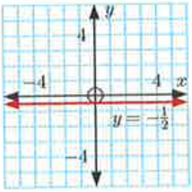
d



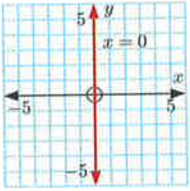
e



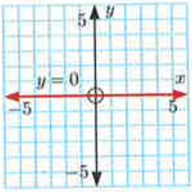
f



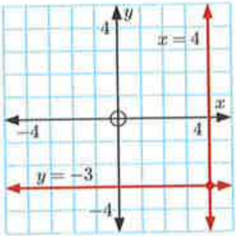
g



h



2 **a**

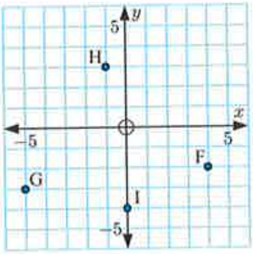


b (4, −3)

REVIEW SET 12A

1 **a** V(−2, −1) **b** W(4, 2) **c** X(4, −2)
d Y(0, 3) **e** Z(−4, 3)

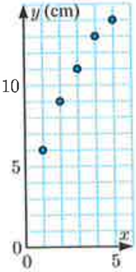
2



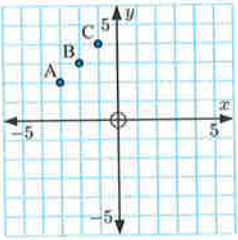
3 **a** x-coordinate of A is 3, x-coordinate of D is −1
b y-coordinate of B is −2, y-coordinate of C is 4
c A(3, −2), B(0, −2), E(3, 0), F(−2, 1)

4 **a** second **b** third **c** on the negative y-axis **d** first

5

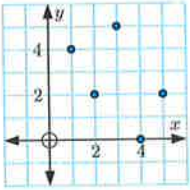


6 **a**

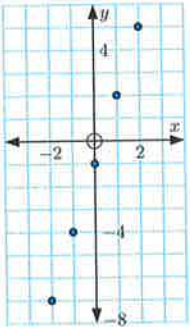


b D(0, 5)

7 **a**



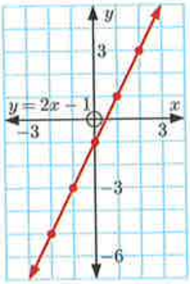
b



8 **a** $y=x-3$ **b** $y=2x$

9

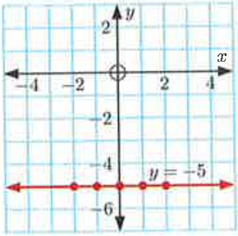
x	−2	−1	0	1	2
y	−5	−3	−1	1	3



10 **a**

x	−2	−1	0	1	2
y	−5	−5	−5	−5	−5

b

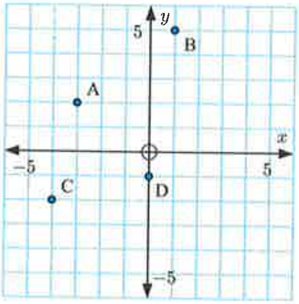


REVIEW SET 12B

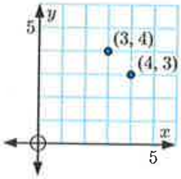
1 **a** E **b** F **c** A **d** B **e** C **f** D

2 third **3** **a** 2 **b** 3 **c** A(3, 2), B(−2, −4)

4

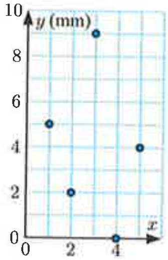


5 No, they are different points.



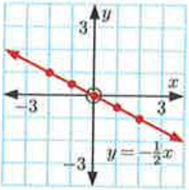
6 **a** $y = x - 2$ **b** (0, −2)

7

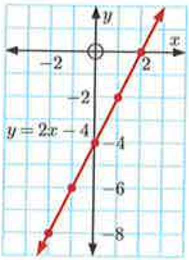


8 −8

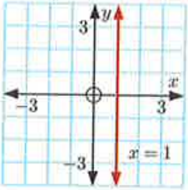
9 **a**



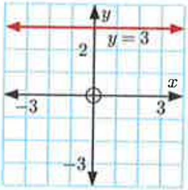
b



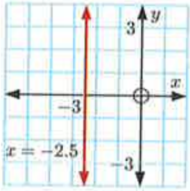
10 **a**



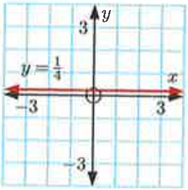
b



c



d



EXERCISE 13A.1

1 **a** 16 units^3 **b** 54 units^3 **c** 36 units^3

2 **a** **b** has the greatest volume. **b** **a** has the least volume.

3 **a** cm^3 **b** cm^3 **c** m^3 **d** mm^3 **e** cm^3
f cm^3 **g** m^3 **h** cm^3 **i** mm^3

EXERCISE 13A.2

1 **a** 48 000 mm^3 **b** 0.029 m^3 **c** 1 200 000 cm^3
d 12.485 cm^3 **e** 450 cm^3 **f** 0.0145 m^3
g 295 mm^3 **h** 0.001 43 cm^3 **i** 5 600 000 mm^3

2 1.5 m^3 **3** 235 000 cm^3 **4** 200 000 pieces

EXERCISE 13B.1

1 **a** 16 units^3 **b** 45 units^3 **c** 36 units^3

2 **a** 720 mm^3 **b** 125 cm^3 **c** 81 m^3

3 **a** 29 600 mm^3 **b** 20.832 m^3

4 **B**, by 16 cm^3 **5** 12 m^3 **6** 1.44 m^3 **7** 4 cm

EXERCISE 13B.2

1 **a** 112 m^3 **b** 780 cm^3 **c** 44 cm^3 **d** 192 mm^3
e 119 m^3 **f** 6.4 cm^3

2 168 m^3 **3** 8 cm

4 **a** 36 m^3 **b** 42 cm^3 **c** 180 cm^3 **d** 135 m^3
e 144 m^3 **f** 480 000 cm^3

5 32.5 cm^3 **6** $271.44

7 **a** 4800 cm^3 **b** 2400 cm^3 **c** $\frac{1}{2}$

EXERCISE 13C

1 **C** **2** **D** **3** **A** **4** **C**

5 **a** 8 L **b** 2000 kL **c** 0.786 kL **d** 0.04 L
e 3950 L **f** 1 000 000 000 mL

6 64 bottles **7** 4.96 kL

EXERCISE 13D

1 **a** 7500 cm^3 **b** 7500 mL **c** 7.5 L

2 12 kL **3** 2.4 L **4** 72 L

5 **a** 1125 cm^2 **b** 337 500 cm^3
c **i** 337.5 L **ii** 0.3375 kL

6 5 cm

EXERCISE 13E

1 **a** g **b** kg **c** g **d** mg **e** t **f** kg

2 **a** 3000 g **b** 6.79 g **c** 0.029 g **d** 540 g
e 1 200 000 g **f** 5.249 g **g** 10 370 g **h** 250 000 g

3 **a** 6000 kg **b** 4 kg **c** 0.35 kg
d 0.4 kg **e** 2400 kg **f** 0.285 kg
g 0.001 436 kg **h** 0.000 05 kg

4 504 g **5** 2000 lollipops **6** 4000 bricks **7** 1200 kg

8 19 kg **9** **a** 80 g **b** 0.997 92 m^2 ≈ 1 m^2 **c** ≈ 5.00 g

EXERCISE 13F

1 6 g **2** 4 kg **3** 3.45 kg

4 **a** 9.6 L **b** 9.6 kg **c** 8.4 kg

EXERCISE 13G

1 **a** 1020 min **b** 23 min **c** 4320 min
d 268 min **e** 7893 min

2 **a** $3287\frac{1}{4}$ days **b** 6 days **c** 48 days **d** 3 days

3 **a** 21 600 s **b** 780 s **c** 44 160 s **d** 4 838 400 s

4 1 h 20 min **5** 134 min **6** ≈ 57 days **7** 2 h 7 min

EXERCISE 13H

1 **a** 4 h 25 min **b** 7 h 16 min **c** 2 h 48 min
d 3 h 46 min **e** 9 h 3 min **f** 1 day 2 h 20 min

2 1 h 42 min **3** 2 h 23 min

4 **a** 8 h 55 min **b** 52 min

5 **a** 37 min **b** 33 min **c** 24 min

6 **a** Deborah **b** Terry
c 1 h 15 min, from 2:15 until 3:30 **d** no **e** £337.50

7 **a** 8:12 pm **b** 4:09 pm **c** 12:30 pm **d** 5:10 pm
e 8:05 am **f** 4:30 am the next day

8 1:45 pm **9** 4:45 pm

10 **a** Otter Odyssey
b 3D Underwater World
c 2:05 pm
d Whale Mania, and 3D Underwater World
e 9:45 am
f 2 h 25 min
g **i** 2 h 55 min
ii

Show	*Time*
Diving Dolphins	1:40
Whale Mania	3:15
Seal of Approval	11:30
Otter Odyssey	10:00
3D Underwater World	12:30
Marine Park Parade	4:00

EXERCISE 13I

1 **a** 7 am **b** 2 pm **c** 8 pm **d** 8 pm

2 **a** 11 am Wednesday **b** 3 am Wednesday
c 8 am Wednesday **d** 7 pm Tuesday

3 **a** 11 pm Thursday **b** 8 am Friday
c 10 am Friday **d** 3 pm Friday

4 5 pm **5** **a** 3 pm **b** 8 pm **c** 4 am the next day

6 10:45 pm **7** 8 am the next day **8** 8 hours

REVIEW SET 13A

1 **a** 5.86 L **b** 6300 min **c** 0.046 kL
d 2500 kg **e** 2 360 000 cm^3 **f** 7.02 L

2 **a** 36 units^3 **b** 93.6 cm^3 **c** 108 cm^3

3 €4438.72 **4** 4500 kg

5 **a** 0.4125 kL **b** 412.5 kg **6** 30 river crossings

7 **a** 1:20 pm **b** 3:46 pm **c** 4:37 pm **d** 8:15 am

8 15.3 cm^3

9 **a** 1 L **b** 1 h 40 min **c** after 4:10 pm

10 **a** 11 am **b** 2 am

REVIEW SET 13B

1 **a** 463 min **b** 32.7 cm^3 **c** 5.7 g
d 3900 mL **e** 50 days **f** 1 200 000 cm^3

2 100 000 dominoes **3** 17 h 4 min

4 **a** 435.6 cm^3 **b** 4368 cm^3 **5** 16.25 L

6 480 L **7** 8:21 am **8** **a** 7 kg **b** 7.95 kg

9 6:30 am

10 **a** **i** 10 hours **ii** $7\frac{1}{2}$ hours **b** Thursday
c 65 hours

EXERCISE 14A

1 **a** 4 : 5 **b** 15 : 8 **c** 1 : 4 **d** 8 : 7
e 9 : 5 **f** 2 : 11

2 **a** 3 : 4 **b** 1 : 7 **c** 2 : 3 **d** 3 : 5

3 **a** 17 : 100 **b** 50 : 60 **c** 1000 : 150
d 9 : 24 **e** 12 : 180 **f** 400 : 1000

4 **a** 152 : 164 **b** 2 : 5 **c** 3 : 500 **d** 20 : 12

5 **a** 8 : 5 **b** 200 : 800 **c** 2000 : 700 **d** 350 : 1000

EXERCISE 14B

1 **a** $\frac{3}{8}$ **b** $\frac{5}{8}$ **2** **a** $\frac{2}{5}$ **b** $\frac{3}{5}$

3 **a** **i** 1 : 3 **ii** $\frac{1}{4}$ **iii** 25%
b **i** 2 : 3 **ii** $\frac{2}{5}$ **iii** 40%
c **i** 3 : 7 **ii** $\frac{3}{10}$ **iii** 30%
d **i** 2 : 1 **ii** $\frac{2}{3}$ **iii** $66\frac{2}{3}\% \approx 66.7\%$
e **i** 3 : 1 **ii** $\frac{3}{4}$ **iii** 75%
f **i** 6 : 2 **ii** $\frac{3}{4}$ (or $\frac{6}{8}$) **iii** 75%

EXERCISE 14C

1 **a** 12 : 30 **b** 2 : 5

2 **a** 1 : 3 **b** 3 : 1 **c** 1 : 5 **d** 3 : 1 **e** 4 : 5
f 7 : 4 **g** 2 : 5 **h** 3 : 2 **i** 2 : 3 **j** 2 : 7

3 **a** 1 : 1 **b** 3 : 2 **c** 3 : 1 **d** 1 : 2 **e** 16 : 3
f 5 : 4

4 **a** 1 : 3 **b** 1 : 2 **c** 1 : 2 **d** 1 : 1 **e** 3 : 5
f 2 : 1

5 **a** 1 : 10 **b** 3 : 10 **c** 1 : 2 **d** 1 : 5 **e** 1 : 4
f 1 : 10 **g** 3 : 2 **h** 1 : 20 **i** 4 : 1 **j** 5 : 7
k 5 : 1 **l** 3 : 5 **m** 2 : 5 **n** 1 : 15 **o** 1 : 120
p 7 : 8

6 **a** equal **b** equal **c** not equal **d** equal
e not equal **f** not equal **g** equal **h** not equal

7 **a** □ = 6 **b** □ = 3 **c** □ = 33 **d** □ = 15
e □ = 18 **f** □ = 8 **g** □ = 4 **h** □ = 2
i □ = 1 **j** □ = 10 **k** □ = 28 **l** □ = 20

EXERCISE 14D

1 **a** 36 doctors **b** 150 nurses **2** 20 teachers

3 21 CDs **4** 56 station wagons

5 **a** 1 : 10 **b** 10 mL of chocolate topping

6 120 g (45 g raisins and 75 g nuts)

EXERCISE 14E

1 **a** **i** $\frac{2}{3}$ **ii** $\frac{1}{3}$
b **i** 12 chocolates **ii** 6 chocolates

2 **a** 125 g beetroot, 75 g yoghurt
b 375 g beetroot, 225 g yoghurt

3 **a** 160 mL pineapple, 240 mL orange
b 400 mL pineapple, 600 mL orange

4 **a** £5, £25 **b** $20, $8 **5** €480 **6** ¥48 000

EXERCISE 14F.1

1 **a** 1 : 100, scale factor is 100
b 1 : 100 000, scale factor is 100 000
c 1 : 3000, scale factor is 3000
d 1 : 2 000 000, scale factor is 2 000 000
e 1 : 250 000, scale factor is 250 000
f 1 : 20 000 000, scale factor is 20 000 000

2 **a** 1 cm represents 2.5 m **b** 1 cm represents 40 m
c 1 cm represents 5 m **d** 1 cm represents 250 m
e 1 cm represents 1.5 km **f** 1 cm represents 220 km

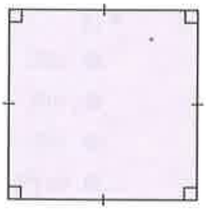
3 **a** 1 : 6000 **b** 1 : 1000

EXERCISE 14F.2

1 **a** 50 cm **b** 13 cm **c** 24 cm **d** 28 cm

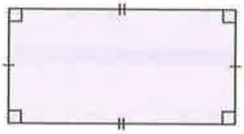
2 **a** 35 m **b** 1.6 km **c** 520 m **d** 64 m

3 **a**



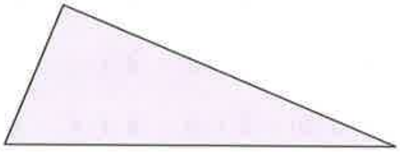
Scale: 1 cm represents 10 m

b



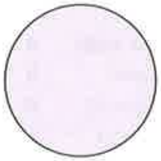
Scale: 1 cm represents 2 km

c



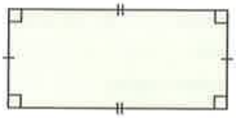
Scale: 1 : 500

d



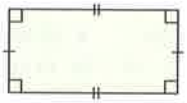
Scale: 1 : 250 000

4 **a**



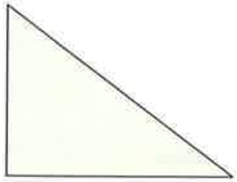
Scale: 1 : 1000

b



Scale: 1 : 200

c



Scale: 1 : 2000

5 **a** height = 50 m, width = 30 m **b** 2.5 m **c** 3.2 mm

6 **a** 38.4 m by 17.6 m **b** **C**

7 **a** 4.88 m **b** 64 cm **c** 1.44 m **d** 1.12 m

8 **a** ≈ 1900 km **b** ≈ 2600 km **c** ≈ 1600 km

9 **a** 1 mm represents 1.25 m **b** 68.75 m **c** 7.5 m

10 **a** 5.5 mm **b** 2.25 mm **c** 1.25 mm

11 **a** 10.8 m by 6.6 m **b** 3.24 m by 1.68 m **c** £2735.64

12 **a** 0.001 25 mm **b** 0.003 mm

REVIEW SET 14A

1 7 : 4 **2** **a** 9 : 4 **b** 5 : 2 **c** 11 : 5

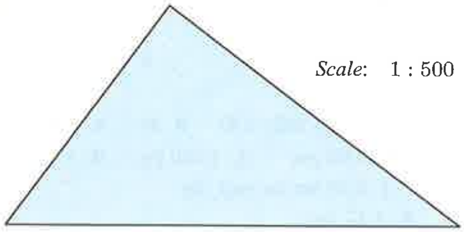
3 **a** 1 : 4 **b** 8 : 5 **c** 4 : 11

4 **a** equal **b** not equal **c** equal **5** **a** $\frac{13}{15}$ **b** $\frac{2}{15}$

6 **a** ☐ = 6 **b** ☐ = 9 **c** ☐ = 45

7 21 trucks **8** 48 sit-ups **9** **a** $\frac{3}{10}$ **b** €90

10



Scale: 1 : 500

REVIEW SET 14B

1 white : red = 7 : 2

2 **a** 53 : 120 **b** 10 : 3 **c** 3 : 20

3 **a** 64 : 24 **b** 8 : 3 **4** **a** 1 : 4 **b** 1 : 8

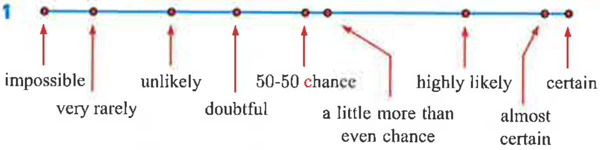
5 27 cm **6** 16 matches **7** 400 mL

8 720 apple, 600 pear **9** 2.6 m

10 **a** 1 mm represents 20 cm **b** 80 cm **c** 3.4 m

EXERCISE 15A

1



2 **a** highly unlikely **b** almost impossible **c** certain
d likely **e** highly unlikely **f** 50-50 chance
g almost certain **h** likely

3 **a** possible **b** possible **c** impossible
d possible **e** impossible **f** possible
g **i** possible **ii** impossible **iii** certain

4 **a** almost certain **b** no
c False, there is 1 in 100 chance that the ticket will be pink.

5 **a** It is more likely that the marble is red, as there are more red marbles than blue.
b true

6 **a** **i** highly likely **ii** 50-50 chance
b No, not enough information is provided.

EXERCISE 15B

1 **a** very likely **b** unlikely **c** even chance
d certain **e** slightly more than even chance

2 **a** 0.2 **b** unlikely

3 **a** **i** slightly less than even chance **ii** likely
b Sunday

4 **a** 0.5 **b** **i** 1 **ii** 0

5 **a** $\frac{1}{10} = 0.1$, $\frac{1}{2} = 0.5$, $\frac{2}{5} = 0.4$
b the 8:10 am train **c** 0.6
d The sum of the probabilities is 1. It is certain that one of these events will occur.

6 **a** **i** 0.1 **ii** 0.2
b **i** Lily (0.45) **ii** Ralph (0.5)
c **i** Lily (0.35), Ralph (0.6)
ii Lily: unlikely, Ralph: slightly more than even chance

d Ralph has greater probability of knocking over 2 or 3 pins, while Lily is more likely to knock over 0 or 1 pins. So, Ralph appears to be better at the game.

e Both add to 1. It is certain that each player will knock over 0, 1, 2, or 3 pins, as these are all of the possible outcomes.

EXERCISE 15C

1 **a** {smile, frown}

b {Monday, Tuesday, Wednesday, Thursday, Friday, Saturday, Sunday}

c {10, 11, 12, 13, 14, 15, 16, 17, 18, 19}

d {January, February, March, April, May, June, July, August, September, October, November, December}

e **i** {1, 2, 3, 4} **ii** {A, B, C, D, E}

iii {blue, red, yellow}

2 **a** 2 outcomes **b** 7 outcomes **c** 10 outcomes

d 12 outcomes

e **i** 4 outcomes **ii** 5 outcomes **iii** 3 outcomes

3 **a** {HH, HT, TH, TT}

b {ABC, ACB, BAC, BCA, CAB, CBA}

c {HHH, HHT, HTH, THH, HTT, THT, TTH, TTT}

d {FFF, FFM, FMF, MFF, MMF, MFM, FMM, MMM}

e {PP, PQ, PR, QP, QQ, QR, RP, RQ, RR}

f {11, 12, 13, ..., 64, 65, 66}

g {WXYZ, WXZY, WYXZ, WYZX, WZXY, WZYX, XWYZ, XWZY, XYWZ, XYZW, XZWY, XZYW, YWXZ, YWZX, YXWZ, YXZW, YZWX, YZXW, ZWXY, ZWYX, ZXWY, ZXYW, ZYWX, ZYXW}

4 **a** 4 outcomes **b** 6 outcomes **c** 8 outcomes

d 8 outcomes **e** 9 outcomes **f** 36 outcomes

g 24 outcomes

5 Label the corners of table A, B, C, D, so sample space is the set of edges {AB, BC, CD, DA}.

EXERCISE 15D

1 **a** {1, 2, 3, 4, 5, 6} **b** 6 outcomes

c **i** $\frac{1}{6}$ **ii** $\frac{3}{6} = \frac{1}{2}$ **iii** $\frac{5}{6}$

2 **a** 26 outcomes **b** **i** $\frac{1}{26}$ **ii** $\frac{4}{26} = \frac{2}{13}$ **iii** $\frac{7}{26}$

3 **a** **i** 3 yellow, 3 red **ii** $\frac{1}{2}$ **iii** $\frac{1}{2}$

b **i** 5 yellow, 3 red **ii** $\frac{5}{8}$ **iii** $\frac{3}{8}$

c **i** 7 yellow, 3 red **ii** $\frac{7}{10}$ **iii** $\frac{3}{10}$

4 **a** $\frac{1}{2}$ **b** 1 **c** $\frac{1}{4}$ **d** $\frac{2}{3}$

5 **a** $\frac{1}{8}$ **b** $\frac{1}{4}$ **c** $\frac{1}{2}$ **d** 0 **e** $\frac{5}{8}$

6 **a** $\frac{1}{12}$ **b** $\frac{1}{6}$ **c** $\frac{2}{3}$ **d** $\frac{1}{3}$ **e** $\frac{7}{12}$

7 **a** $\frac{1}{8}$ **b** $\frac{1}{4}$ **c** $\frac{5}{8}$ **d** 0 **e** $\frac{7}{8}$

f $\frac{3}{4}$ **g** $\frac{5}{8}$ **h** 1 **i** 0

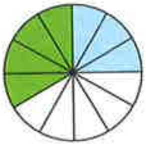
8 **B**

9 **a** $\frac{1}{52}$ **b** $\frac{1}{4}$ **c** $\frac{1}{13}$ **d** $\frac{1}{26}$ **e** $\frac{3}{13}$

f $\frac{1}{2}$ **g** $\frac{1}{26}$ **h** $\frac{3}{26}$ **i** $\frac{2}{13}$

10 **a**

b



c

11 **a** {HH, HT, TH, TT} **b** 4 outcomes

c **i** $\frac{1}{4}$ **ii** $\frac{1}{4}$ **iii** $\frac{1}{2}$ **iv** $\frac{3}{4}$

12 **a** {BBB, BBG, BGB, GBB, GGB, GBG, BGG, GGG}

b **i** $\frac{1}{8}$ **ii** $\frac{1}{8}$ **iii** $\frac{1}{8}$ **iv** $\frac{3}{8}$ **v** $\frac{1}{2}$ **vi** $\frac{7}{8}$

13 **a** $\frac{1}{3}$ **b** $\frac{1}{6}$ **c** $\frac{1}{3}$ **d** $\frac{2}{3}$

14 **a** $\frac{1}{9}$ **b** $\frac{4}{9}$ **c** $\frac{2}{9}$ **d** $\frac{5}{9}$

EXERCISE 15E

1 **a** Terry will not go to school tomorrow.

b Jennifer has less than 3 pets.

c When selecting a ball from this bag, the result is neither red nor blue.

2 $\text{P}(A') = 0.3$, $\text{P}(B') = 0.88$ **3** **a** $\frac{1}{7}$ **b** $\frac{6}{7}$

4 **C** and **E**

5 **a** F' = Fran will remember to get her diary signed tonight

b $\text{P}(F') = 0.77$ **c** F' is more likely to occur.

6 $\frac{24}{29}$ **7** **a** $\frac{1}{10}$ **b** $\frac{9}{10}$

8 **a** {HHHH, HHHT, HHTH, HTHH, THHH, HHTT, HTHT, HTTH, THHT, THTH, TTHH, HTTT, THTT, TTHT, TTTH, TTTT}

b **i** $\frac{1}{16}$ **ii** $\frac{15}{16}$

9 **a** $\text{P}(A) = \frac{2}{5}$, $\text{P}(B) = \frac{3}{5}$

b $\text{P}(A) + \text{P}(B) = \frac{2}{5} + \frac{3}{5} = 1$

c No, A and B are not complementary events as B is not the event that A does not occur, and vice versa. Two events are only complementary if exactly one of them must occur, which is not the case for events A and B.

REVIEW SET 15A

1 **a** unlikely **b** almost certain

2 **a** unlikely **b** extremely likely **c** impossible

d '50-50' chance **e** certain **f** probable

3 **a** {peppermint, caramel, jelly baby, marshmallow}

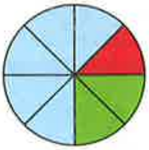
b {23, 29, 31, 37}

4 **a** $\frac{4}{9}$ **b** $\frac{1}{3}$ **c** $\frac{7}{9}$ **d** $\frac{2}{3}$ **e** $\frac{1}{3}$

5 **a** $\frac{1}{2}$ **b** $\frac{1}{2}$ **c** $\frac{5}{8}$ **d** $\frac{1}{3}$

6 **a** $\frac{1}{100}$ **b** $\frac{1}{2}$ **c** $\frac{9}{20}$

7



8 **a** $\frac{1}{26}$

b $\frac{5}{26}$

c $\frac{4}{13}$

9 **a** $\text{P}(A') = 0.06$

b **i** almost certain **ii** highly unlikely

10 **a** {Teegan, Casey, Trish, Deb, Skye, Wendy, Donna}

b **i** $\frac{2}{7}$ **ii** $\frac{3}{7}$

c 0, as there are no 10 year old males in the choir.

REVIEW SET 15B

1 **a** possible **b** certain **c** impossible

2 **a** almost certain **b** impossible

c slightly more than even chance

3 **a** {Mercury, Venus, Earth, Mars, Jupiter, Saturn, Uranus, Neptune}
b {L2, L7, L4, C2, C7, C4, N2, N7, N4}

4 $\frac{1}{8}$ **5** $\frac{7}{10}$

6 No, as the probabilities add to $\frac{41}{40}$ which is more than 1.

7 **a** 0 **b** black **c** pink and white

8 **a** E' = Jacqueline will not win **b** $P(E') = 0.4$

9 **a** {A, B, C, D, E, F, G, H, I, J, K, L, M, N, O, P, R, S, T, W}
b yes **c** **i** $\frac{1}{20}$ **ii** $\frac{1}{5}$ **iii** $\frac{1}{10}$

10 **a** {11, 12, 13, 14, 21, 22, 23, 24, 31, 32, 33, 34, 41, 42, 43, 44}
b **i** $\frac{1}{16}$ **ii** $\frac{1}{4}$ **iii** $\frac{7}{16}$

EXERCISE 16A

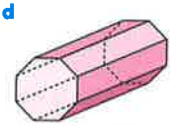
1 **a** cylinder **b** triangular-based pyramid (tetrahedron)
c pentagonal prism

2 **a**

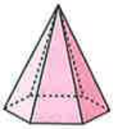
b

c

d



e



3 **a** a cylinder **b** a sphere **c** a rectangular prism
d a cone **e** a triangular-based pyramid **f** a cylinder

4 **a** rectangle **b** triangle

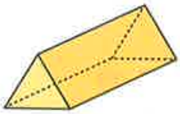
5 **a**

b

c

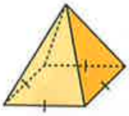
EXERCISE 16B

1 **a**



a triangular prism

b

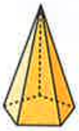


a square-based pyramid

c

a cylinder

d

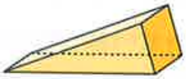


a pentagonal-based pyramid

e

a triangular-based pyramid (tetrahedron)

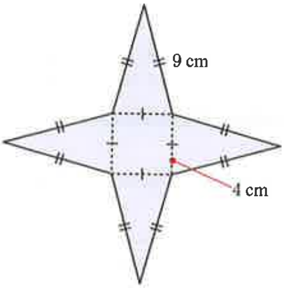
f



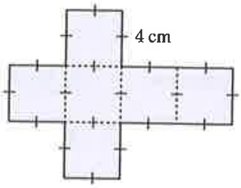
a triangular prism

2 **Note:** Other answers are possible.

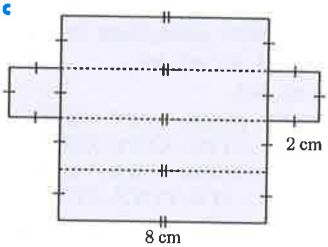
a



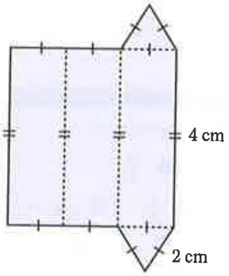
b



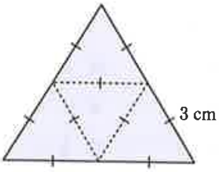
c



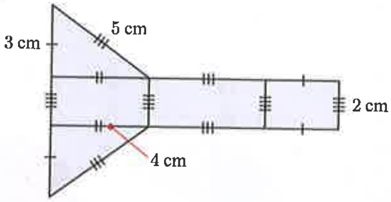
d



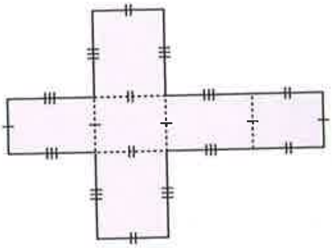
e



f

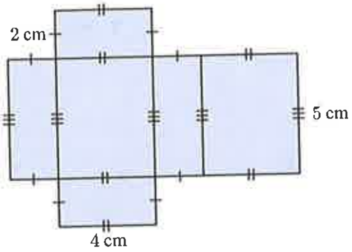


3



4 **D**

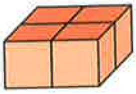
5 **a** **Note:** Other answers are possible.



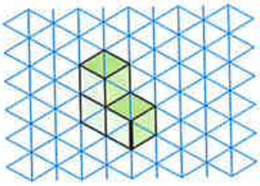
b 76 cm^2

EXERCISE 16C

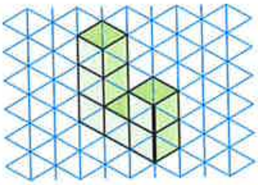
1



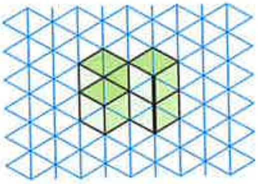
2 **a**



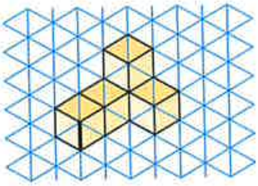
b



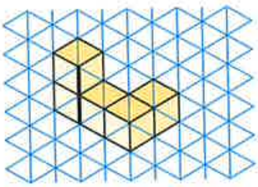
c



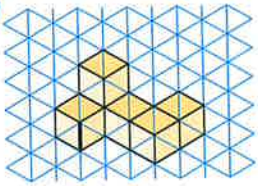
d



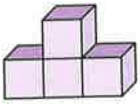
e



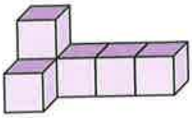
f



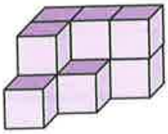
3 **a**



b

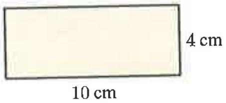


c

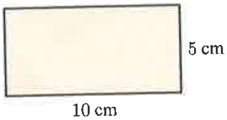


EXERCISE 16D.1

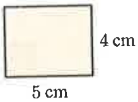
1 **a**



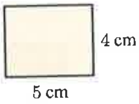
b



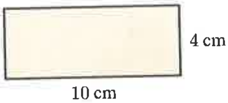
c



d



e



2 **a** 3

b **i**

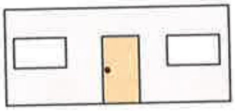
ii

iii

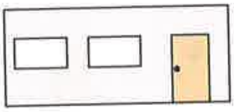
iv

v

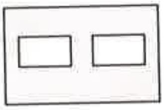
3 **a**



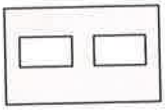
b



c



d



4 **a**

b

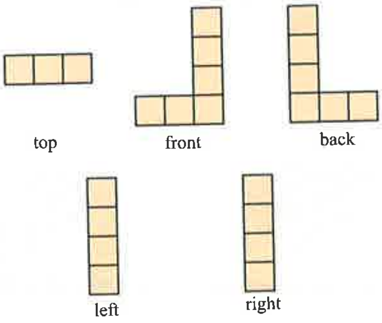
c

d

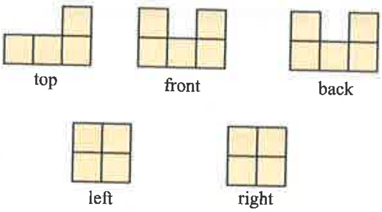
e

EXERCISE 16D.2

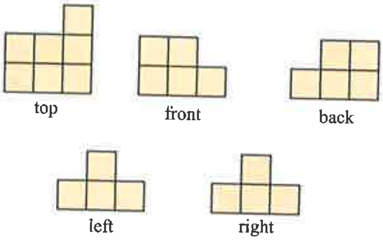
1 **a**



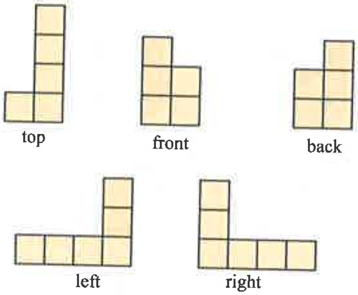
b



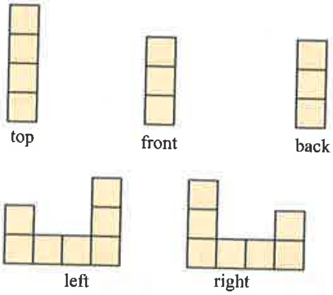
c



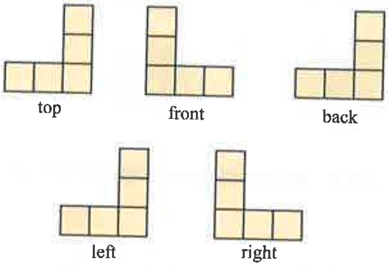
d



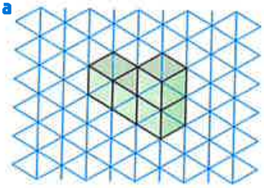
e



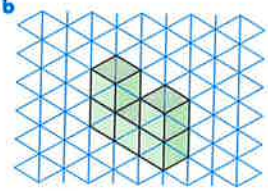
f



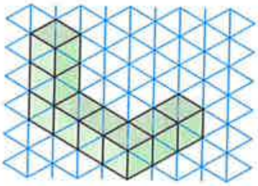
2 **a**



b



c

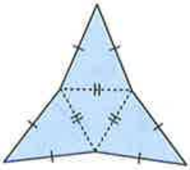


3 **a** C **b** A **c** D **d** B

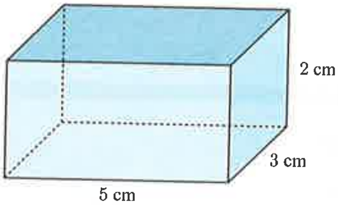
REVIEW SET 16A

1 **a** triangular prism **b** pentagonal-based pyramid

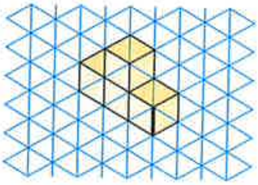
2 **Note:** Other answers are possible. **3** rectangular prism



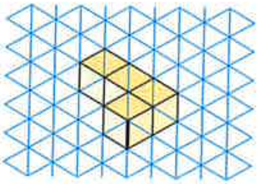
4



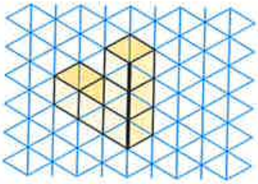
5 **a**



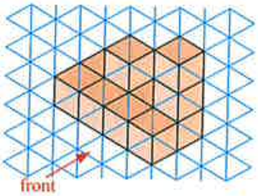
b



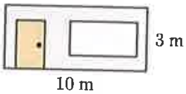
c



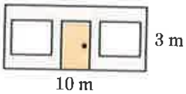
6



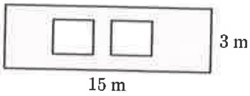
7 **a**



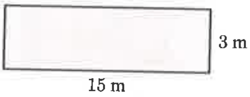
b



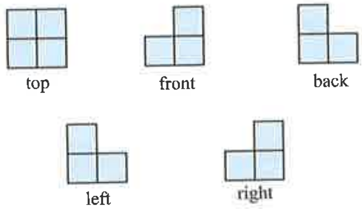
c



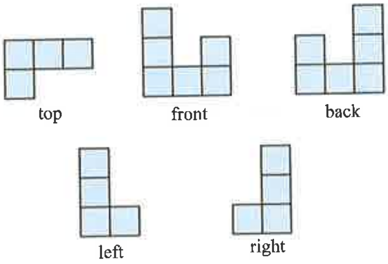
d



8 **a**



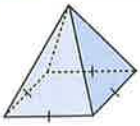
b



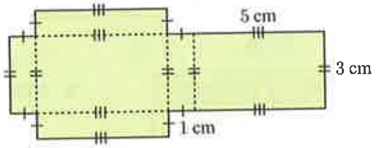
REVIEW SET 16B

1 **a**

b

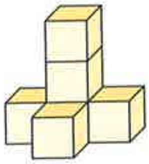


2 **Note:** Other answers are possible.

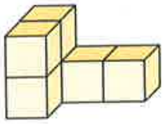


3 a cube

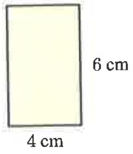
4 **a**



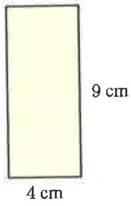
b



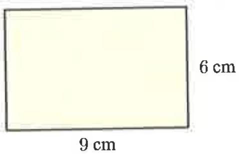
5 **a**



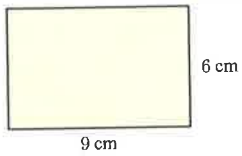
b



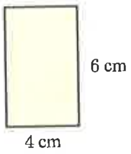
c



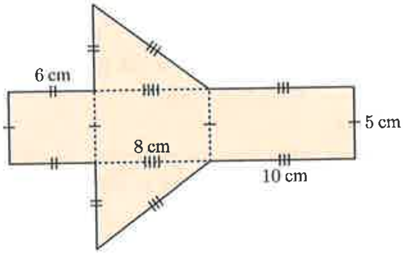
d



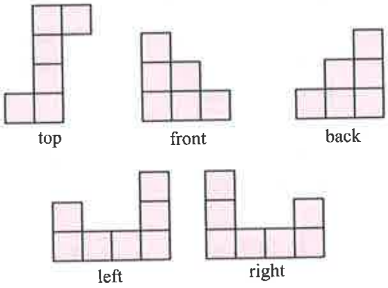
e



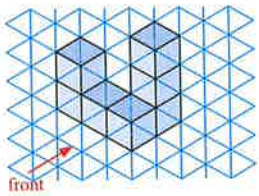
6 **a** **Note:** Other answers are possible. **b** 168 cm^2



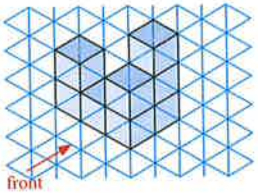
7



8



or



EXERCISE 17A

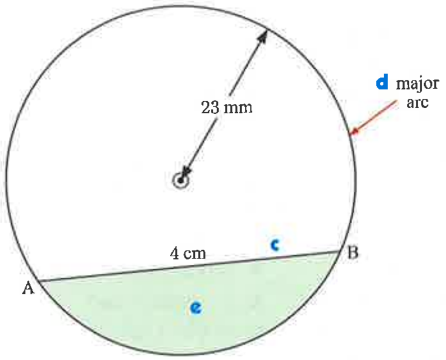
1 **a** F **b** D **c** A **d** I **e** L **f** G
g C **h** E **i** K **j** J **k** B **l** H

2 a diameter

3 **a** A diameter is made up of two radii, one on each side of the centre.
b **i** 8 cm **ii** 6 cm

4 **a** 3 cm **b** 5 cm **c** 6 cm **d** 10 cm
e 2 cm **f** 8 cm

5 **a**



b 46 mm

EXERCISE 17B

1 **a** ≈ 15.7 cm **b** ≈ 22.0 cm **c** ≈ 18.8 m
d ≈ 44.0 cm **e** ≈ 34.5 m **f** ≈ 56.5 cm

2 **a** ≈ 31.4 cm **b** ≈ 50.3 m **c** ≈ 75.4 km
d ≈ 9.4 m **e** ≈ 28.3 cm **f** ≈ 94.2 cm
g ≈ 81.7 m **h** ≈ 157.1 m **i** ≈ 91.1 m

3 ≈ 251.3 cm **4** **a** ≈ 25.1 m **b** 26 lengths **c** €650

5 **a** ≈ 219.9 cm **b** ≈ 219.9 km

6 **a** green, blue, red
b The blue path is longer than the green path, which is 50 m, but shorter than the red path, which is 100 m.
c ≈ 78.5 m

7 ≈ 0.23 m

EXERCISE 17C

1 **a** ≈ 28.3 cm^2 **b** ≈ 201.1 cm^2 **c** ≈ 153.9 m^2
d ≈ 113.1 cm^2 **e** ≈ 22.9 km^2 **f** ≈ 31 415.9 m^2

2 ≈ 380.1 m^2 **3** ≈ 452.4 cm^2

4 **a** ≈ 25.1 m^2 **b** ≈ 39.3 cm^2 **c** ≈ 38.5 cm^2
d ≈ 50.1 cm^2 **e** ≈ 13.7 m^2 **f** ≈ 23.1 cm^2

5 ≈ 9818.3 m^2 **6** **a** 220 cm **b** ≈ 1.7 m^2

7 ≈ 2.0 m^2

8 **a** 160 m **b** 1600 m^2 **c** ≈ 1256.6 m^2
d ≈ 343.4 m^2 **e** ≈ 125.7 m

EXERCISE 17D

1 **a** ≈ 226.2 cm^3 **b** ≈ 754.0 mm^3 **c** ≈ 56.5 m^3
d ≈ 942.5 cm^3 **e** ≈ 15.7 m^3 **f** ≈ 4.0 m^3

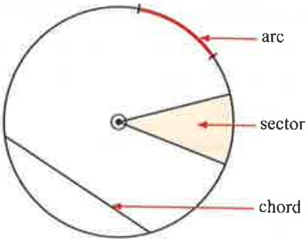
2 ≈ 8143.0 cm^3 **3** ≈ 9236.3 cm^3 **4** ≈ 1021.0 cm^3

5 ≈ 4319.7 cm^3 **6** ≈ 14.1 kL **7** ≈ 226.2 mL

8 Yes, as the petrol can holds ≈ 5.6 L of petrol, and Frank only needs $60 \div 15 = 4$ L of petrol to reach the petrol station.

REVIEW SET 17A

1 **a** An **arc** is a part of a circle. It joins any two different points on the circle.



b A **sector** of a circle is the region between two radii and the circle.
c A **chord** of a circle is a line which joins any two points of the circle.

2 **a** ≈ 18.8 m **b** ≈ 50.3 cm **c** ≈ 11.9 m

3 ≈ 251.3 cm

4 **a** True, the minor arc is always shorter than the major arc, so it must also be shorter than the semi-circle.
b False, a chord may be shorter than the radius if the endpoints of the chord are close to each other.

5 **a** ≈ 12.6 m^2 **b** ≈ 176.7 cm^2 **c** ≈ 78.5 m^2

6 **a** ≈ 1256.6 cm^2 **b** ≈ 50 265.5 cm^3

7 **a** ≈ 254.5 cm^3 **b** ≈ 137.4 m^3 **8** ≈ 628.3 mm^3

REVIEW SET 17B

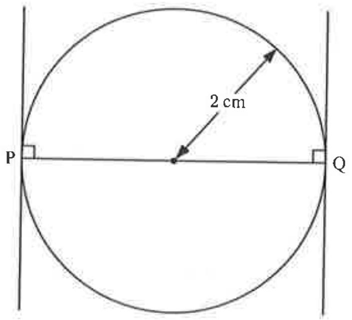
1 **a** ≈ 37.7 cm **b** ≈ 11.9 cm **c** ≈ 36.4 m

2 **a** 6.5 cm **b** ≈ 40.8 cm **c** ≈ 132.7 cm^2

3 **a** ≈ 40.9 cm^2 **b** ≈ 24.8 m^2 **c** ≈ 25.1 m^2

4 ≈ 377.0 m

5 **a, b, c**



d The tangents at P and Q are both perpendicular to the diameter [PQ].
∴ the tangents at P and Q are parallel.
{co-interior angles are supplementary}

6 **a** ≈ 4.5 m^2 **b** ≈ 13.7 m^2

7 **a** ≈ 88.0 cm^3 **b** ≈ 125.7 m^3 **8** ≈ 4.7 L

EXERCISE 18A.1

1 **a**

Level of achievement	*Tally*	*Frequency*
A	|||	3
B	𝍸	5
C	𝍸 𝍸 𝍸 |	16
D	|||	3
E	|	1
	Total	28

b **i** 16 students **ii** $\frac{5}{28}$ **c** C

2 **a**

Sport	*Tally*	*Frequency*
Tennis	𝍸 |	6
Swimming	||||	4
Cricket	𝍸	5
Basketball	||	2
Athletics	𝍸 |||	8
	Total	25

b athletics

3 **a**

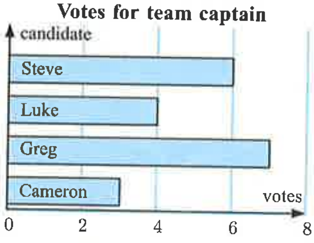
Attraction	*Tally*	*Frequency*
Side shows	𝍸 ||||	9
Farm animals	|||	3
Ring events	𝍸 ||	7
Dogs and cats	||	2
Wood chopping	||||	4
	Total	25

b side shows

EXERCISE 18A.2

1 **a** 5 cats **b** 25 animals **c** 16% **d** dog

2 **a**



b **i** Greg **ii** Cameron **c** **i** 20% **ii** 65%

3 **a**

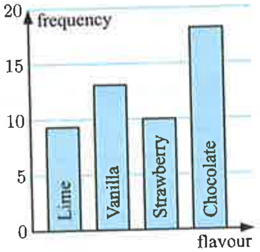
Ice cream flavour	*Tally*	*Frequency*
Chocolate	𝍸 𝍸 𝍸 |||	18
Strawberry	𝍸 𝍸	10
Vanilla	𝍸 𝍸 |||	13
Lime	𝍸 ||||	9
	Total	50

b 13 students

c 18%

d chocolate

e **Favourite ice cream flavours**

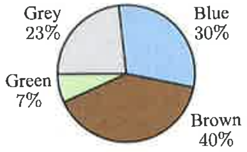


4 **a** False, the most common fine is for speeding.

b true **c** true

d False, there were more expired licence fines than traffic light offence fines.

5 **a** **Eye colour**



b **i** ≈ 6.67% **ii** ≈ 53.3%

EXERCISE 18B

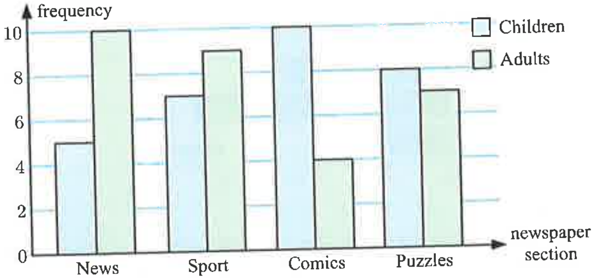
1 **a** 4 students **b** 6 students **c** **i** east **ii** south

d class A

2 **a** 10 students **b** Redstone School

c **i** Redstone School **ii** Hillsvale School

3 **a** **Most enjoyed newspaper section**



b children: comics; adults: news

c News and comics, as adults are more interested in the news whilst children are more interested in comics.

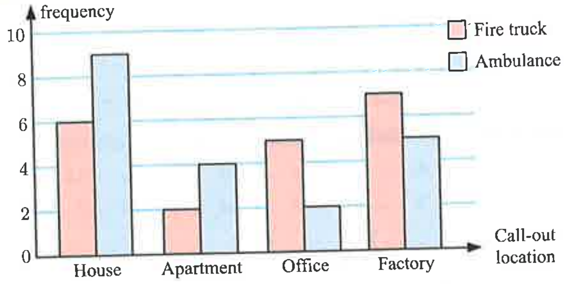
4 **a** **Call-out location for fire truck**

Location	*Tally*	*Frequency*
House (H)	𝍸 |	6
Apartment (A)	||	2
Office (O)	𝍸	5
Factory (F)	𝍸 ||	7
	Total	20

Call-out location for ambulance

Location	*Tally*	*Frequency*
House (H)	𝍸 ||||	9
Apartment (A)	||||	4
Office (O)	||	2
Factory (F)	𝍸	5
	Total	20

b **Location of call-outs**



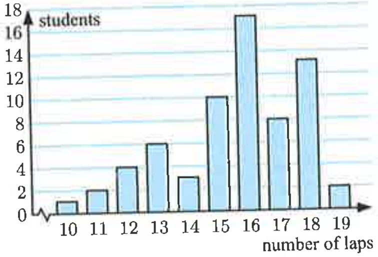
c fire truck: factory; ambulance: house **d** fire truck

EXERCISE 18C

1 **a** 30 workers **b** 5 workers **c** 30%

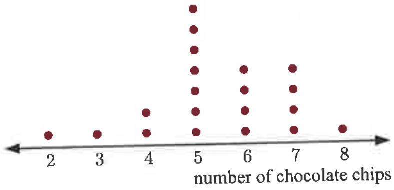
d 8 times is the outlier

2 **a** **Number of laps run by students**



b 16 laps **c** 7 students **d** $\frac{53}{66}$

3 **a** **Number of chocolate chips in biscuits**



b 5 chocolate chips **c** 8 chocolate chips

d 7 biscuits **e** 20%

4 **a** 16 people **b** 2 slices **c** 4 people **d** yes, 10 slices

5 **a** 20 schools **b** 2 schools **c** 111 students **d** 8 schools

6 **a** **Unordered**

0	7 2 5
1	2 1 9 4 2 3 6
2	7 9 2 2 8 1
3	0 5 7 2
4	2 9 3 0
5	1 9
6	2 2 9
7	4

b **Ordered**

0	2 5 7
1	1 2 2 3 4 6 9
2	1 2 2 7 8 9
3	0 2 5 7
4	0 2 3 9
5	1 9
6	2 2 9
7	4

Scale: 1 | 2 means 12 runs

c 17 times **d** **i** 2 runs **ii** 74 runs

7 **a**

Unordered

7	8 8
8	6 8 0 8 6 2
9	4 6 2 6 2 2 4
10	8 0 0 4 6
11	2 2 6 4 6
12	2 4
13	
14	
15	
16	0

Ordered

7	8 8
8	0 2 6 6 8 8
9	2 2 2 4 4 6 6
10	0 0 4 6 8
11	2 2 4 6 6
12	2 4
13	
14	
15	
16	0

Scale: 12 | 2 means 122 pages

b 13 newspapers **c** ≈ 46.4% **d** yes, 160 pages

EXERCISE 18D.1

1 **a** 8 **b** 11 **c** 4 **d** 3.5 **e** 5.64 **f** 5

2 **a** 8 **b** 12 **c** 3 **d** 6 **e** 2.2 **f** 4.1

3 **a** **i** 4 **ii** 3 **iii** 0

b No, as the mode is 0 which is also the minimum. It does not give an accurate measure of the "centre".

4 **a** 204.8 **b** 200.5

5 **a** 6 texts **b** 4 texts **c** 3 texts

6 **a** **i** 5 **ii** 4.7 **iii** 5

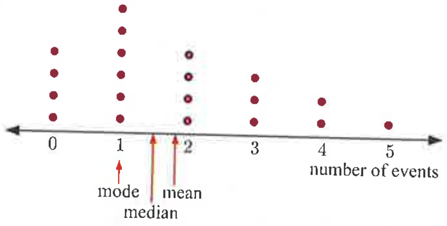
b **i** 4 **ii** ≈ 2.54 **iii** 3

c **i** 3.6 **ii** ≈ 3.01 **iii** 3.05

7 **a** 4 students

b **i** 1 event **ii** 1.5 events **iii** 1.8 events

c



8 **a** *Group X*: 6.5 marks, *Group Y*: ≈ 7.64 marks

b No. It is fair as the mean calculates the average score per student.

c Group Y

9 **a** *Zach*: mean = 7.2 fish, median = 7.5 fish

Ed: mean = 6.5 fish, median = 6.5 fish

b Zach, as he has a higher mean and median.

10 **a** 47 hot dogs

b *Josh*: mean ≈ 34.4 hot dogs, median = 34 hot dogs

Eugene: mean ≈ 49.4 hot dogs, median = 50 hot dogs

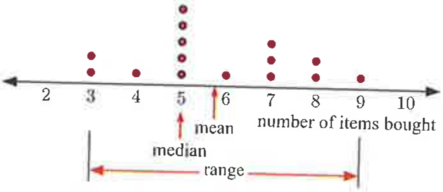
c Eugene, as he has a higher mean and median.

EXERCISE 18D.2

1 **a** 11 **b** 2 **c** 8 **d** 14 **e** 29 **f** 5.1

2 **a** 5 **b** 8 **c** 33

3 **a**



b mean = 5.8 items, median = 5 items, range = 6 items

4 **a**

City	*Range*
Beijing	11°C
Berlin	6°C
Cairo	2°C
Lima	1°C
Moscow	9°C
Ottawa	14°C
Reykjavik	8°C
Wellington	3°C

b **i** Ottawa

ii Lima

EXERCISE 18E

1 **a** a census **b** a sample **c** a sample **d** a census

2 **a** In general, people at a swimming pool are more likely to be able to swim. People in other places do not get asked.

b Could be biased since sunlight may not reach the plants because of the barn and so growth could be much slower.

c Workers in New York are likely to take longer to travel to work than workers in the rest of the United States, as New York is a very busy location.

3 **a** 192 members

b Not reliable as the sample is very small.

4 **a** 70% preferred tea

b It would depend on the size of the youth group, but the estimate appears to be reliable.

5 **a** ≈ 4.37 meals per week

b No, people who live in the CBD are likely to eat out more than the average person, and the sample size is too small to be reliable.

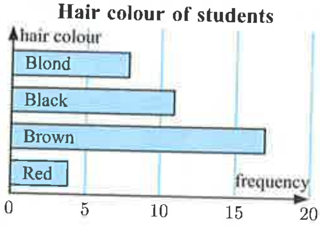
REVIEW SET 18A

1 **a** True, as type O blood has the largest sector.

b False, as the angle for the type B blood sector is less than $\frac{1}{4}$ of 360° = 90°.

c True, as more than half of the pie chart represents blood types other than type O.

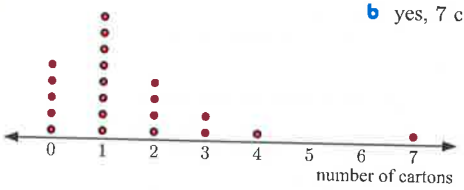
2 **a**



b red

c No, the sample is too small to make reliable conclusions and is unlikely to be representative of all students.

3 **a**



b yes, 7 cartons

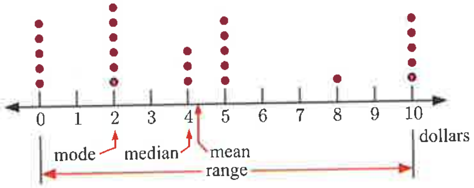
4 **a** ≈ 26.3% **b** 51 pages **5** 217.5 m

6 **a** **i** 3 calf injuries **ii** 5 calf injuries **b** Panthers

c Panthers: hamstring injuries, Tigers: knee injuries

7 57 **8** **a** a census **b** a sample

9 **a, c** Pocket money received each week



b **i** \$2 **ii** \$4.28 **iii** \$4 **iv** \$10

10 **a** The sample is too small and does not take into account the views of older students. Also, this sample only takes one location into account so it is not representative of all high school students in general.

b Diners on other days and times are not represented in the sample.

REVIEW SET 18B

1 Birthday money

2 **a** 10 **b** 9 **c** 6 **3** categorical

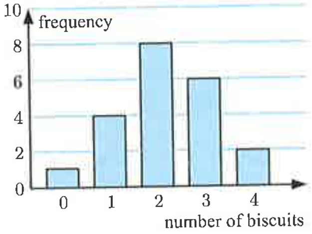
4 **a** 30% **b** 1 drink

5 **a**

Number of biscuits	*Tally*	*Frequency*
0	|	1
1	||||	4
2	𝍸 |||	8
3	𝍸 |	6
4	||	2
	Total	21

b 21 days

c Biscuits eaten



6 **a** mean = 11.75, median = 11.5, mode = 11, range = 4

b mean = 3.4, median = 4, mode = 4, range = 4

7 Alyssa

8 **a**

Unordered

2	6 8 3
3	2 5 9 9 0 2
4	3 1 6 5
5	9 3 4
6	0 7 5 5

Ordered

2	3 6 8
3	0 2 2 5 9 9
4	1 3 5 6
5	3 4 9
6	0 5 5 7

Scale: 2 | 3 means 23

b 9 children **c** 25%

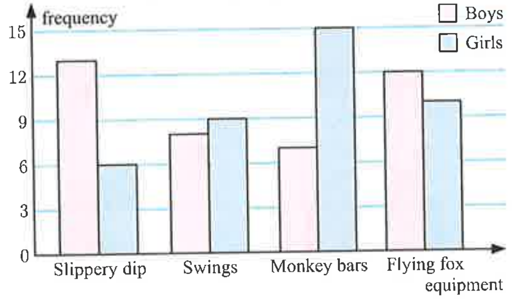
d **i** 44.1 **ii** 42 **iii** 44

9 **a** 20.6 **b** 10.5

c No, patrons leaving a children's movie are likely to be younger than the average patron.

10 **a** 12 boys **b** 6 girls

c Favourite piece of playground equipment



d **i** slippery dip **ii** monkey bars **e** boys

EXERCISE 19A

1 **a** 3 units right, 2 units down **b** 4 units right, 3 units up

c 4 units down **d** 4 units left

e 3 units left, 2 units down **f** 3 units right, 4 units down

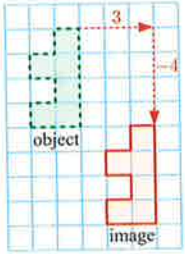
2 **a** 3 units right, 1 unit up **b** 3 units left, 1 unit down

c 3 units right, 3 units down **d** 3 units left, 3 units up

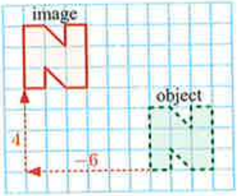
e 5 units right, 1 unit up **f** 5 units left, 1 unit down

g 2 units left, 4 units down **h** 2 units right, 4 units up

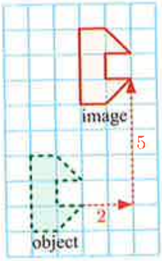
3 **a**



b

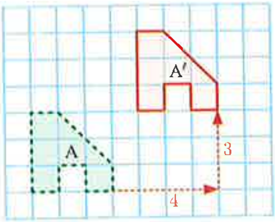


c



4 **a** figure D **b** 3 units left, 4 units down

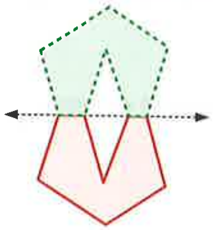
5 **a**



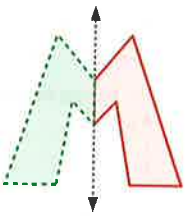
b 4 units left, 3 units down

EXERCISE 19B.1

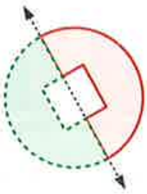
1 **a**



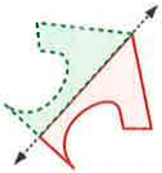
b



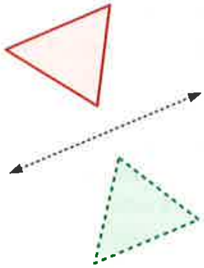
c



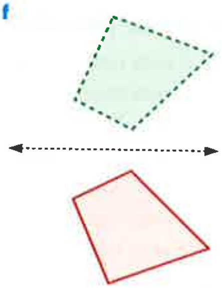
d



e

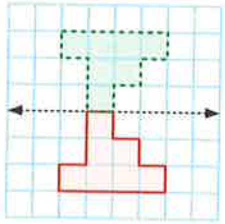


f

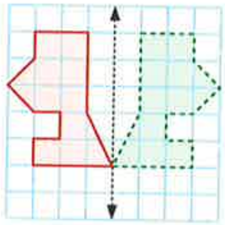


2 **D**

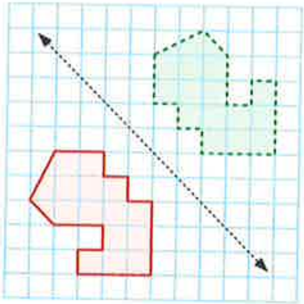
3 **a**



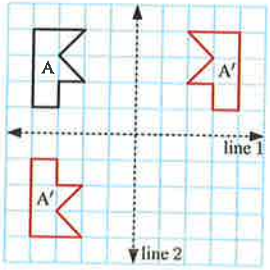
b



c

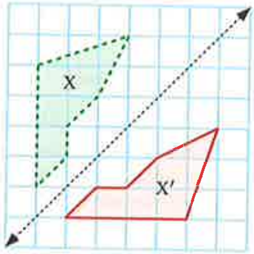


4

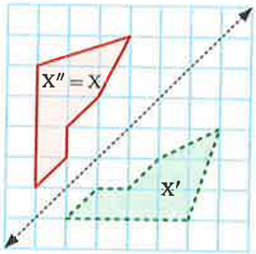


5 **Note:** Other answers are possible.
The original figure X is obtained.

Reflecting X to get X′:

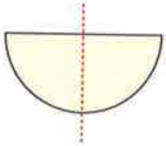


Reflecting X′ in the same mirror line:

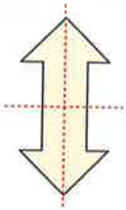


EXERCISE 19B.2

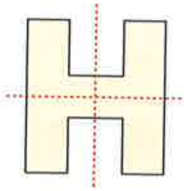
1 **a**



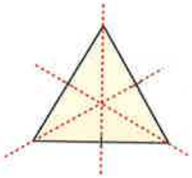
b



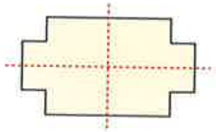
c



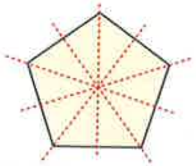
d



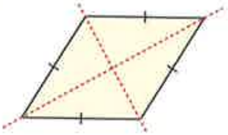
e



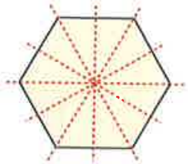
f



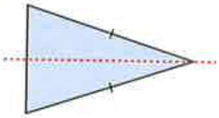
g



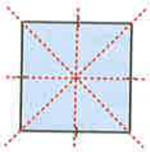
h



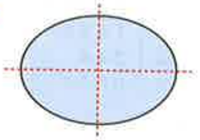
2 **a** **i**



ii



iii



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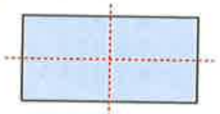
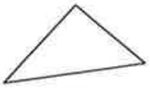


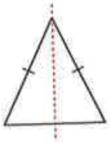
b the square

3 **a** one line of symmetry **b** one line of symmetry

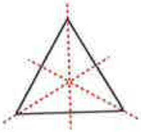
4 **a**



a scalene triangle has no lines of symmetry

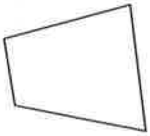


an isosceles triangle has one line of symmetry

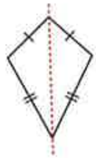


an equilateral triangle has 3 lines of symmetry

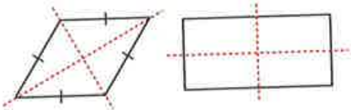
b



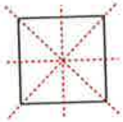
an irregular quadrilateral has no lines of symmetry



a kite has one line of symmetry



a rhombus and a rectangle have two lines of symmetry



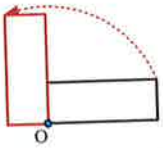
a square has four lines of symmetry

c A circle has infinitely many lines of symmetry.

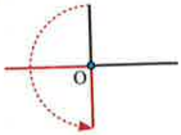
EXERCISE 19C.1

1 **a** **B** **b** **D** **c** **A** **d** **C**

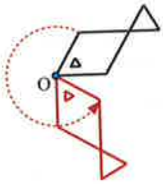
2 **a**



b



c

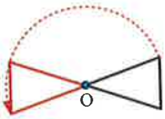


d

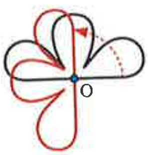
e

f

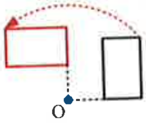
g



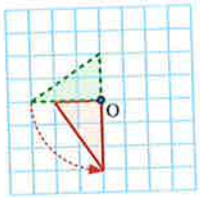
h



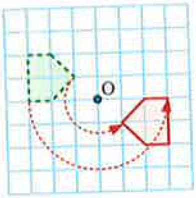
i



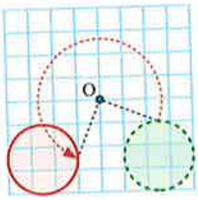
3 **a**



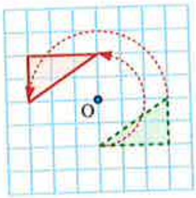
b



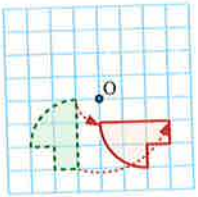
c



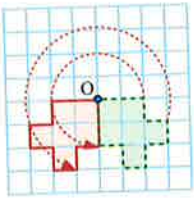
d



e



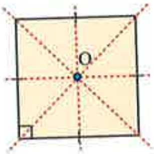
f



EXERCISE 19C.2

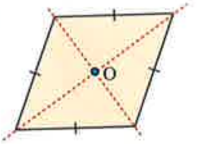
1 **b**, **d**, and **f** have rotational symmetry.

2 **a** **i**



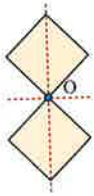
ii 4

b **i**



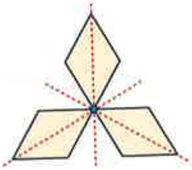
ii 2

c **i**



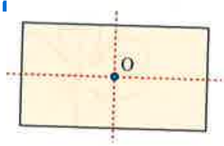
ii 2

d **i**



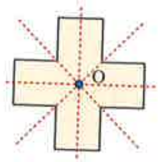
ii 3

e **i**



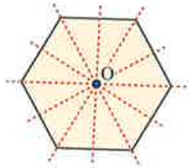
ii 2

f **i**



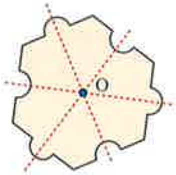
ii 4

g **i**



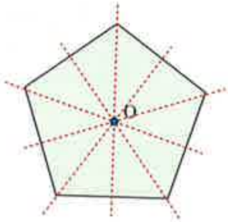
ii 6

h **i**

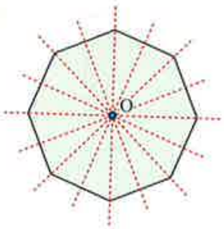


ii 3

3 **a**



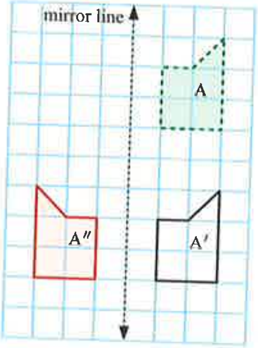
b



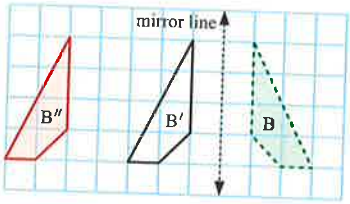
Note: There may be other answers.

EXERCISE 19D

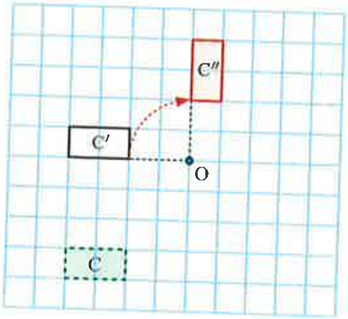
1 **a**



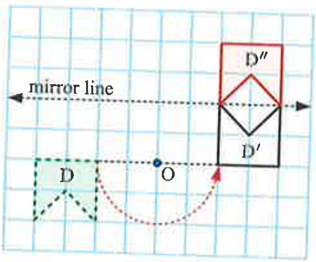
b



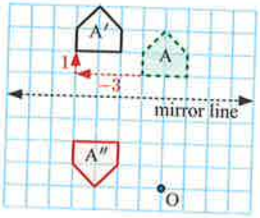
c



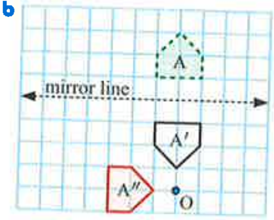
d



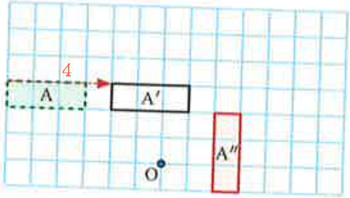
2 **a**



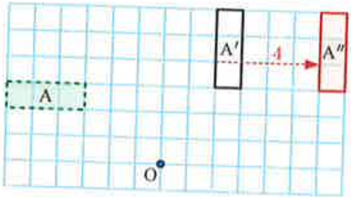
b



3 **a**

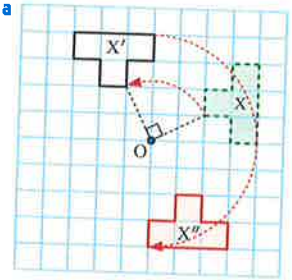


b



No, the results are not the same.

4 **a**



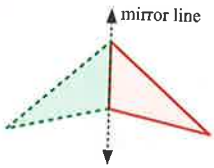
b a rotation of 90° clockwise about O

5 **a** Reflect point X in line 1, then rotate 90° anticlockwise about O, then reflect in line 2.

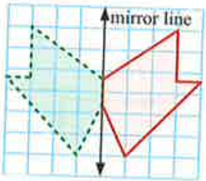
b Reflect point X in line 1, then rotate 180° (anticlockwise or clockwise) about O, then reflect in line 2.

REVIEW SET 19A

1 **a**

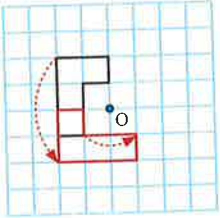


b



2 **a** 2 units right, 3 units up **b** 2 units left, 3 units down
c 4 units right, 2 units down **d** 6 units left, 1 unit down

3

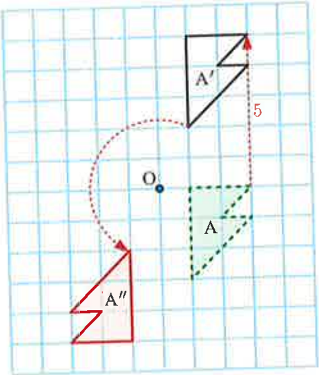


4 **a** Yes, one line of symmetry.

b no

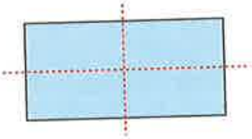
5 **a** 4 **b** 3

6

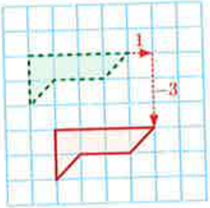


REVIEW SET 19B

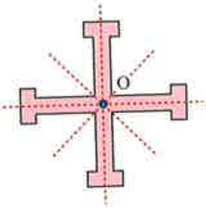
1



2

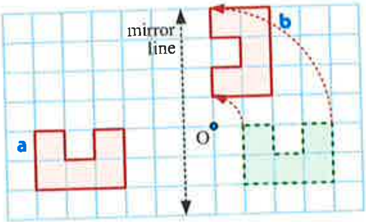


3 **a**

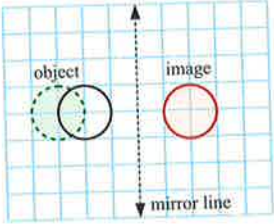


b 4

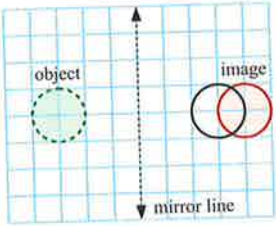
4



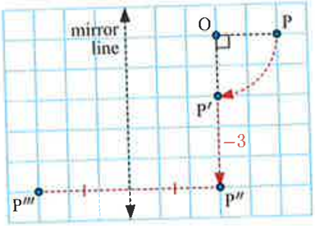
5 **a**



b



6



EXERCISE 20A

1 **a** 5 km in 1 hour **b** 15 dollars in 1 hour
c 7 litres in 1 second **d** 99 cents in 1 litre
e 30 kg in 1 hour **f** 14 grams in 1 minute
g 96 dollars in 1 day **h** 66 metres in 1 second
i 21 mL in 1 hour

2 **a** \$/h **b** km/h **c** cents/L **d** words/min **e** °C/min

3 **a** 16 km per litre **b** 52 km per hour **c** 3.5 L per s
d €1.09 per L **e** £17 per hour

4 75 beats/min **5** 39 megajoules/day

6 **a** Annie: ≈ 0.56 km per min, Victoria: 0.5 km per min
b train

7 **a** Xinsong: \$21 per hour, Jay: \$22 per hour **b** Jay

8 **a** 400 L per day **b** 8000 L

9 **a** \$12.60 per hour **b** \$239.40 **10** 24 300 L

11 **a** \$20 per metre **b** \$540 **c** 220 m

12 **a** **i** 18.5 km/L **ii** ≈ 5.41 L/100 km
b 80 L **c** \$108

EXERCISE 20B.1

1 **a** 25 km/h **b** 30 km/h **c** 12 km/h **d** 900 km/h

2 Jason's average speed is 105 km/h which is over the speed limit.
∴ he has broken the law.

3 **a** 216 km **b** 45 minutes **4** ≈ 1.6 m/s

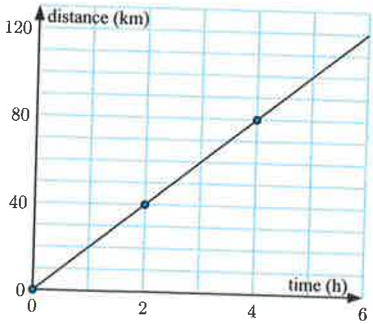
5 **a** 350 km **b** 2 hours

6 **a** Yiren: ≈ 2.67 m/s, Sean: 2.5 m/s **b** Yiren by 50 s

EXERCISE 20B.2

1 **a** Yes, as the travel graph is a straight line.
b 200 km **c** 100 km/h

2 **a**



b 60 km
c 5 hours
d 20 km/h

3 **a** 100 km **b** 2 hours **c** 50 km/h **d** 100 km
e 1 hour **f** 100 km/h **g** 200 km **h** 3 hours
i ≈ 66.67 km/h

EXERCISE 20C

1 **a** 4 g per cm^3 **b** 22.6 g per cm^3 **c** ≈ 1.06 g per cm^3

2 **a** 11.3 times **b** ≈ 7.93 times **c** ≈ 1.47 times

3 die **B** (6.25 g/cm^3 compared with ≈ 5.79 g/cm^3)

4 Petrol has a lower density than water so it will float on water. So, the upper layer is petrol.

5 The density of the doorstop is ≈ 1.33 g/cm^3, which is higher than the density of water. So, the doorstop will sink in water.

EXERCISE 20D

1 **a** \$3.80 per ball **b** \$1.69 per kg **c** 0.96 cents per g
d 79.2 cents per L **e** \$2 per m **f** 124 pence per L

2 **a** 400 g at ≈ \$1.28 per 100 g
b 200 mL at ≈ €0.90 per 100 mL
c 4 boxes at ≈ \$0.51 per box
d 36 pack at ≈ \$0.25 per tablet
e 50 m at £0.73 per 10 m
f 250 g at 8.6 cents per 10 g

3 **a** small toothpaste: \$0.29 per 10 g
large toothpaste: ≈ \$0.25 per 10 g
b 160 g **c** Yes, rate is ≈ \$0.19 per 10 g.

EXERCISE 20E

1 **a** 110 Australian dollars **b** 275 Australian dollars
c 50 US dollars **d** 1000 US dollars

2 **a** 48 Swiss francs **b** 324 Swiss francs
c 75 euros **d** 1250 euros

3 3400 yen

4 No, Estelle has 560 British pounds but would need 600 British pounds for a 5 night stay.

5 **a** 2300 New Zealand dollars **b** 920 New Zealand dollars
c 800 Canadian dollars

EXERCISE 20F.1

1 10 800 L/hour

2 **a** 1 beat/s **b** 3600 beats/h **c** 86 400 beats/day

3 **a** 125 mL/s **b** 450 L/h

4 **a** 0.3 m/day **b** 0.0125 m/h **c** 12.5 mm/h

5 **a** 1225 g/week **b** 1.225 kg/week **6** 6800 kg/m^3

EXERCISE 20F.2

1 **a** 108 km/h **b** 252 km/h **c** 18 km/h

2 **a** 20 m/s **b** 40 m/s **c** 2.5 m/s

3 **a** 720 km/h **b** 162 km/h **c** 97.2 km/h
d 2880 km/h

4 **a** ≈ 13.89 m/s **b** ≈ 30.56 m/s **c** ≈ 5.83 m/s
d 150 m/s

5 **a** ≈ 10.42 m/s **b** ≈ 37.52 km/h

6 **a** ≈ 37.11 km/h **b** ≈ 62.07 km/h **c** ≈ 55.38 km/h
d 6 km/h

REVIEW SET 20A

1 14 L per minute **2** **a** 2.5 beats/s **b** 194.4 km/h

3 train (96.25 km/h versus ≈ 84.71 km/h)

4 10 minutes **5** **a** 150 km **b** 2 hours **c** 50 km/h

6 **a** 13 km/h **b** ≈ 3.61 m/s **7** 2.8 g/cm^3

8 **a** 300 Chinese yuan **b** 400 Singapore dollars

9 0.014 g/cm^3 **10** 80 g at ≈ \$0.27 per 10 g

REVIEW SET 20B

1 €27.50/h **2** **a** ≈ 22.22 m/s **b** 12.5 mm/month

3 **a** \$8.50 per kg **b** \$21.25

4 **a** 66 km/h **b** 18 km/L **5** 41 cents per bar

6 Density of object ≈ 0.71 g/cm^3, which is lower than the density of water. So, the object would float.

7 **a** 1.3 kg at ≈ \$4.23 per kg **b** 1.3 kg packet

8 **a** 20 km **b** 30 min **c** 40 km/h **d** 52 km/h
e at C and D

9 6550 pesos

10 **a** 50 km/h **b** 6.5 hours **c** ≈ 13.89 m/s

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